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Portfolio Optimization of the Mean-Absolute Deviation Model of Some Stocks using the Singular Covariance Matrix

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Abstract*: Investing in the stock sector, investors often face risk problems. Usually, forming an investment portfolio is done to minimize risk. In this research, investment portfolio optimization is discussed. The data analyzed are 8 shares traded on the capital market in Indonesia through the Indonesia Stock Exchange (IDX). Optimization is performed using the Mean-Absolute Deviation model with the singular covariance matrix to determine the optimal weights. The results of portfolio optimization Mean-Absolute Deviation model with singular covariance matrix method, was obtained optimal portfolio weights that is of 17.22% for BBCA shares; 26.64% for TKIM shares; 9.96% for BBRI shares; 9.96% for BBNI shares; 8.70% for BMRI shares; 3.75% for ADRO shares; 6.52% for GGRM shares; and 17.25% for UNTR shares. Where the optimal portfolio composition is obtained the expected rate of return (expected return) of 0.18% with a portfolio risk level (standard deviation) of 0.07%.

***Index Terms*: Return, Risk, Mean-Standard Deviation model, singular covariance matrix method, surface efficient.**

I. INTRODUCTION

Investment is to invest several capitals in the form of shares or assets to carry out business activities to get the expected profit in the future [1]. In general, investing in securities can be done through the money market or the capital market [2] [3]. Generally, investors will consider the rate of return on investment [4] [5]. Stocks that provide higher investment return expectations will be chosen by investors [6]. Therefore, every investment has a risk, investors should not only consider high returns [7] [8]. In investing, every investor has a risk tolerance under their respective preferences [9] [10]. The formation of a portfolio generally will provide optimum results, i.e. provide maximum returns on certain risks, or provide certain returns on minimum risks. Therefore an investment portfolio optimization analysis needs to be carried out [11].

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Optimization of the Mean-Variance model portfolio has been widely studied by previous researchers. For example in the study of Stempien and Chan [6] expanded the mean-variance model by forming in a portfolio, to solve the problems of the trilemma energy system. to obtain policy packages that support the possibility of economic growth. Besides, in the research of Santos et al. [4], determined the optimal repeatability alternative using the theory of mean-variance Markowitz portfolio optimization on a combination of the generation of fields in Spain. Similar studies on Tayaki and Tolun [12], investigate the effects of portfolio optimization using the mean-variance model by reducing non-negative dimensions. The development of the Mean-variance model has been widely carried out by previous researchers. As in the study of Grechuk and Zabarankin [13], choosing a reverse portfolio, by determining the amount of deviation on observations with the mean-deviation model to determine the optimal portfolio owned by an investor. As well as in Li's research [14], optimizing the portfolio of stock data of Chinese small cap using the asymmetry robust mean absolute deviation model. Also in Qin's study [9], modeling portfolio selection under conditions of hybrid uncertainty, using fuzzy random variables to describe stochastic returns using the mean-absolute deviation model.

In the portfolio optimization process, a covariance matrix is formed which is obtained from the estimated covariance of each share [10]. Next, the inverse of the covariance matrix is determined. A matrix has an inverse if nonsingular (the determinant of the matrix is not equal to zero) [15]. Whereas for singular matrices (the determinant of the matrix is zero) cannot be determined using ordinary inverses [16]. However, Pseudo inverse can provide a solution approach of determining the inverse of a singular matrix [17] [18] [19].

Based on the description above, this paper discusses the optimization of the mean-standard deviation of some stocks using the singular covariance matrix method. The object analyzed is several shares traded on the capital market in Indonesia. The goal is to obtain an allocation of investment portfolio weights, so can obtain the optimum return and risk of investment portfolios.

II. MATERIAL AND METHOD

This section discusses the materials and methods used in the study, as follows.

A. Material

The data used in this study are shares traded in Indonesia. The stock data was obtained from https://finance.yahoo.com for the period from October 2015 to October 2018. The data used consisted of 8 (eight) shares in the form of BBCA, TKIM, BBNI, BBRI, BMRI, ADRO, GGRM and UNTR.

B. Method

The model used in stock portfolio optimization is the mean-standard deviation, using a singular covariance matrix. Discussions include stock returns, singular covariance matrices, and mean-standard deviation models.

III. MATHEMATICAL MODELS

A. Stock Returns

Return on Assets is a reward from the results of investment activities carried out. Usually expressed as a percentage of the initial investment price [20] [21]. Mathematically it can be stated as follows:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{1}$$

where R_t declares stock returns in period t, P_t states stock prices in period t, and $P_{(t-1)}$ states stock prices in period t-1. Next, the expected return of a stock is mathematically defined as follows:

$$\mu_i = E[R_i] = \int_{-\infty}^{\infty} r_i f(r_i) dr_i \tag{2}$$

Determination of stock variance can be obtained from the root of the variance. The variance of a stock is mathematically defined as follows:

$$\sigma_i^2 = E\left[(R_i - \mu_i)^2\right] = \int_{-\infty}^{\infty} (r_i - \mu_i)^2 f(r_i) dr_i \tag{3}$$

The estimated value of covariance between shares i and j is expressed as σ_{ij} . Mathematically, is defined as follows:

$$\sigma_{ij} = E\left[(R_i - \mu_i)\ (R_j - \mu_j)\right] = \rho_{ij}\sigma_i\sigma_j \tag{4}$$

B. Portfolio formation

From the expected return of each stock, the average vector is formed as follows:

$$\boldsymbol{\mu}^T = (\mu_1, \mu_2, \cdots, \mu_N).$$

Since N represents the number of shares in the portfolio, so the unit e element vector consists of number 1 as many as N, it can be stated:

$$\mathbf{e}^T = (1,1,\cdots,1)$$

From the estimation results of variance and covariance between shares, a variance-covariance matrix is formed, organized as follows:

$$\mathbf{M} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_N^2 \end{pmatrix}$$

With a stock weight vector

$$\mathbf{w}^T = (w_1, w_2, \cdots, w_N)$$

If w_i states the proportion (weight) of funds invested in stock i, then return vector weighting vector $\mathbf{w}$, then the return of the portfolio R_p is given as follows:

$$R_p = \sum_{i=1}^{N} w_i R_i \tag{5}$$

Based on equation (5), the average (expected return) of a portfolio is given as follows:

$$\mu_p = E[R_p] = \sum_{i=1}^{N} w_i E[R_i] = \sum_{i=1}^{N} w_i \mu_i = \mathbf{w}^T \boldsymbol{\mu} \tag{6}$$

Based on equation (5) obtained portfolio return variance, given as follows:

$$\sigma_p^2 = E\left[\left(R_p - E[R_p]\right)^2\right] = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} = \mathbf{w}^\mathbf{T}\mathbf{M}\mathbf{w} \tag{7}$$

Portfolio return expectations indicate how much investors will benefit in the future from their investment returns. While the portfolio risk counter can be done by determining the standard deviation obtained from the roots of the portfolio return variance.

C. Pseudo invers

In general, a square matrix of size $n \times n$ and nonsingular has an inverse. But it is different for matrices of size $m \times n$ or $n \times n$ singular which do not have an inverse. But it can be done by generalizing from a singular inverse matrix $m \times n$ or $n \times n$ [5] [7]. Generalization of singular matrix inverse can be done using the pseudo inverse method.

Definition 1 [8]

If A matrix $n \times m$ is a real or complex number, there is a unique $\mathbf{M}^\dagger$ matrix $m \times n$, then $\mathbf{M}^\dagger$ is a pseudo inverse of the matrix $\mathbf{M}$ if it meets:

1) $\mathbf{M}\mathbf{M}^\dagger\mathbf{M} = \mathbf{M}$ *3)* $\left(\mathbf{M}\mathbf{M}^\dagger\right)^T = \mathbf{M}\mathbf{M}^\dagger$

2) $\mathbf{M}^\dagger\mathbf{M}\mathbf{M}^\dagger = \mathbf{M}^\dagger$ *4)* $\left(\mathbf{M}^\dagger\mathbf{M}\right)^\mathbf{T} = \mathbf{M}^\dagger\mathbf{M}$

D. Optimization of the Mean-Standard Deviation Model Portfolio

The problem of optimizing investment portfolios by maximizing return expectations using the Mean-Standard Deviation model, and considering the risk tolerance provided by investors is formulated as follows:

$$\max\{2\tau\mu_p - \sigma_p\}$$

$$\text{Obstacles} \quad \sum_{i=1}^{N} w_i = 1$$

or in the form of vector-matrix

$$\max\{2\tau\mathbf{w}^T\boldsymbol{\mu} - (\mathbf{w}^T\mathbf{M}\mathbf{w})^{1/2}\}$$

$$\text{Obstacles} \quad \mathbf{w}^T\mathbf{e} = 1$$

where τ is the risk tolerance given by the investor.

The Lagrange multiplier function of the above portfolio optimization issues is given as follows:

$$L = 2\tau\mathbf{w}^T\boldsymbol{\mu} - (\mathbf{w}^T\mathbf{M}\mathbf{w})^{1/2} + \lambda(\mathbf{w}^T\mathbf{e} - 1)$$

$$\frac{\partial L}{\partial w} = 2\tau\boldsymbol{\mu} - \frac{1}{2}2\frac{\mathbf{M}\mathbf{w}}{(\mathbf{w}^T\mathbf{M}\mathbf{w})^{1/2}} + \lambda\mathbf{e} = 0 \tag{8}$$

$$\frac{\partial L}{\partial \lambda} = \mathbf{w}^T\mathbf{e} - 1 = 0$$

(9)

Based on equation (8) is obtained

$$\frac{\mathbf{Mw}}{(\mathbf{w}^{T}\mathbf{Mw})^{1/2}} = 2\tau\boldsymbol{\mu} + \lambda\mathbf{e} \qquad (10)$$

Equation (10) the two segments multiplied by $\mathbf{M}^{\dagger}$, are obtained

$$\frac{\mathbf{w}}{(\mathbf{w}^{T}\mathbf{Mw})^{1/2}} = 2\tau\mathbf{M}^{\dagger}\boldsymbol{\mu} + \lambda\mathbf{M}^{\dagger}\mathbf{e} \qquad (11)$$

Based on equation (11) the two segments times $\mathbf{e}^{T}$, are obtained:

$$\frac{\mathbf{we}^{T}}{(\mathbf{w}^{T}\mathbf{Mw})^{1/2}} = 2\tau\mathbf{e}^{T}\mathbf{M}^{\dagger}\boldsymbol{\mu} + \lambda\mathbf{e}^{T}\mathbf{M}^{\dagger}\mathbf{e}$$

$$\frac{1}{(\mathbf{w}^{T}\mathbf{Mw})^{1/2}} = 2\tau\mathbf{e}^{T}\mathbf{M}^{\dagger}\boldsymbol{\mu} + \lambda\mathbf{e}^{T}\mathbf{M}^{\dagger}\mathbf{e} \qquad (12)$$

Equation (12) substitution to equation (11), obtained weighting vector as follows:

$$\frac{\mathbf{w}}{(\mathbf{w}^{T}\mathbf{Mw})^{1/2}} \times \frac{(\mathbf{w}^{T}\mathbf{Mw})^{1/2}}{1} = \frac{2\tau\mathbf{M}^{\dagger}\boldsymbol{\mu} + \lambda\mathbf{M}^{\dagger}\mathbf{e}}{2\tau\mathbf{e}^{T}\mathbf{M}^{\dagger}\boldsymbol{\mu} + \lambda\mathbf{e}^{T}\mathbf{M}^{\dagger}\mathbf{e}}$$

$$\mathbf{w} = \frac{2\tau\mathbf{M}^{\dagger}\boldsymbol{\mu} + \lambda\mathbf{M}^{\dagger}\mathbf{e}}{2\tau\mathbf{e}^{T}\mathbf{M}^{\dagger}\boldsymbol{\mu} + \lambda\mathbf{e}^{T}\mathbf{M}^{\dagger}\mathbf{e}} \qquad (13)$$

Equation (10) multiplied by $\mathbf{w}^{T}$, obtained:

$$\frac{\mathbf{w}^{T}\mathbf{Mw}}{(\mathbf{w}^{T}\mathbf{Mw})^{1/2}} = 2\tau\mathbf{w}^{T}\boldsymbol{\mu} + \lambda\mathbf{w}^{T}\mathbf{e}$$

$$(\mathbf{w}^{T}\mathbf{Mw})^{1/2} = 2\tau\boldsymbol{\mu}^{T}\mathbf{w} + \lambda \qquad (14)$$

Equations (12) and (13) substitute to equation (14), the equation is obtained as follows:

$$\frac{1}{2\tau\mathbf{e}^{T}\mathbf{M}^{\dagger}\boldsymbol{\mu} + \lambda\mathbf{e}^{T}\mathbf{M}^{\dagger}\mathbf{e}} = 2\tau\boldsymbol{\mu}^{T}\mathbf{w} + \lambda$$

$$\frac{1}{2\tau\mathbf{e}^{T}\mathbf{M}^{\dagger}\boldsymbol{\mu} + \lambda\mathbf{e}^{T}\mathbf{M}^{\dagger}\mathbf{e}} = 2\tau\boldsymbol{\mu}^{T}\frac{2\tau\mathbf{M}^{\dagger}\boldsymbol{\mu} + \lambda\mathbf{M}^{\dagger}\mathbf{e}}{2\tau\mathbf{e}^{T}\mathbf{M}^{\dagger}\boldsymbol{\mu} + \lambda\mathbf{e}^{T}\mathbf{M}^{\dagger}\mathbf{e}} + \lambda$$

$$\frac{1}{2\tau\mathbf{e}^{T}\mathbf{M}^{\dagger}\boldsymbol{\mu} + \lambda\mathbf{e}^{T}\mathbf{M}^{\dagger}\mathbf{e}} = 2\tau\boldsymbol{\mu}^{T}\frac{2\tau\mathbf{M}^{\dagger}\boldsymbol{\mu} + \lambda\mathbf{M}^{\dagger}\mathbf{e}}{2\tau\mathbf{e}^{T}\mathbf{M}^{\dagger}\boldsymbol{\mu} + \lambda\mathbf{e}^{T}\mathbf{M}^{\dagger}\mathbf{e}} + \frac{\lambda(2\tau\mathbf{e}^{T}\mathbf{M}^{\dagger}\boldsymbol{\mu} + \lambda\mathbf{e}^{T}\mathbf{M}^{\dagger}\mathbf{e})}{2\tau\mathbf{e}^{T}\mathbf{M}^{\dagger}\boldsymbol{\mu} + \lambda\mathbf{e}^{T}\mathbf{M}^{\dagger}\mathbf{e}}$$

$$1 = 2\tau\boldsymbol{\mu}^{T}\left(2\tau\mathbf{M}^{\dagger}\boldsymbol{\mu} + \lambda\mathbf{M}^{\dagger}\mathbf{e}\right) + \lambda 2\tau\mathbf{e}^{T}\mathbf{M}^{\dagger}\boldsymbol{\mu} + \lambda^{2}\mathbf{e}^{T}\mathbf{M}^{\dagger}\mathbf{e}$$

$$1 = 4\tau^{2}\boldsymbol{\mu}^{T}\mathbf{M}^{\dagger}\boldsymbol{\mu} + 2\tau\boldsymbol{\mu}^{T}\lambda\mathbf{M}^{\dagger}\mathbf{e} + \lambda 2\tau\mathbf{e}^{T}\mathbf{M}^{\dagger}\boldsymbol{\mu} + \lambda^{2}\mathbf{e}^{T}\mathbf{M}^{\dagger}\mathbf{e}$$

$$4\tau^{2}\boldsymbol{\mu}^{T}\mathbf{M}^{\dagger}\boldsymbol{\mu} - 1 + \lambda(2\tau\boldsymbol{\mu}^{T}\mathbf{M}^{\dagger}\mathbf{e} + 2\tau\mathbf{e}^{T}\mathbf{M}^{\dagger}\boldsymbol{\mu}) + \lambda^{2}\mathbf{e}^{T}\mathbf{M}^{\dagger}\mathbf{e} = 0$$

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a};\ \text{for } \lambda > 0 \qquad (15)$$

with

$a = \mathbf{e}^{T}\mathbf{M}^{\dagger}\mathbf{e}$, $b = (2\tau\boldsymbol{\mu}^{T}\mathbf{M}^{\dagger}\mathbf{e} + 2\tau\mathbf{e}^{T}\mathbf{M}^{\dagger}\boldsymbol{\mu})$ and

$c = 4\tau^{2}\boldsymbol{\mu}^{T}\mathbf{M}^{\dagger}\boldsymbol{\mu} - 1$

$\mathbf{M}^{\dagger}$ is the Pseudo inverse of the matrix $\mathbf{M}$.

IV. RESULTS AND DISCUSSION

This section is intended to show how to apply the model numerically from the results of formulations, in analyzing stock data formed in portfolios. There are eight stocks compiled in the portfolio: BBCA, TKIM, BBNI, BBRI, BMRI, ADRO, GGRM and UNTR.

From the estimation of stock data, the expected return data for each stock is compiled in the average transpose vector $\boldsymbol{\mu}^{T}$. As for the results of the data covariance estimator for each share, the **M** covariance matrix is formed in a row arranged as follows:

$$\boldsymbol{\mu}^{T} = (0.0009214\ 0.004454\ 0.0008002\ 0.0007747\ 0.0006919\ 0.001660\ 0.0007055\ 0.0008675)$$

$$\mathbf{M} = \begin{bmatrix} 0.00016 & -0.00044 & 0.00024 & 0.00024 & 0.00024 & -0.00038 & -0.00025 & 0.00030 \\ -0.00044 & 0.00122 & -0.00066 & -0.00066 & -0.00065 & 0.00104 & 0.00068 & -0.00083 \\ 0.00024 & -0.00066 & 0.00036 & 0.00036 & 0.00035 & -0.00056 & -0.00037 & 0.00045 \\ 0.00024 & -0.00066 & 0.00036 & 0.00036 & 0.00035 & -0.00056 & -0.00037 & 0.00045 \\ 0.00024 & -0.00065 & 0.00035 & 0.00035 & 0.00035 & -0.00055 & -0.00036 & 0.00044 \\ -0.00038 & 0.00104 & -0.00056 & -0.00056 & -0.00055 & 0.00088 & 0.00058 & -0.00071 \\ -0.00025 & 0.00068 & -0.00037 & -0.00037 & -0.00036 & 0.00058 & 0.00038 & -0.00046 \\ 0.00030 & -0.00083 & 0.00045 & 0.00045 & 0.00044 & -0.00071 & -0.00046 & 0.00057 \end{bmatrix}$$

From the calculation, the determinant of the covariance matrix is zero (a), so the covariance matrix is singular. Determination of the inverse of the singular covariance matrix uses the Pseudo Inverse method. The determination of the Pseudo inverse of a singular covariance matrix satisfies the four conditions in Definition 1. The inverse of the singular covariance matrix M is given as follows:

$$\mathbf{M}^{\dagger} = 10^{5} \times \begin{bmatrix} 0.50 & 1.50 & -0.25 & -0.25 & 0.50 & -0.50 & -1.00 & 0.50 \\ 1.50 & 1.50 & 0.25 & 0.25 & 0.00 & 0.00 & 0.00 & 1.00 \\ -0.25 & 0.25 & 0.63 & 0.63 & 0.25 & 0.75 & -0.00 & 0.25 \\ -0.25 & 0.25 & 0.63 & 0.63 & 0.25 & 0.75 & 1.00 & 0.25 \\ 0.50 & -0.00 & 0.25 & 0.25 & 1.00 & -0.50 & 1.00 & -0.00 \\ -0.50 & -0.00 & 0.75 & 0.75 & 0.50 & -0.00 & -1.00 & -0.50 \\ -1.00 & 0.00 & -0.00 & -0.00 & 1.00 & 1.00 & -0.00 & 1.00 \\ 0.50 & 1.00 & 0.25 & 0.25 & 0.00 & -0.50 & 1.00 & 1.00 \end{bmatrix}$$

Because the formation of the portfolio consists of eight stocks to be analyzed, the unit element vector is defined as $\mathbf{e}^{T} = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]$.

The portfolio optimization process uses the unit vector e, the average vector n and the inverse matrix covariance m. The portfolio optimization process is carried out using the Mean-Standard Deviation model. In determining portfolio weights using equation (13). Software *Matlab R2010a* is used in the process of numerical analysis on investment portfolio optimization. For the optimization process of the investment portfolio Mean-Standard Deviation, risk tolerance factors are considered by investors. The results of numerical analysis in the portfolio optimization process of the Mean-Standard Deviation model using the singular covariance matrix method are given in **Table 1**.

TABLE 1. Results of the Portfolio Optimization Process using the Singular Covariance Matrix Method

τ	Weights (w_i) for stocks								μ_P	σ_P	Ratio $= \frac{\mu_P}{\sigma_P}$
	BBCA	TKIM	BBRI	BBNI	BMRI	ADRO	GGRM	UNTR			
0,00	0.046512	0.209302	0.116279	0.116279	0.162791	0.093023	0.093023	0.162791	0.001632	0.000682	2.3932
0,02	0.058412	0.214706	0.114702	0.114702	0.155611	0.087766	0.090386	0.163716	0.00165	0.000682	2.4180
0,04	0.07035	0.220127	0.113119	0.113119	0.148408	0.082493	0.08774	0.164644	0.001668	0.000683	2.4403
0,06	0.082363	0.225581	0.111527	0.111527	0.14116	0.077186	0.085078	0.165578	0.001686	0.000685	2.4601
0,08	0.09449	0.231088	0.109919	0.109919	0.133843	0.071829	0.082391	0.166521	0.001704	0.000688	2.4774
0,10	0.106773	0.236666	0.108291	0.108291	0.126432	0.066403	0.079669	0.167476	0.001722	0.000691	2.4921
0,12	0.119255	0.242333	0.106636	0.106636	0.118902	0.060889	0.076902	0.168447	0.001741	0.000695	2.5043
0,14	0.131984	0.248113	0.104949	0.104949	0.111222	0.055266	0.074082	0.169436	0.00176	0.000700	2.5138
0,16	0.14501	0.254028	0.103222	0.103222	0.103363	0.049512	0.071195	0.170449	0.001779	0.000706	2.5205
0,18	0.15839	0.260103	0.101448	0.101448	0.09529	0.043601	0.06823	0.17149	0.001799	0.000713	2.5245
0,20	**0.172189**	**0.266369**	**0.099619**	**0.099619**	**0.086965**	**0.037505**	**0.065172**	**0.172563**	**0.00182**	**0.000721**	**2.5257**
0,22	0.186479	0.272858	0.097725	0.097725	0.078343	0.031192	0.062005	0.173674	0.001841	0.000730	2.5239
0,24	0.201344	0.279608	0.095754	0.095754	0.069374	0.024626	0.058711	0.174829	0.001864	0.000740	2.5191
0,26	0.216881	0.286663	0.093694	0.093694	0.06	0.017762	0.055267	0.176038	0.001887	0.000751	2.5110
0,28	0.233207	0.294076	0.09153	0.09153	0.05015	0.01055	0.05165	0.177307	0.001911	0.000765	2.4996
0,30	0.250458	0.301909	0.089243	0.089243	0.039742	0.00293	0.047827	0.178648	0.001937	0.000780	2.4847
0,31	0.259481	0.306006	0.088047	0.088047	0.034298	-0.00106	0.045827	0.17935	0.001951	0.000788	2.4759

Taking into account the results of **Table 1**, it can be seen that the value of risk tolerance is in the range of $0 \le \tau \le 0.30$ and the magnitude of the increase in risk tolerance is 0.02. This is because the value of risk tolerance $\tau > 0.30$ results in a negative portfolio weight. The investment portfolio optimization process is presented in the form of a mean-standard deviation efficient portfolio graph as shown in Figure 1. The graph of the ratio between the average and standard deviation of the portfolio given by Figure 2.

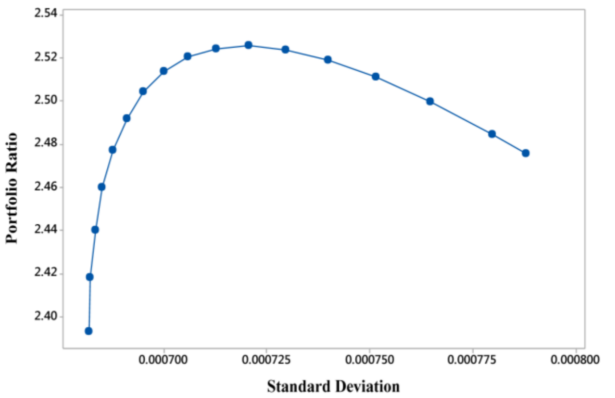

Figure 1 Graph of Mean Standard Deviation Efficient Portfolio

Based on Figure 1, the results show that efficient portfolios are located along the efficient surface line with risk tolerance in the range $0 \le \tau \le 0.30$. In Figure 1, the magnitude of the increase in portfolio return expectations is followed by an increase in the standard deviation. This also obtained the expectation of a maximum portfolio return of 0.001937 with the magnitude of the standard deviation (risk) of 0.000780. While the expected minimum portfolio returns of 0.001632 with the magnitude of standard deviation (risk) is 0.000682.

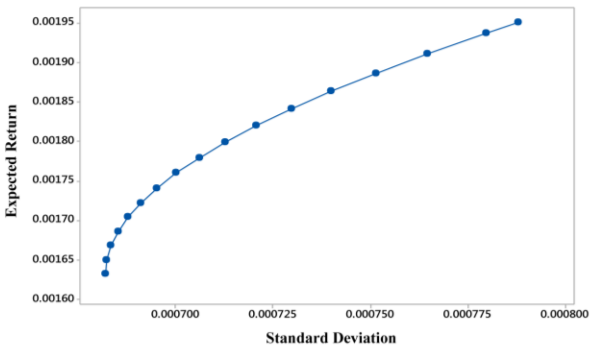

Figure 2 Graph the ratio between average and standard deviation of portfolio

In Figure 2 and Table 1, it is seen that the ratio between the expected return and the largest standard deviation of the portfolio is 2.5257 or is obtained when the risk tolerance = 0.20. Portfolio ratios continue to increase in the range of risk tolerance intervals of $0 \le \tau \le 0.20$ and have decreased in the range of risk tolerance intervals $0.20 \le \tau \le 0.30$. The optimal portfolio weight composition, given by Figure 3.

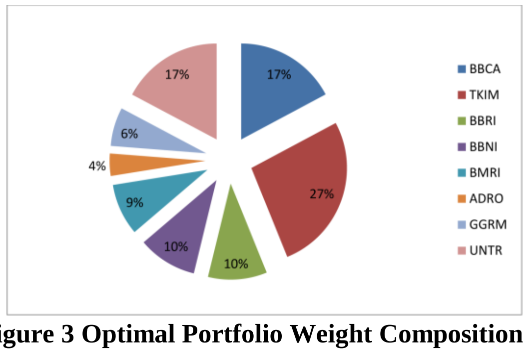

Figure 3 Optimal Portfolio Weight Composition

Based on Figure 3, the optimal portfolio composition of 8 (eight) shares that is 17.22% for BBCA shares; 26.64% for TKIM shares; 9.96% for BBRI shares; 9.96% for BBNI shares; 8.70% for BMRI shares; 3.75% for ADRO shares; 6.52% for GGRM shares; and 17.25% for UNTR shares. Where the optimal portfolio composition is obtained the expected rate of return (expected return) of 0.18% with a portfolio risk level (standard deviation) of 0.07%.

V. CONCLUSION

The optimal solution of the investment portfolio of the mean-standard deviation model with risk tolerance is expressed in the form of a weighting vector. Based on the analysis of eight stocks, the composition of the optimal global portfolio weight obtained is BBCA stock shares amounted to 0.172189, TKIM shares amounted to 0.266369, BBRI shares amounted to 0.099619, BBNI shares amounted to 0.099619, BMRI shares amounted to 0.086965, ADRO shares amounted to 0.037505, GGRM shares amounted to 0.065172 and UNTR shares amounted to 0.172563. The amount of expected return obtained by 0.00182 and portfolio standard deviation (risk) of 0.000721. The optimal portfolio is reached when the portfolio ratio is 2.5257 with a risk tolerance of 0.20. It has the optimal global portfolio with the largest ratio of mean to risk.

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