

# Three Level Hysteresis Function Based Sliding Mode Control of Three Phase PWM Inverter for Shunt Active Power Filter Application in Distribution Network



P. Balamurugan, N. Senthil Kumar

**Abstract:** In this paper, a Three-Level Hysteresis based Sliding (TLHSM) function approach to regulate output currents of three-phase capacitor powered Voltage Source Inverter (VSI) is analyzed and implemented. VSI functions as shunt active power filter to compensate for harmonic currents generated by a nonlinear load in the power distribution network. The performance of the control is compared with conventional two-level hysteresis-based control through simulation in MATLAB/Simulink. Sliding Mode Control (SMC) is recognized as robust controller with a high stability over a wide range of load and source variations. The set-back of unfixed switching frequency problem with conventional two-level hysteresis control is disregarded in this approach. Switching frequency prediction is observed to be accurate. The tracking error and Total Harmonic Distortion (THD) in source currents are the measurable parameters for evaluation of control effort and is within the limits specified by IEEE519:2014 recommendations.

**Keywords:** Sliding mode control, Three phase inverter, three-level hysteresis function, THD.

## I. INTRODUCTION

Voltage source inverters plays an important role in daily life. It is aimed to provide voltage and frequency stability, high efficiency and low total harmonic distortion (THD) even during load unbalance, load variation and inferences. Several algorithms have been proposed to improve performance of inverters like PID, repetitive, predictive, linear feedback and sliding mode control.

PID is effective only for nominal loads. Repetitive control has a large lag in control. Linear feedback control depends on the mathematical model to be accurate in order to have an effective control. Predictive control requires large computations which in turn increases the cost. A logarithmic function-based formulation of switching surface for the control of single-phase inverters was proposed in [1]. The

performance of the control exhibits better dynamic response at variable switching frequencies.

Inverters for UPS can be controlled by taking output voltage as well as inductor current as a variable under control. Such control results in multi-loop control, regulating output voltage and limiting infinite switching frequency. Variable structure control technique for UPS inverters, employing multi-loop control was implemented in [2].

Sliding mode control is a nonlinear control method. It alters the dynamics of a nonlinear system by the application of discontinuous control signal. It is a Variable Structure System which switches between structures to get desired performance. Some advantages provided by sliding mode control are that the gain parameters can be calculated mathematically [3], it is insensitive and robust to load variations and line variations.

Second order SMC applied to control a 4-leg, 3 level neutral point inverter topology interface with non-conventional energy sources. An experimental validation was performed for validation of the control strategy yielding better performance [4].

This paper introduces a new approach of using hysteresis function for switching between the sliding modes for a three phase Voltage Source Inverter. The method of hysteresis function implemented in this paper, causes the switches in the inverter to be ON for half cycle. Hence the switching frequency is also reduced to half the value which would be used normally. A discrete sliding mode controller employing feedforward compensation for inverter application is implemented in [5]. It provides the tracking of sinusoidal voltage at the output of the UPS.

## II. MODELING OF INVERTER

A three phase three leg voltage source converter is powered by a dc source (battery or fully charged capacitor) consists of six semiconductor switches (S1 - S6) may be MOSFET or IGBT with anti-parallel diodes. The ac voltage output of the inverter is available at the midpoint of each leg. The output of the inverter is a stepped wave or pulse wave depending on the modulation control strategy. To eliminate the switching harmonics and for wave-shaping a filter network (LC or LCL) is usually connected between the inverter output and the grid / load as in Fig. 1. Let  $V_{sr}$ ,  $V_{sy}$ ,  $V_{sb}$  are the inverter pole voltages measured with respect to ground.

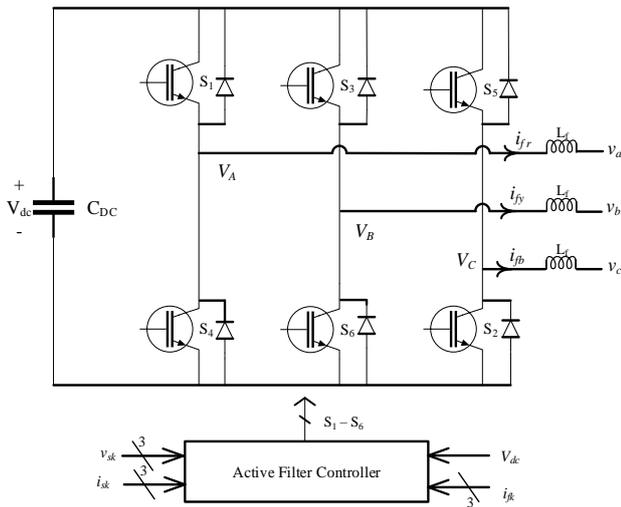
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**Fig.1 Circuit of Capacitor Powered Three Phase Voltage Source Inverter**

Applying Kirchoff's circuit laws, we get

$$V_{ab} = L \frac{di_{ab}}{dt} + r i_{ab} + V_{AB} \quad (1)$$

$$V_{bc} = L \frac{di_{bc}}{dt} + r i_{bc} + V_{BC} \quad (2)$$

$$V_{ca} = L \frac{di_{ca}}{dt} + r i_{ca} + V_{CA} \quad (3)$$

$$i_{ab} = C \frac{dV_{AB}}{dt} + \frac{V_{AB}}{R} \quad (4)$$

$$i_{bc} = C \frac{dV_{BC}}{dt} + \frac{V_{BC}}{R} \quad (5)$$

$$i_{ca} = C \frac{dV_{CA}}{dt} + \frac{V_{CA}}{R} \quad (6)$$

Where 'r' is the effective series resistance of inductor,  $i_a, i_b, i_c$ , are the pole currents of the inverter and  $V_{AB}, V_{BC}, V_{CA}$  are output line voltages after filter which appears across the load. The equations (1) – (6) in matrix form can be expressed as follows:

$$\begin{pmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{pmatrix} = \begin{pmatrix} v_a - v_b \\ v_b - v_c \\ v_c - v_a \end{pmatrix} = L \frac{d}{dt} \begin{pmatrix} i_a - i_b \\ i_b - i_c \\ i_c - i_a \end{pmatrix} + r \begin{pmatrix} i_a - i_b \\ i_b - i_c \\ i_c - i_a \end{pmatrix} + \begin{pmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{pmatrix} \quad (7)$$

$$\begin{pmatrix} i_{ab} \\ i_{bc} \\ i_{ca} \end{pmatrix} = C \frac{d}{dt} \begin{pmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{pmatrix} + \frac{1}{R} \begin{pmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{pmatrix} \quad (8)$$

The objective of this modeling is to develop a controller which can be able to track the inverter output voltage or current in accordance to a desired reference signal [6-7].

### III. TLH SLIDING MODE CONTROL

To develop a nonlinear type controller for tracking reference current at the output, let us formulate the sliding function for each phase. Consider phase AB initially. The LC filter is connected across the grid / load. The voltage impressed across the filter inductor and current through the filter capacitor is expressed as

$$L \frac{di}{dt} = V_i - V_o \quad (9)$$

$$i = i_c + i_o \quad (10)$$

Where,  $V_i = uV_s$ , where 'u' is the duty ratio of the switch.

Defining the state variables  $x_1 = i_f - i_f^*$  and  $x_2 = i_f - i_f^*$

From above equations,

$$\dot{x}_1 = x_2 \quad (11)$$

$$\dot{x}_2 = \omega_o^2 (-x_1 + uV_{DC} + DV_{DC}) \quad (12)$$

Where  $\omega_o = \frac{1}{\sqrt{LC}}$  and  $D$  is the disturbance term for a reference of  $i_f^*$ .

The sliding surface is given by,

$$\sigma = \alpha x_1 + x_2 + \beta x_3 \quad (13)$$

Where,  $x_3 = \int x_1$ . It eliminates the steady state error in output voltage.

The sliding mode ( $\sigma = 0$ ) is given by

$$\dot{x}_1 = -\alpha x_1 - \beta x_3 \quad (14)$$

$$\dot{x}_3 = x_1 \quad (15)$$

The sliding mode is stable if  $\alpha$  and  $\beta$  are positive.

$$\dot{\sigma} = (\beta - \omega_o^2)x_1 + \sigma x_2 + \omega_o^2 V_{DC}(u + D) \quad (16)$$

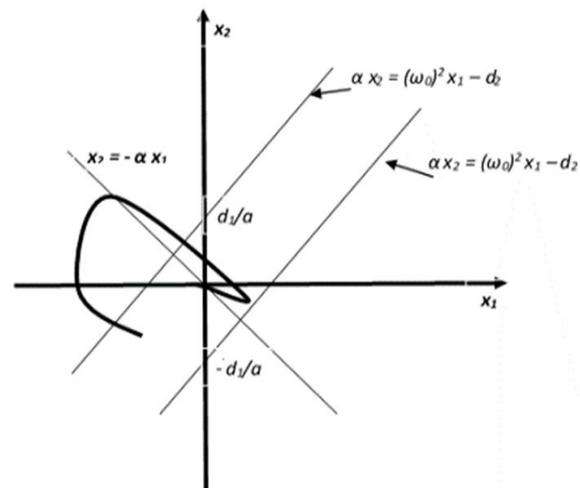
For,  $u = \text{sign}(\sigma)$ , if  $u = 1$ , then

$$[(\beta - \omega_o^2)x_1 + \sigma x_2 + \omega_o^2 V_{DC}(u + D)] > 0$$

If  $u = -1$ , then

$$[(\beta - \omega_o^2)x_1 + \sigma x_2 + \omega_o^2 V_{DC}(u + D)] < 0$$

The reaching condition for the sliding surface is shown in Fig. 2.

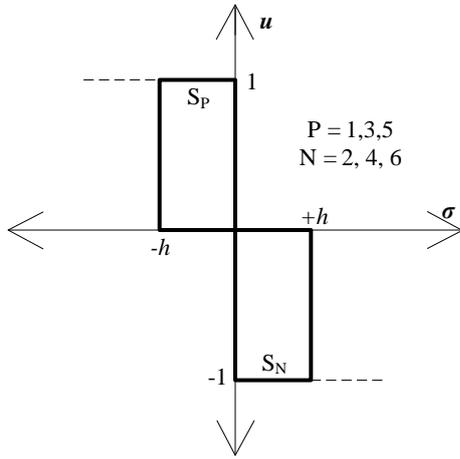


**Fig. 2. Stability region of reaching mode**

$$\text{Here, } d_1(t) = \omega_o^2 V_{DC}(1 + D(t)) \quad (17)$$

$$d_2(t) = \omega_o^2 V_{DC}(-1 + D(t)) \quad (18)$$

Under the condition of disturbance, the error in tracking becomes large resulting in very high switching frequency. This effect of high switching frequencies in the sliding mode is called chattering. To overcome it a hysteresis function with zero level access to output voltage is proposed.



**Fig. 3. Hysteresis Sliding Function for a Leg of Inverter**

The voltage error derivative used in sliding mode function amplifies high frequency in signal. To overcome this capacitor current is feedback is used. Modified control law is given by

$$x_2 = (i_c - i_c^*)/C, \quad (19)$$

$$\text{where } i_c^* = C v_0 \quad (20)$$

Since, the capacitance value is uncertain, a nominal capacitance  $C_o$ .

The new parameters,  $\alpha' = \frac{\alpha}{1+\varepsilon}$  and  $\beta' = \beta/(1 + \varepsilon)$ , where  $\varepsilon = (C - C_o)/C_o$ . The sliding function is found for all legs of the inverter using similar calculations.

#### IV. COMPUTATION OF HARMONIC CURRENTS AND REFERENCE CURRENT GENERATION

The harmonic current references are generated based on instantaneous real and reactive power theory proposed by H. Akagi in [10]. The instantaneous source voltages and load currents are transformed in to  $\alpha\beta 0$ - domain using Clarke's transformation. Without change in power, the Clarke's transformation of instantaneous voltages and currents are governed by the following equation

$$\begin{bmatrix} v_\alpha \\ v_\beta \\ v_0 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} v_r \\ v_y \\ v_b \end{bmatrix} \quad (21a)$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} i_r \\ i_y \\ i_b \end{bmatrix} \quad (21b)$$

The instantaneous real power ( $p$ ) and reactive power ( $q$ ) is calculated in the transformed domain as in equation (22).

$$p = v_\alpha i_\alpha + v_\beta i_\beta + v_0 i_0 \quad (22a)$$

$$q = v_\beta i_\alpha - v_\alpha i_\beta \quad (22b)$$

Also, it is observed that the instantaneous powers constitute of two components namely average and oscillating components as represented in equation (23).

$$p = \bar{p} + \tilde{p}; \quad q = \bar{q} + \tilde{q} \quad (23)$$

The components of power include both fundamental and harmonic powers. The average and oscillating components of power are separated from the computed power by a second order Butterworth low-pass filter with a cut-off frequency around twice the supply frequency.

The compensating currents for the SAPF are computed from (23) after separating the average and oscillating components. The currents are calculated as in (24).

$$\begin{bmatrix} i_{c\alpha}^* \\ i_{c\beta}^* \end{bmatrix} = \frac{1}{\sqrt{v_\alpha^2 + v_\beta^2}} \begin{bmatrix} v_\alpha & v_\beta \\ v_\beta & -v_\alpha \end{bmatrix} \begin{bmatrix} p_c^* \\ q_c^* \end{bmatrix} \quad (24)$$

The compensation currents calculated using equation 4 cannot be used directly and must be transformed back to the time domain using inverse Clarke's transformation dictated by (25).

$$\begin{bmatrix} i_{cr}^* \\ i_{cy}^* \\ i_{cb}^* \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 \\ -1/2 & \sqrt{3}/2 \\ -1/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} i_{c\alpha}^* \\ i_{c\beta}^* \end{bmatrix} \quad (25)$$

The currents computed using equation 5 is used as the reference current for the SAPF.

The compensation is possible only when the dc-link of the VSI is maintained constant. Hence to regulate the dc-link, considerable amount of active power is drawn from the grid and is achieved by a proportional integral (PI) controller. The dc link voltage is compared to a reference dc voltage ( $V_{DC}^*$ ) and the voltage error is processed by the PI controller and added to the reference active power component in (23).

#### V. SIMULATION OF THE PROPOSED CONTROL STRATEGY

The performance of the control strategy is to be evaluated for its control effort. Hence, the TLHSM controller with SAPF is simulated in MATLAB/Simulink environment and simulation results obtained are analyzed. The objective function is to track non-sinusoidal reference currents computed using instantaneous real and reactive power theory [10].

The parameters used for simulation are given in Table-I. The source voltage is assumed pure sinusoidal at the grid in the absence of nonlinear load.

Table-I. Simulation Parameters of the SAPF

Parameters	Values
<b>Source Parameters:</b>	
Source Voltage ( $V_s$ ):	440 V, RMS, 3 $\Phi$
Frequency ( $f_s$ ):	50 Hz
Line resistance ( $R_s$ ):	0.1 $\Omega$
Line inductance ( $L_s$ ):	12 mH
<b>Load Parameters:</b>	
Three Phase diode bridge rectifier:	R= 20 $\Omega$ L= 30mH
<b>Filter Parameters:</b>	
DC Link voltage ( $V_{dc}$ ):	700V
DC Link capacitor ( $C_{dc}$ ):	400 $\mu$ F
Interface Inductor ( $L_f$ ):	5 mH

The Simulink implementation of the system is shown in Fig. 4.

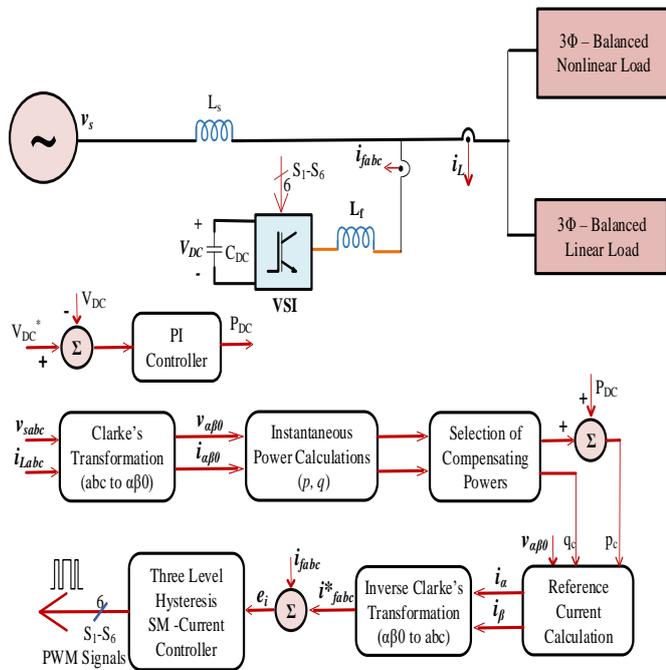


Fig4. Simulink Implementation of TLHSM Controlled SAPF

The sliding mode hysteresis function-based switching control for the switches in single leg of the inverter is shown in Fig. 5. The sliding surface expressed in the previous section is implemented to generate the gating pulses for the switches of the inverter.

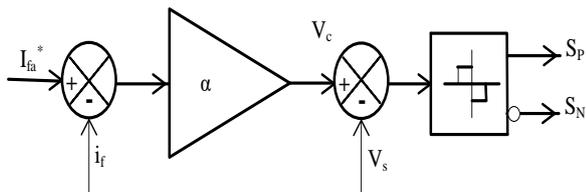


Fig. 5. Implementation of TLHSM Control for One Leg of Inverter

The simulation results of TLHSM controlled VSI for harmonic current compensation is shown in Figure 6. The waveforms of three phase source voltage ( $v_{sabc}$ ), source current ( $i_s$ ), filter injected currents ( $i_f$ ), and dc-link voltage ( $V_{DC}$ ). The nonlinear load considered is a diode bridge rectifier delivering RL type of load. The current drawn by the nonlinear load is non-sinusoidal which introduces harmonic voltages as it propagates into the distribution network. The source current is non-sinusoidal and is equal to nonlinear load current when the filter is switched 'OFF'. With filter switched 'ON' the source current is sinusoidal at fundamental frequency. The compensated source current is shown in Figure 6(b) and the nonlinear load current is illustrated in 6(c). The injected filter currents for compensation is shown in figure 6(d), and the dc-link voltage in figure 6(e). It is clear the compensation is achieved less than one cycle of source voltage.

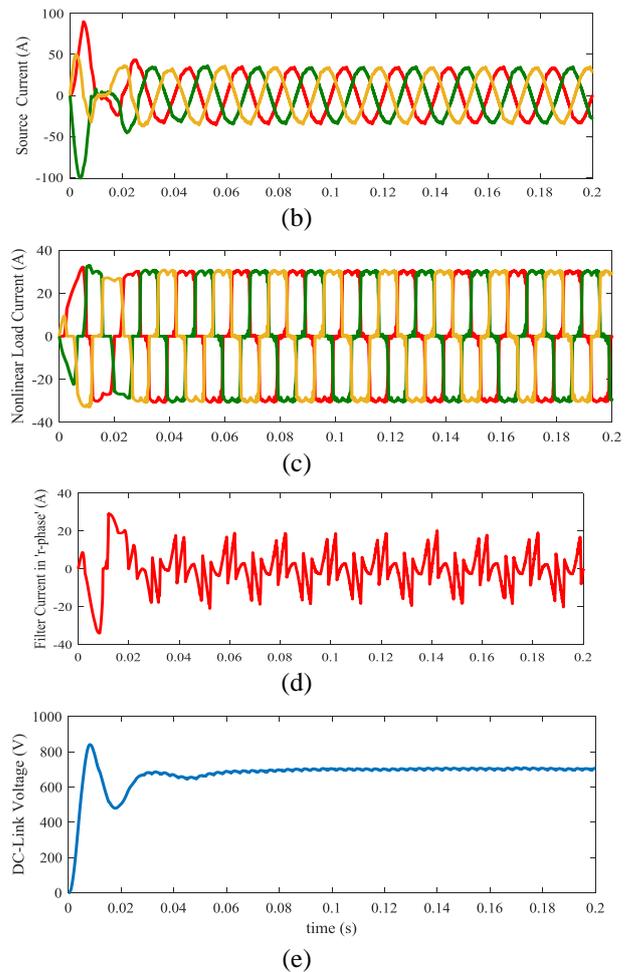
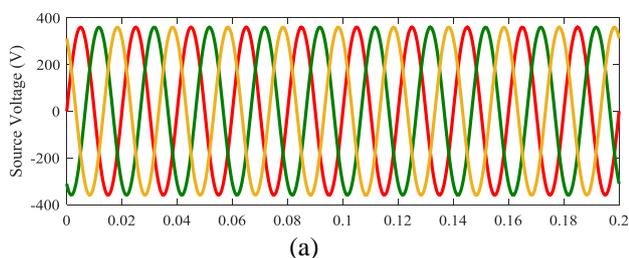
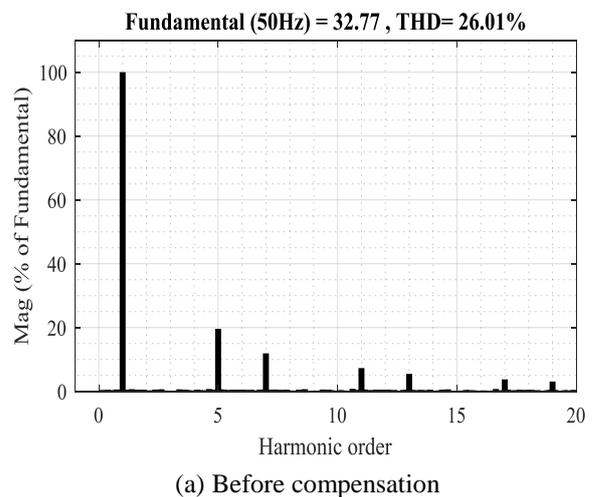
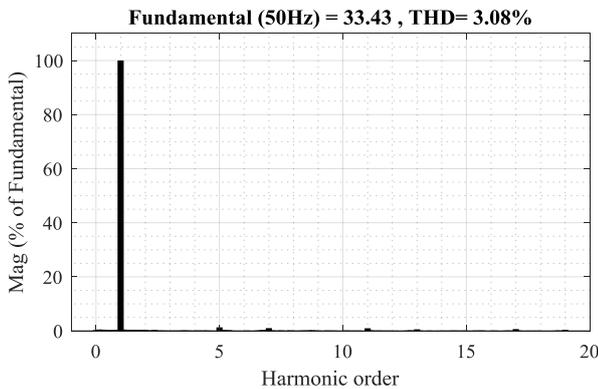


Fig. 6. Waveforms of (a) Source voltage (b) Source current (c) Nonlinear load current (d) Injected Filter Currents (e) DC-link Voltage



(a) Before compensation



(b) After compensation

**Figure 7. Harmonic spectrum of source current**

The harmonic spectrum of the source current is shown in Fig. 7. The THD of source current in the absence of filter is 26.01% with 5<sup>th</sup>, 7<sup>th</sup> and 11<sup>th</sup> order harmonics as dominant. With filter switched into the network, THD reduces to 3.08% and the dominant lower order harmonics are compensated by shunt active filter due to the control action of TLHSM controller. The performance of the TLHSM controller is satisfied in terms of tracking error. THD is less when the harmonic components of currents are suppressed. With shunt active power filter, the source current is sinusoidal and is in phase with the source voltage resulting in unity power factor at the grid.

The results of TLHSM control is compared with conventional two-level hysteresis control for the harmonic distortion levels in the source current and tabulated in Table-II.

Table II. Comparison of %THD and tracking error

Type of Control	% THD in $I_s$	Tracking error (mean)
Two level Hysteresis	5.68	0.0116
TLHSM	3.08	0.0056

## VI. CONCLUSION

In this paper, sliding mode control based on three-level hysteresis function is applied to capacitor powered three phase VSI to function as shunt active power filter. The control of objective of harmonic current regulation in source current is achieved with minimum THD in compliance with IEEE519:2014 recommendations. The controller is capable of tracking a non-sinusoidal current reference when applied. The current tracking is error free and is chattering free. This chattering free control results in unpredictable high switching frequencies in case of large errors. Compared with conventional two-level hysteresis control, the switching frequency is reduced to half in three level hysteresis function.

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