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Existence of Second Order Impulsive Neutral Integro-Differential Evolution Control Systems with an Infinite Delay

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Abstract: ***This article, we study sufficient conditions for the controllability of second-order impulsive neutral integro-differential evolution systems with an infinite delay in Banach spaces by using the theory of cosine families of bounded linear operators and fixed point theorem.***

Keywords: Nonlinear ordinary differential operators, Impulsive optimal control problems; evolution equations.

2000 Subject Classification: 34L30, 49N25, 37L05.

I. INTRODUCTION

The study of impulsive functional differential systems is related to their utility in simulating processes and phenomena subject to short-time perturbations during their evolution. The perturbations are executed discretely and their duration is negligible in comparison with the total duration of the processes. That is why the perturbations are considered to take place instantaneously in the form of impulses. The theory of impulsive systems provides a common frame work for mathematical modeling of many real world phenomena. Moreover, these impulsive phenomena can also be found in fields such as information science, electronics, fed-batch culture in fermentative production, robotics and telecommunications (see [1, 5, 14, 16, 18, 19, 21] and references therein).

In recent years, the study of impulsive control systems has received increasing interest. Due to its importance several authors have investigated the controllability of impulsive systems (see [2, 6, 7, 10, 20]).

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Motivated by the effort of the after mentioned papers [2, 11, 18], the primary inspiration driving this manuscript is mainly concerned with the study of controllability of second order impulsive partial neutral integro-differential system of the form

$$\frac{d}{d\grave{u}}\left[\vartheta'(\grave{u})-\theta_1\left(\grave{u},\vartheta_{\grave{u}},\int_0^{\grave{u}}a_1(\grave{u},s,\vartheta_s)ds\right)\right]=A(\grave{u})\vartheta(\grave{u})+Bu(\grave{u})+\theta_2\left(\grave{u},\vartheta_{\grave{u}},\int_0^{\grave{u}}a_2(\grave{u},s,\vartheta_s)ds\right),$$
$$\grave{u}\in J=[0,a],\ \grave{u}\neq\grave{u}_k,\ k=1,2,...,m, \tag{1}$$
$$\vartheta_0=\phi\in\mathcal{B},\quad \vartheta'(0)=\xi\in X, \tag{2}$$
$$\Delta\vartheta(u`_k)=I_k(\vartheta_{u`k}),\qquad k=1,2,\dots,m, \tag{3}$$
$$\Delta\vartheta'(u`_k)=J_k(\vartheta_{u`k}),\qquad k=1,2,\dots,m, \tag{4}$$

The control function $u(\cdot)$ is given in $L^2(J,U)$, a Banach space of admissible control functions with U as a Banach space and $B : U \to X$ as a bounded linear operator; For $`u \in J,\vartheta_{u`}$ represents the function $\vartheta_{u`} : (-\infty,0] \to X$ defined by $\vartheta_{u`}(\theta) = \vartheta(u` + \theta), -\infty< \theta \le 0$ which belongs to some abstract phase space B defined axiomatically, $\theta_1,\theta_2 : J \times B \times X \to X$, $a_1,a_2 : J \times J \times B \to X$ $I_k : B \to X, J_k : B \to X$ are appropriate functions and will be specified later. $0 < u`_1 < \dots < u`_n < a$ are fixed numbers and the symbol $\Delta\xi(u`)$ represents the jump of a function ξ at $`u$ , which is defined by $\Delta\xi(u`) = \xi(u`^+)-\xi(u`^-)$. Throughout the text we will assume that $A(\cdot)$ generates an evolution operator $S(u,s`)$.

II. PRELIMINARIES

This section we review some basic concepts, notation, and properties required to find our main results. Nowadays there has been an increasing interest in studying the theoretical non-autonomous second order initial value problem

where $A(u`) : D(A(u`)) \subseteq \vartheta \to X$, $`u \in J = [0,a]$ is a closed densely defined operator and $f : J \to X$ is an suitable function. Equations of this form have been considered in several papers. We refer the reader to [15, 17] and the references therein. In the majority of works, the existence of results to the problem (5)-(6) is related to the existence of an evolution operator $S(u,s`)$ for the homogeneous equation,

$\vartheta''(u`) = A(u`)\vartheta(u`)$, results to the problem (5)-(6) is related to the existence of an evolution operator $S(u,s`)$ for the homogeneous equation,

$$\vartheta''(u`) = A(u`)\vartheta(u`),\qquad 0 \le s, u` \le a. \tag{7}$$

(5)-(6) is related to the existence of an evolution operator $S(u,s`)$ for the homogeneous equation,

$$\vartheta''(u`) = A(u`)\vartheta(u`), \qquad 0 \le s, u` \le a.$$

Let as assume that the domain of $A(u`)$ is a subspace of D dense in X and not dependent

of $`u$, and for each $\vartheta \in D$ the function $`u \mapsto A(u`)\vartheta$ is continuous. The fundamental solution for the second-order evolution equation (7), has been developed by Kozak [13], and we will use the following concept of evolution operator.

Definition 2.1 *A family S of a bounded linear operator $S(u,s`)$: $J \times J \to L(\vartheta)$ is called an evolution operator for (7), if the following conditions are satisfied:*

(Z1) *For each $\vartheta \in X$, the mappings $(u,s`) \in [0,a] \times [0,a]$*

$\to S(u,s`)\vartheta \in X$ of class C^1 and

(i) *For each $u` \in [0,a]$, $S(u,`u) = 0$,*

(ii) *For all $u,s` \in [0,a]$, and for each $\vartheta \in X$,*

$$\boxed{\frac{\partial}{\partial \grave{u}} S(\grave{u}, s)\vartheta|_{\grave{u}=s} = \vartheta} \qquad \boxed{\frac{\partial}{\partial s} S(\grave{u}, s)\vartheta|_{\grave{u}=s} = -\vartheta}$$

(Z2) *For all $u,s` \in [0,a]$ if $\vartheta \in D(A)$, then $S(u,s`)\vartheta \in D(A)$, the mappings $(u,s`) \in [0,a] \times [0,a] \to S(u,s`)\vartheta \in \vartheta$ is of class C^2 and*

$$(i) \quad \frac{\partial^2}{\partial \grave{u}^2} S(\grave{u}, s)\vartheta = A(\grave{u})S(\grave{u}, s)\vartheta,$$

$$(ii) \quad \frac{\partial^2}{\partial s^2} S(\grave{u}, s)\vartheta = S(\grave{u}, s)A(s)\vartheta,$$

$$(iii) \quad \frac{\partial}{\partial s}\frac{\partial}{\partial \grave{u}} S(\grave{u}, s)\vartheta|_{\grave{u}=s} = 0.$$

(Z3) *For all $u,s` \in [0,a]$ if $\vartheta \in D(A)$, then $\frac{\partial}{\partial s} S(\grave{u}, s)\vartheta \in D(A)$, there exists $\frac{\partial^2}{\partial \grave{u}^2}\frac{\partial}{\partial s} S(\grave{u}, s)\vartheta$, $\frac{\partial^2}{\partial s^2}\frac{\partial}{\partial \grave{u}} S(\grave{u}, s)\vartheta$ and*

$$(i) \quad \frac{\partial^2}{\partial u^2}\frac{\partial}{\partial s} S(\grave{u}, s)\vartheta = A(\grave{u})\frac{\partial}{\partial s} S(\grave{u}, s)\vartheta,$$

$$(ii) \quad \frac{\partial^2}{\partial s^2}\frac{\partial}{\partial \grave{u}} S(\grave{u}, s)\vartheta = \frac{\partial}{\partial \grave{u}} S(\grave{u}, s)A(s)\vartheta$$

and the mapping $[0,a] \times [0,a] \ni (\grave{u}, s) \to A(\grave{u})\frac{\partial}{\partial s} S(\grave{u}, s)\vartheta$ *is continuous.*

Throughout this problem we assume that there exists an evolution operator $S(u,s`)$ associated to the operator $A(u`)$. To abbreviate the text, we introduce the operator $C(\grave{u}, s) = -\frac{\partial S(\grave{u},s)}{\partial s}$.

In addition, we set N and N for positive constants such that $\sup_{0<s,u<a`} \|S(u,s`)\| \le N$ and $\sup_{0<s,\grave{u}<a} \|C(\grave{u}, s)\| \le \widetilde{N}$. In addition, we denote by N_1 and $Nf1$ is a positive constant such that $\|S(u`+h,s) - S(u,s`)\| \le N_1|h|$ and $\|C(u`+h,s) - C(u,s`)\| \le Nf1\,|h|$, (8) for all $s,u,`u`+h \in [0,a]$.

Assuming that $f : J \to X$ is an integrable function, the mild solution $\vartheta: [0,a] \to X$ of the problem (5)-(6) is given by,

$$\vartheta(\grave{u}) = C(\grave{u}, s)v + S(\grave{u}, s)w + \int_s^{\grave{u}} S(\grave{u}, \tau)f(\tau)d\tau. \qquad (7)$$

In the literature a number of methods have been discussed to establish the existence of the evolution operator $S(.,.)$. In particular, a very studied situation is that $A(u`)$ is that perturbation of an operator A that generates a cosine operator function. In this reason, below we briefly analysis some essential properties of the theory of cosine functions. Let $A : D(A) \subseteq \vartheta \to X$ be the infinitesimal generator of a strongly continuous cosine family of bounded linear operators $(C(u`))_{u`\in R}$ on Banach space ϑ. We denote by $(S(u`))_{u`\in R}$ the sine function associated with $(C(u`))_{u`\in R}$ which is defined by $S(\grave{u})\vartheta = \int_0^{\grave{u}} C(s)\vartheta ds$, for $\vartheta \in X$ and $u` \in R$. We refer them to [3, 23] for the necessary concepts about cosine functions. After that we only mention a few results and notations about this matter needed to establish our results. It is immediate that

$$C(\grave{u})\vartheta - \vartheta = A\int_0^{\grave{u}} S(s)\vartheta ds,$$

for all $\vartheta \in X$. The notation $D(A)$ stands for the domain of the operator A endowed with the graph norm $\|\vartheta\|_A = \|\vartheta\| + \|A\vartheta\|, \vartheta \in D(A)$. Moreover, in this work, E is the space formed by the vectors $\vartheta \in \vartheta$ for which $C(.)\vartheta$ is of class C^1 on R. It was proved by Kisynski [12] that E endowed with the norm $\|\vartheta\|_E = \|\vartheta\| + \sup_{0\le t\le 1} \|AS(t)\vartheta\|, \vartheta \in E$, is a Banach space. The operator-valued function

$$\mathcal{H}(\grave{u}) = \begin{bmatrix} C(\grave{u}) & S(\grave{u}) \\ AS(\grave{u}) & C(\grave{u}), \end{bmatrix}$$

is a strongly continuous group of bounded linear operators on the space $E \times \vartheta$ generated by the operator

$$\mathcal{H}(\grave{u}) = \begin{bmatrix} 0 & 1 \\ A & 0 \end{bmatrix}$$

defined on $D(A) \times E$. From this, it follows that $S(u`) : X \to E$ is a bounded linear map such that the operator valued maps $S(.)$ is strongly continuous and $AS(u`) : E \to X$ is a bounded linear operator such that $A(S)\vartheta \to 0$ as $`u \to 0$, for each $\vartheta \in E$. Furthermore, if $\vartheta: [0,\infty) \to X$ is a locally integrable function, then the function $y(\grave{u}) = \int_0^{\grave{u}} S(\grave{u} - s)\vartheta(s)ds$ defines an E-valued continuous function. The existence of solutions for the second order abstract Cauchy problem,

$$\vartheta''(u`) = A\vartheta(u`) + h(u`), \qquad 0 \le u` \le a \qquad (11)$$

$$v, \vartheta'(s) = \omega, \qquad (12)$$

where $h : J \to X$ is an integrable function, has been discussed in [24]. Similarly the existence of solutions of semilinear second order abstract Cauchy problems has been treated in [25]. We only mention here that the function $\vartheta(\cdot)$ given by

$$\vartheta(\grave{u}) = C(\grave{u} - s)v + S(\grave{u} - s)\omega + \int_s^{\grave{u}} S(\grave{u} - \tau)h(\tau)d\tau, \quad 0 \le \grave{u} \le a,$$

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is called a mild solution of (7)-(8) and that when $\upsilon \in E, \vartheta(.)$ is continuously differentiable and

$$\vartheta'(\grave{u}) = AS(\grave{u}-s)\upsilon + C(\grave{u}-s)\omega + \int_s^{\grave{u}} C(\grave{u}-\tau)h(\tau)d\tau \quad 0 \le \grave{u} \le a.$$

In addition ,if $\upsilon \in D(A), \omega \in E$ and f is a continuously differentiable function , then the function $\vartheta(\cdot)$ is a solution of the initial value problem (11)-(12).

the function $`u \, 7\!\!\longrightarrow B(u`)\vartheta$is continuously differentiable ine e $\vartheta$for each $\vartheta \in E$. It has been Assume now that $A(u`) = A + B(u`)$ where $B(\cdot): R \to L(E,\vartheta)$ is a map such that established by serizawa [22] that for each (e $\upsilon,\omega) \in D(A) \times E$ the non-autonomous abstract Cauchy problem

$$\vartheta''(u`) = (A + B(u`))\vartheta(u`), u` \in R,$$

$$\vartheta(0) = \upsilon, \qquad \vartheta e'(0) = \omega,$$

has a unique solution $\vartheta(\cdot)$ such that the function $`u \, 7\!\!\longrightarrow \vartheta(u`)$ is continuously differentiable in E. It is clear that the same argument allows us to conclude that Eq.(14), with the initial condition (12) has a unique solution $\vartheta(\cdot,s)$ such that the function $`u \, 7\!\!\longrightarrow \vartheta(u,s`)$ is continuously differentiable in E. It follows from (13) that

$$\vartheta(\grave{u},s) = C(\grave{u}-s)\upsilon + S(\grave{u}-s)\omega + \int_s^{\grave{u}} S(\grave{u}-\tau)\widetilde{B}(\tau)\vartheta(\tau,s)d\tau$$

$$\vartheta(\grave{u},s) = S(\grave{u}-s)\omega + \int_s^{\grave{u}} S(\grave{u}-\tau)\widetilde{B}(\tau)\vartheta(\tau,s)d\tau.$$

In particular ,for $\upsilon = 0$ we have

(16) Consequently,

$$\|\vartheta(\grave{u},s)\|_1 \le \|S(\grave{u}-s)\|_{\mathcal{L}(\vartheta,E)}\|\omega\| + \int_s^{\grave{u}} \|S(\grave{u},\tau)\|_{\mathcal{L}(\vartheta,E)} \left\|\widetilde{B}(\tau)\right\|_{\mathcal{L}(\vartheta,E)} \|\vartheta(\tau,s\|_1 d\tau$$

and, applying the Gronwall - Bellman lemma we infer that

$$k\vartheta(u,s` \quad )k_1 \le Mfk\omega k, \qquad s,u` \in J.(17)$$

We define the operator $S(u,s`)\omega = \vartheta(u,s`)$. It follows from the previous estimate that $S(u,s`)$ is a bounded linear map on $E$. Since $E$ is dense in $X$, we can extend $S(u,s`)$ to X. We keep the notation $S(u,s`)$ for this extension. It is well known that, exception the case $dim(X) < \infty$, the cosine function $C(u`)$ cannot be compact for all $`u \in$ R. By contrast, for the cosine functions that arise in specific applications, the sine function $S(u`)$ is very often a compact operator for all $`u \in$ R. This motivates the result [[8], Theorem 1.2].

We now consider some notations and definitions concerning impulsive differential equations. A function $\vartheta: [\sigma,\tau] \to X$ is said to be a normalized piece wise continuous function on $[\sigma, \tau]$ if $\vartheta$is piece wise continuous and left continuous on $(\sigma,\tau]$. We denote by PC $([\sigma,\tau],X)$the space of normalized piecewise continuous functions from$[\sigma,\tau]$ into X. In particular, we introduce the space PC formed by all normalized piece wise continuous functions $\vartheta: [0,a] \to X$ such that $\vartheta(\cdot)$ is continuous at $`u \, 6 = u`_k, \vartheta(u`_{k^-}) = \vartheta(u`_k)$ and $\vartheta(u`^+_k)$ exists, for $k = 1,2,...,m$. In this paper, we always assume that PC is endowed with the norm

$\|\vartheta\|_{\mathcal{PC}} = \sup_{s\in J} \|\vartheta(s)\|$. It is clear that (PC,$k\cdot k_{PC}$) is a Banach space.

In what follows, we put $`u_0 = 0, u`_{n+1} = a$ and, for $\vartheta \in$ PC, we denote by $\vartheta^{\sim}_k$, for $k = 0,1,...,m$, the function $\vartheta^{\sim}_k \in C([u`_k,u`_{k+1}];\vartheta)$ given by $\vartheta^{\sim}_k(u`) = \vartheta(u`)$ for $`u \in (u`_k,u`_{k+1})$ and $\vartheta^{\sim}_k(u`_k) = \lim \vartheta(u`)$. Moreover, for a set $E \subseteq$ PC, we denote by $E^{\sim}_k$, for $k = 0,1,...,m$, the$u` \to u`^+$ set $E^{\sim}_k = \vartheta^{\sim}_k : \vartheta \in E$.

Lemma 2.1 *A set $E \subseteq$ PC is relatively compact in PC if and only if each $E_k, k = 0,1,...,m$,*

is relatively compact in $C([u`_k,u`_{k+1}];\vartheta)$. (14)

(15)

In this work we will employ an axiomatic definition of the phase space B, similar to the one used in [9] and suitably modify to treat retarded impulsive differential equations. More precisely, B will denote the vector space of functions defined from $(-\infty,0]$ into ϑendowed with a seminorm denoted $k\cdot k_B$ and such that the following axioms are hold:

(A) If $\vartheta: (-\infty,\mu + b] \to \vartheta, b > 0$, is such that $\vartheta_\mu \in$ B and $\vartheta|_{[\mu,\mu+b]} \in$ PC$([\mu,\mu + b],X)$ then, for every $`u \in [\mu,\mu + b)$, the following conditions are hold:and

(i) $\vartheta_{u`}$ is $\in$ B,

(ii) $k\vartheta(u`)k \le H k\vartheta_{u`}k_B$

(iii) $k\vartheta_{u`}k_B \le K(u` - \mu)\sup\{k\vartheta(s)k : \mu \le s \le u`\} + M(u` - \mu)k\vartheta_\mu k_B$,

where $H > 0$ is a constant; $K,M : [0,\infty) \to [1,\infty)$, K is continuous, M is locally bounded and H,K,M are independent of $\vartheta(\cdot)$.

(B) The space B is complete.

Remark 2.1 *In impulsive functional differential systems, the map $[\mu,\mu+b] \to$ B, $u` \to \vartheta_t$, is in general discontinuous. For this reason, this property has been omitted from our description of the phase space* B.

Now we include that some of our proofs are based on the following well-known result [[4], Theroem 6.5.4].

Lemma 2.2 *(Leray-Schauder Alternative) Let D be a closed convex subsets of a normed lined space X such that $0 \in D$. Let $F : D \to D$ be a completely continuous map. Then the set $\{\vartheta \in D : \vartheta = \lambda F(\vartheta)$, for some$0 < \lambda < 1\}$ is unbounded or the map F has a fixed point in D.*

The terminology and notations are generally used in functional analysis. In particular, for Banach spaces $(Z,k\cdot k),(W,k\cdot k_w)$, the notation L$(Z,W)$ stands for the Banach space of bounded linear operators from Z into W and we abbreviate to L(Z) whenever $Z = W$. By $\sigma(A)$ (respectively ,$\rho(A)$) we denote the spectrum (respectively ,the resolvent set)of a linear operator A. Moreover$B_r(\vartheta,Z)$ denotes the closed ball with center at $\vartheta$and radius $r > 0$ in the space Z.

Remark 2.2 *In what follows the notation $g(a)$ stands for the space* $g(a) = \{y : (-\infty,a] \to \vartheta : y|_J \in PC, y_0 = 0\}$.

by $\varphi e0 = \varphi$ *and* $\varphi e(u`) = C(u,`0)\varphi(0) + S(u,`0)\zeta$, *for* $u` \geq 0$.e *endowed with the sup norm. In addition, we denote by* $\varphi : (-\infty,a] \to \vartheta$ *the function defined*

Definition 2.2 *A function* $\vartheta: (-\infty,a] \to X$ *is called a mild solution of the abstract Cauchy problem (1)-(4), if* $\vartheta_0 = \varphi \in$ B, $\vartheta|_J \in PC$, *the impulsive conditions* $\Delta\vartheta(u`_k) = I_k(\vartheta_{u`k})$,

$\Delta\vartheta(u`_k) = J_k(\vartheta_{u`k})$, $k = 1,2\ldots,m$, *are satisfied and the following integral equation*

$$\vartheta(\grave{u}) = C(\grave{u},0)\phi(0) + S(\grave{u},0)\Big(\vartheta - \theta_1(0,\phi,0)\Big) + \int_0^{\grave{u}} C(\grave{u},s)\theta_1(s,\vartheta_s,\int_0^s a_1(s,\tau,\vartheta_\tau)d\tau)ds$$
$$+ \int_0^{\grave{u}} S(\grave{u},s)\Big[Bu(s) + \theta_2(s,\vartheta_s,\int_0^s a_2(s,\tau,\vartheta_\tau)d\tau)\Big]ds + \sum_{0<\grave{u}_k<\grave{u}} C(\grave{u},\grave{u}_k)I_k(\vartheta_{\grave{u}_k})$$

$$+ X_k \quad S(u,`u`_k)J_k(\vartheta_{u`k}), \; 0 < u < a.` \; 0<u`<u`$$

is verified.

III. CONTROLLABILITY RESULT

To establish our result, we introduce the following assumptions on system (1)-(4):

(H1) There exists a constant $L_{a_i} > 0, \widetilde{L_{a_i}} > 0$ such that $\|a_i(u,s,y`_1) - a_i(u,s,y`_2)\| \leq L_{ai}\|y_1 - y_2\|_B, u,s \in J, y_i \in B.$

$\widetilde{a_i} = \| \int_0^{\grave{u}} a_i(\grave{u},s,0)ds\|, i = 1$ L,2.

(H2) $\theta_1 : J \times B \times X$
The function $\to X$ is continuous and there exist constants $L_{\theta i} > 0, \widetilde{L_{\theta_i}} > 0$ for $\psi_i \in \mathcal{B}, \grave{u} \in J, x, y \in X$, such that $\|\theta_i(u,\psi`_1,x) - \theta_i(u,\psi`_2,y)\| \leq L_{\theta i}(\|\psi_1 - \psi_2\|_B + \|x - y\|)$

and $Le\theta_i = \sup_{u` \in J}\|\theta_i(u,`0,0)\|$, $i = 1,2$.

(H3) B is a continuous operator from U to X and the linear operator $W : L^2(J,U) \to X$, defined by

$$Wu = \int_0^a S(a,s)Bu(s)ds$$

has a bounded invertible operator W^{-1} which takes values in $L^2(J,U)/kerW$ and there exist positive constant M such that $\|BW^{-1}\| \leq M_1$.

(H4) The impulsive functions satisfy the following conditions:

$$L_I > 0, \widetilde{L}_I > 0 \text{ for } \psi_1, \psi_2 \in \mathcal{B}$$

$\|I_k($

(i) The maps I_k : B $\to X$, $k = 1,2,\ldots,m$ is continuous and there exist constants such that $\psi_1) - I_k(\psi_2)\| \leq L_I\|\psi_1 - \psi_2\|$ and $\widetilde{L}_I = \|I_k(0)\|$.

(ii) The maps J_k : B $\to X$, $k = 1,2,\ldots,m$ is continuous and there exists constants $L_J > 0, \widetilde{L}_J > 0$ for $\psi_1, \psi_2 \in \mathcal{B}$ such that $\|J_k(\psi_1) - J_k(\psi_2)\| \leq L_J\|\psi_1 - \psi_2\|$ and $LeJ = \|J_k(0)\|$.(H5)

$$Let \quad N[\|\zeta\| + aL_{\theta_1}\|\phi\| + \widetilde{L_{\theta_1}}] + \tilde{N}a[L_{\theta_1}[(1 + L_{\theta_1})(K_a r + c_1 + \widetilde{L_{\theta_1}})] + \widetilde{L_{\theta_1}}]$$
$$+ Na[L_{\theta_2}[(1 + L_{\theta_2})(K_a r + c_1 + \widetilde{L_{\theta_2}})] + \widetilde{L_{\theta_2}}] + aNA_0 + \sum_{k=1}^{m}(\tilde{N}\widetilde{L}_I$$
$$+ N\widetilde{L}_J) + \sum_{k=1}^{M}(\tilde{N}L_J)[K_a r + \left\|\widetilde{\phi}_{\grave{u}_k}\right\|] \leq r, for \;\; some \;\; r > 0.$$

(H6) Let $\mu = K_a(1 + aNM_1)\Big[a(\tilde{N}L_{\theta_1}(1 + L_{a_1}) + aNL_{\theta_2}(1 + L_{a_2})\sum_{k=1}^{m}((\tilde{N}L_I + NL_J)\Big] < 1$ be such that $0 \leq \mu < 1$.

Definition 3.3 *The system (14-17) is said to be controllable on the interval J, if for every* $\vartheta_0 = \varphi \in$ B, $\vartheta'(0) = \zeta$ *and* $z_1 \in X$, *there exists a control* $u \in L^2(J,U)$ *such that the mild solution* $\vartheta(\cdot)$ *of (1)-(4) satisfies* $\vartheta(a) = z_1$.

The following results is an immediate application of the contraction principle of Banach. To simplify the text, we denote $K_a = \sup_{0 \leq u` \leq a} K(u`)$.

Since $\left\|\widetilde{\phi}_k\right\|_{\mathcal{B}} = \widetilde{N}\|\phi(0)\| + N\|\zeta\| + M\|\phi\|_{\mathcal{B}} < \infty, 0 \leq t \leq a$, we denote,

$c_1 = \sup_{0 \leq \grave{u} \leq a}\|\phi_{\grave{u}}\|_{\mathcal{B}}$ and $\left\|y_{\grave{u}} + \widetilde{\phi}_{\grave{u}}\right\| \leq K_a\|y_{\grave{u}}\| + \left\|\widetilde{\phi}\right\| \leq K_a r + c_1 = \rho$

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Theorem 3.1 *If the hypothesis (H1)-(H6) are satisfied, then the impulsive second order system (1)-(4) is controllable on J.*

Proof. Using the assumption (H3), we define the control function

$$u(\grave{u}) = W^{-1}[z_1 - C(a,0)\phi(0) - S(a,0)[\zeta - \theta_1(0,\phi,0)]$$
$$+ \int_0^a C(a,s)\theta_1(s,\vartheta_s,\int_0^s a_1(s,\tau,\vartheta_\tau)d\tau)ds - \int_0^a S(a,s)\theta_2(s,\vartheta_s,\int_0^s a_2(s,\tau,\vartheta_\tau)d\tau)ds$$
$$- \sum_{k=1}^{m} C(a,\grave{u}_k)I_k(\vartheta_{\grave{u}_k}) - \sum_{k=1}^{m} S(a,\grave{u}_k)J_K(\vartheta_{\grave{u}_k})](\grave{u})$$

Let $B_r = \{\vartheta \in X, \|\vartheta\| \leq r\}$ for some $r > 0$.

we shall now show that when using this control the operator Γ on the space $g(a)$ defined by $(\Gamma y)_0 = 0$ and

$$
\begin{aligned}
(\Gamma y)(\grave{u}) &= S(\grave{u},0)[\zeta-\theta_1(0,\phi,0)] - \int_0^{\grave{u}} C(\grave{u},s)\theta_1(s,y_s+\widetilde{\phi}_s,\int_0^s a_1(s,\tau,y_\tau+\widetilde{\phi}_\tau)d\tau)ds \\
&+ \int_0^{\grave{u}} S(\grave{u},s)\theta_2(s,y_s+\widetilde{\phi}_s,\int_0^s a_2(s,\tau,y_\tau+\widetilde{\phi}_\tau)d\tau)ds \\
&+ \int_0^{\grave{u}} S(\grave{u},\eta)BW^{-1}\Big[z_1 - C(a,0)\phi(0) - S(a,0)[\zeta-\theta_1(0,\phi,0)] \\
&+ \int_0^a C(a,s)\theta_1(s,\vartheta_s,\int_0^s a_1(s,\tau,\vartheta_\tau)d\tau)ds \\
&- \int_0^a S(a,s)\theta_2(s,y_s+\widetilde{\phi}_s,\int_0^s a_2(s,\tau,y_\tau+\widetilde{\phi}_\tau)d\tau)ds - \sum_{k=1}^m C(a,\grave{u}_k)I_k(y_{\grave{u}_k}+\widetilde{\phi}_{\grave{u}_k}) \\
&- \sum_{k=1}^m S(a,\grave{u}_k)J_k(y_{\grave{u}_k}+\widetilde{\phi}_{\grave{u}_k})\Big](\eta)d\eta + \sum_{0<\grave{u}_k<\grave{u}} C(\grave{u},\grave{u}_k)I_k(y_{\grave{u}_k}+\widetilde{\phi}_{\grave{u}_k}) \\
&+ \sum_{0<\grave{u}_k<\grave{u}} S(\grave{u},\grave{u}_k)J_k(y_{\grave{u}_k}+\widetilde{\phi}_{\grave{u}_k}), \qquad \grave{u}\in J,
\end{aligned}
$$

(18)

has a fixed point $\vartheta(\cdot)$. This fixed point is then a mild solution of the system (1)-(4). Clearly $(\Gamma\vartheta)(a) = z_1$ which means that the control u steers the system from the initial state φ to z_1 in time a, provided we can obtain a fixed point of the operator Γ which implies that the system is controllable. From the assumptions, it is easy to see that Γ is well defined and continuous. For convenience let us take,

$$
\begin{aligned}
\|Bu(s)\| \le M_1[\|z_1\| &+ \widetilde{N}\,\|\phi(0)\| + N[\|\zeta\| + aL_{\theta_1}\,\|\phi\| + \widetilde{L_{\theta_1}}] \\
&+ \widetilde{N}a[L_{\theta_1}[(1+L_{\theta_1})(K_a r + c_1 + \widetilde{L_{\theta_1}})] + \widetilde{L_{\theta_1}}] \\
&+ Na[L_{\theta_2}[(1+L_{\theta_2})(K_a r + c_1 + \widetilde{L_{\theta_2}})] + \widetilde{L_{\theta_2}}] \\
&+ \widetilde{N}\sum_{k=1}^m [L_I(K_a r + \left\|\widetilde{\phi}_{\grave{u}_k}\right\|) + \widetilde{L}_I] + N\sum_{k=1}^m [L_J(K_a r + \left\|\widetilde{\phi}\right\|) + \widetilde{L}_J] = A_0
\end{aligned}
$$

First we show that Γ maps $B_r(0,g(a))$ into $B_r(0,g(a))$. To this end, from the definition of the operator Γ in (18) and our hypotheses, we obtain

$$
\begin{aligned}
\|(\Gamma y)(\grave{u})\| \le N[\|\zeta\| &+ aL_{\theta_1}\,\|\phi\| + \widetilde{L_{\theta_1}}] + \widetilde{N}a[L_{\theta_1}[(1+L_{\theta_1})(K_a r + c_1 + \widetilde{L_{\theta_1}})] + \widetilde{L_{\theta_1}}] \\
&+ Na[L_{\theta_2}[(1+L_{\theta_2})(K_a r + c_1 + \widetilde{L_{\theta_2}})] + \widetilde{L_{\theta_2}}] + aNA_0 \\
&+ \sum_{k=1}^m (\widetilde{N}\widetilde{L}_I + N\widetilde{L}_J) + \sum_{k=1}^M (\widetilde{N}L_J)[K_a r + \left\|\widetilde{\phi}_{\grave{u}_k}\right\|] \le r.
\end{aligned}
$$

for $y \in g(a)$ and $\grave{u} \in J$. Hence $k\Gamma yk_a \le r$. Therefore, Γ maps $B_r(0,g(a))$ into itself. Now for $y,z \in B_r(0,g(a))$, we have

$$
\begin{aligned}
\|(\Gamma y)(\grave{u}) - (\Gamma z)(\grave{u})\| &\le K_a(1+aNM_1)\Big[a(\widetilde{N}L_{\theta_1}(1+L_{a_1}) + aNL_{\theta_2}(1+L_{a_2}) \\
&\quad + \sum_{k=1}^m((\widetilde{N}L_I + NL_J)\Big]\|y-z\| \\
&\le \mu\|y-z\|_a,
\end{aligned}
$$

which implies that Γ is a contraction on Br(0,g(a)). Hence by the Banach fixed point we obtain that $\vartheta(\cdot)$ is a mild solution of the problem (1)-(4) and the proof is complete.e theorem, Γ has a unique fixed point y in g(a). Defining $\vartheta(\grave{u}) = y(\grave{u}) + \varphi(\grave{u}), -\infty < \grave{u} \le a$,

We use the below condition instead of (H1) to avoid the Lipschitz continuity of θ1,θ2 used in Theorem 3.1.

(A1)The function ai : J × J × B → X satisfy the following conditions:

(i) The function φ → ai(u,s,φ` ) : is continuous almost everywhere for all `u,s ∈ J.

(ii) The function (`u,s) → ai(u,s,φ`) is strongly measurable for each φ ∈ B.

(iii) There is a positive continuous function pi : J → [0,∞) and a nondecreasing integrable positive function Ωi : R → [0,∞) such that kai(u,s,φ`)k ≤ pi(s)Ωi(φ)

for all t,s,φ ∈ J × J × B, where i = 1,2.

(A2)The function θi : J × B × X → X satisfy the following conditions:

(i) The function (`u,ϑu`) → θi(u,ϑ` u`,x) is continuous for almost everywhere `u ∈ J.

(ii) For each `u ∈ J, the function θi(u,` ·,·) : B → X is continuous and the function u` → θi(u,ϑ` u`,x) is strongly measurable.

(iii) There is a positive continuous function mi : J → [0,∞) and a continuous non decreasing function ψi : R → (0,∞) such that kθi(u,φ,x`)k ≤ mi(u`)ψ(kφkB) + kxk,(u,ψ,x`) ∈ J × B × X,i = 1,2.

Also, we have the following condition.

(A3)

$$
\begin{aligned}
\Big[\|z_1\| &+ \widetilde{N}\|\phi(0)\| + N[\|\zeta\| + \widetilde{L_{\theta_1}}] + \widetilde{N}a\big(m_1(s)\psi(\rho) + p_1(s)\Omega_1(\rho)\big) \\
&+ Na\big(m_2(s)\psi(\rho) + p_2(s)\Omega_2(\rho)\big) + \widetilde{N}\sum_{k=1}^m(\alpha_k^1(\rho) + \alpha_k^2) + N\sum_{k=1}^m(\beta_k^1(\rho) + \beta_k^2)\Big] = M^*
\end{aligned}
$$

Theorem 3.2 *Assume that θ_1 verify condition (A1) and condition (A2),(H3) are satisfied.*

Also, the following condition hold:

(a) For every u` ∈J and every r >0, the set $U_1(r,u`)$ = {$C(u,s`)\theta_1(u,\gamma,x`)$,$U_2(r,u`)$ = {$S(u,s`)\theta_2(u,\gamma,x`)$: s ∈ [0,a], γ,x ∈ B_r(0,B)} is relatively compact in X.

(b) The maps I_k,J_k : B → X are completely continuous and there exist positive constants α_k^i, β_k^i, i = 1,2,k = 1,2,...,m, such that kI_k(γ)k ≤ α_k^1 kγk$_B$ + α_k^2 and kJ_kγk ≤ β_k^1 kγk$_B$ + β_k^2, for all γ ∈ B.

(c) The constant $c = \frac{1}{1-\mu}[N[\|\zeta\| + L_{\theta_1}] + aNM_1\,[M^*] + \sup_{0\le\grave{u}_k\le\grave{u}}\|\phi_s\|_{\mathcal{B}} + K_a \sum (N\alpha_k^2 + N\beta_k^2)]$

$$\mu = K_a\sum_{k=1}^m(\tilde{N}\alpha_k^1 + N\beta_k^1) < 1 \ and \int_c^\infty \frac{ds}{\Omega(s)} > \frac{K_\alpha N}{1-\mu}\int_0^a p(s)ds$$

g e 0<u`k<u` e *where.*

Then the (1)-(4) is controllable on J.

Proof. we define the map Γ on the space $g(a)$ as in eq (18). To prove the controllability of the problem (1)-(4), we must show that the operator Γ has a fixed point. This fixed point is then a mild solution of the system (1)-(4). From the assumptions, it is easy to see that Γ is well defined and continuous.

In order to apply Lemma 2.2, we need to obtain a priori bound for the solutions of the integral equation $y = \lambda\Gamma(y)$, $\lambda \in (0,1)$. To this end, let y^λ be a solution of $\lambda\Gamma(y) = y$, $\lambda \in (0,1)$. Using the notation,

$$\nu^\lambda(t) = \sup_{\le s\le u}\left\|y_s^\lambda + \tilde{\phi}_s\right\|_{\mathcal{B}} \le K_a\left\|y^\lambda\right\|_s + \left\|\tilde{\phi}_s\right\|_{\mathcal{B},a} \le K_a r + c_1 = \rho$$

we observe that

$$
\begin{aligned}
\left\|y^\lambda(\grave{u})\right\| \le N[\|\zeta\| + \widetilde{L_{\theta_1}}] &+ \widetilde{N}\int_0^{\grave{u}}(m_1(s)\psi_1(v^\lambda(s)) + p_1(s)\Omega_1(v^\lambda(s)))ds \\
&+ N\int_0^{\grave{u}}(m_2(s)\psi_2(v^\lambda(s)) + p_2(s)\Omega_2(v^\lambda(s)))ds + aNM_1\,[M^*] \\
&+ \sum_{0<\grave{u}_k<\grave{u}}(\widetilde{N}\alpha_k^1 + N\beta_k^1)v^\lambda(\grave{u}_k) + \sum_{0<\grave{u}_k<\grave{u}}(\widetilde{N}\alpha_k^2 + N\beta_k^2)
\end{aligned}
$$

Hence follows that

$$
\begin{aligned}
v^\lambda(\grave{u}) \le N[\|\zeta\| + \widetilde{L_{\theta_1}}] &+ aNM_1\,[M^*] + \sup_{0\le\grave{u}_k\le\grave{u}}\|\tilde{\phi}_s\|_{\mathcal{B}} + K_a\sum_{0<\grave{u}_k<\grave{u}}(\tilde{N}\alpha_k^2 + N\beta_k^2) + \mu v^\lambda(\grave{u}_k) \\
&+ K_a\int_0^{\grave{u}}\{\tilde{N}(m_1(s)\psi_1(v^\lambda(s)) + p_1(s)\Omega_1(v^\lambda(s))) + N(m_2(s)\psi_2(v^\lambda(s)) + p_2(s)\Omega_2(v^\lambda(s)))\}ds
\end{aligned}
$$

which yields

$$v^\lambda(\grave{u}) \le c + \frac{K_a N}{1-\mu}\int_0^{\grave{u}}\{\tilde{N}(m_1(s)\psi_1(v^\lambda(s)) + p_1(s)\Omega_1(v^\lambda(s))) + N(m_2(s)\psi_2(v^\lambda(s)) + p_2(s)\Omega_2(v^\lambda(s)))\}ds.$$

Denoting by $\omega_\lambda(t)$ the right-hand side of the previous inequality, we see that

$$\omega'_\lambda(\grave{u}) \leq \frac{KaN}{1-\mu}\{\widetilde{N}(m_1(t)\psi_1(\omega_\lambda(t)) + p_1(t)\Omega_1(\omega_\lambda(t))) + N(m_2(t)\psi_2(\omega_\lambda(t)) + p_2(t)\Omega_2(\omega_\lambda(t)))\}$$
$$\leq \frac{KaN}{1-\mu}[p(t)\Omega(\omega_\lambda(t))],$$

and subsequently, upon integrating over $[0,\grave{u}]$, we obtain

$$\int_c^{\omega_\lambda(\grave{u})} \frac{ds}{\Omega(s)} \leq \frac{K_aN}{1-\mu}\int_0^{\grave{u}} p(s)ds \leq \frac{K_aN}{1-\mu}\int_0^a p(s)ds < \int_c^{\infty}\frac{ds}{\Omega(s)}.$$

This estimate permits us to conclude that the set of functions $\{\omega_\lambda : \lambda \in (0,1)\}$ is bounded and, in turn, that $\{y^\lambda : \lambda \in (0,1)\}$ is bounded in $\theta_2(a)$. Next we show that Γ is completely continuous. To clarify this proof, we decompose Γ in the form $\Gamma = \Gamma_1 + \Gamma_2$, where

$$\Gamma_1 y(\grave{u}) = \int_0^{\grave{u}} S(\grave{u},s)\Big[\theta_2(s, y_s + \widetilde{\phi}_s, \int_0^s a_2(s,\tau,y_\tau + \widetilde{\phi}_\tau)d\tau) + Bu(s)\Big]ds$$
$$- \int_0^{\grave{u}} C(\grave{u},s)\theta_1(s, y_s + \widetilde{\phi}_s, \int_0^s a_1(s,\tau,y_\tau + \widetilde{\phi}_\tau)d\tau)ds,$$
$$\Gamma_2 y(\grave{u}) = S(\grave{u},0)[\zeta - \theta_1(0,\phi,0)] + \sum_{0<\grave{u}_k<\grave{u}} C(\grave{u},\grave{u}_k)I_k(y_{\grave{u}_k} + \widetilde{\phi}_{\grave{u}_k})$$
$$+ \sum_{0<\grave{u}_k<\grave{u}} S(\grave{u},\grave{u}_k)J_k(y_{\grave{u}_k} + \widetilde{\phi}_{\grave{u}_k}),\ \grave{u} \in J$$

Using the hypotheses, condition (b) and Lemma 2.1, we obtain that Γ_1 is continuous and that Γ_2 is completely continuous. In order to use the Ascoli-Arzela theorem we prove that Γ_1 takes bounded sets into relatively compact ones. As above, B_r = $B_r(0,g(a))$ and $\left\|y_{\grave{u}} + \widetilde{\phi}_{\grave{u}}\right\|_{\mathcal{B}} \leq K_a r + c_1 = \rho \quad for \quad \grave{u} \in J$. And also, $\|(Bu)(s)\| \leq B_0$.

From the mean value theorem , we get the set $\{\Gamma_1 y(\grave{u}) : y \in B_r(0,g(a))\}$ is relatively compact for each $\grave{u} \in J$Moreover, from

$$\Gamma_1 y(\grave{u}+h) - \Gamma_1 y(\grave{u}) = \int_0^{\grave{u}} [S(\grave{u}+h,s) - S(\grave{u},s)]\Big[\theta_2(s, y_s + \widetilde{\phi}_s, \int_0^s a_2(s,\tau,y_\tau + \widetilde{\phi}_\tau)d\tau) + Bu(s)\Big]ds$$
$$+ \int_{\grave{u}}^{\grave{u}+h} S(\grave{u},s)\Big[\theta_2(s, y_s + \widetilde{\phi}_s, \int_0^s a_2(s,\tau,y_\tau + \widetilde{\phi}_\tau)d\tau) + Bu(s)\Big]ds$$
$$- \int_0^{\grave{u}} [C(\grave{u}+h,s) - C(\grave{u},s)]\theta_1(s, y_s + \widetilde{\phi}_s, \int_0^s a_1(s,\tau,y_\tau + \widetilde{\phi}_\tau)d\tau)ds$$
$$- \int_{\grave{u}}^{\grave{u}+h} C(\grave{u},s)\theta_1(s, y_s + \widetilde{\phi}_s, \int_0^s a_1(s,\tau,y_\tau + \phi_\tau)d\tau)ds$$

and using that $S(.,s)$ and $C(.,s)$ verifies a Lipschitz condition, we obtain that

$$\|\Gamma_1 y(\grave{u}+h) - \Gamma_1 y(\grave{u})\| \leq |h|\,N_1 \int_0^a [(m_2(s)\psi_2(v^\lambda(s)) + p_2(s)\Omega_2(v^\lambda(s))) + B_0]ds$$
$$+ N\int_{\grave{u}}^{\grave{u}+h} [(m_2(s)\psi_2(v^\lambda(s)) + p_2(s)\Omega_2(v^\lambda(s))) + B_0]ds$$
$$+ |h|\,\widetilde{N}\int_0^a (m_1(s)\psi_1(v^\lambda(s)) + p_1(s)\Omega_1(v^\lambda(s)))ds$$
$$+ \widetilde{N}\int_{\grave{u}}^{\grave{u}+h} (m_1(s)\psi_1(v^\lambda(s)) + p_1(s)\Omega_1(v^\lambda(s)))ds$$

which implies that $\|\Gamma_1 y(\grave{u}+h) - \Gamma_1 y(\grave{u})\| \to 0$ as $h \to 0$uniformly for y $\in B_r(0,g(a))$. From this we infer that $\Gamma_1 y(\grave{u})$: $y \in B_r(0,g(a)$ is relatively compact in $g(a)$ and consequently that Γ_1 is completely continuous. This completes the proof of the assertion that the map Γ is completely continuous.

By an application of Lemma 2.1, we conclude that there exists a fixed point y of Γ. It is clear that the function $\vartheta = y + \tilde{\varphi}$ is a mild solution of the system (1)-(4). This completes the proof.

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IV. CONCLUSION

A conclusion section is not required. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions.

APPENDIX

It is optional. Appendixes, if needed, appear before the acknowledgment.

ACKNOWLEDGMENT

It is optional. The preferred spelling of the word "acknowledgment" in American English is without an "e" after the "g." Use the singular heading even if you have many acknowledgments. Avoid expressions such as "One of us (S.B.A.) would like to thank ... ." Instead, write "F. A. Author thanks " *Sponsor and financial support acknowledgments are placed in the unnumbered footnote on the first page.*

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