

Semigraphs and Goldbach Conjecture



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Abstract: It is interesting to learn pure mathematics through graphs as graphs makes the study simple and easier to understand. We have the proof of Goldbach conjecture using graphs and hypergraphs.[1,2]. The author is motivated to study the proof of the conjecture using semigraphs. In this paper, using semigraphs, we discuss the Goldbach statement A number which is both even and not less than 2 can be written as a sum of 2 numbers which are not composites and we also show that every composite number can be expressed as sum of 3 or more primes.

IndexTerms—PVSEWSG, UTSPVEWSG/STSPVEWSG, CPVEWSG, ETEWE

I. INTRODUCTION

The new concept Semigraphs, the general version of graphs was coined by E.Sampathkumar. They are powerful working models than ordinary graphs in all the applied areas where an edge connects several points in the graph, in place of an edge joining only two vertices. The sequence in which the points occur in an edge is the most vital thing in semigraphs.[4] In the second section, we discuss the preliminaries on semigraphs,[3] In the last two sections, we learn new constructions on semigraphs and we see the Goldbach conjecture and also the expressions of composite numbers in terms of primes using semigraphs.

II. DEFINITION

A. Semigraph

A Semigraph G is an ordered pair (V, X) where the first element of the ordered pair namely the point set or vertex set is not an empty set whose elements are called as points/vertices/bullets of G and the second element of the ordered pair namely the line set or edge set is a set of n tuples called as lines/edges of G . These lines of distinct points/vertices, for various tuples $(n \geq 2)$ obeys the conditions given below:

i) Every two edges have a maximum of one vertex in common ii) Two edges (u_1, u_2, \dots, u_n) and (v_1, v_2, \dots, v_m) are considered to be equal if a) $m = n$ and b) either $u_i = v_i$ for $i = 1, 2, \dots, n$ or $u_i = v_{n+1-i}$ for $i = 1, 2, \dots, n$. For the edge $E = (u_1, u_2, \dots, u_n)$ u_1 and u_n are the end vertices and are the u_2, \dots, u_{n-1} are the middle end vertices of E . [4]

A. Neighbouring points/vertices

Two vertices in a semigraph are said to be adjacent/neighbours if they are of the same edge and are said to be successively adjacent if in addition, in sequence also they are successive.

B. Size

Size or order or cardinal number of a line/an edge E in a semigraph G is the exact number of points belonging to that edge.

C. Neighbouring lines/edges

Any two edges in a semigraph are said to be neighbouring edges if they have a point/vertex in common.

D. Non full line/edge

Non full edge or a part of an edge or nf-edge of E is a $k-j+1$ tuple $E'' = (v_{ij}, v_{ij+1}, \dots, v_{ik})$ where $1 \leq j < k \leq n$

E. Complete line/edge

Any edge of a semigraph is called complete line/edge or c-edge.

F. c/nf-line/edge

c/nf-line/edge is a line which is either an nf-edge or a c edge.

G. Uniform vertex semigraph

A semigraph is said to be uniform vertex semigraph if all its vertices are not of different degrees.

H. Edge completeness semigraph

A semigraph is said to be edge completeness if there is an edge between every pair of vertices. The edge completeness semigraph of n vertices is denoted by K_{sn} .

I. Symbol or weight graph

A numbered graph corresponds a label/symbol/weight with every line in the graph. Weights are mostly real quantities. They may be restricted to rational numbers or integers.

III. CONSTRUCTION

A. Non composite point line labelled semigraph

Let $SG(V, E)$ be a semigraph having at least one edge which should be pendant, definitely not containing edges that are parallel. Let $V = \{V_1, V_2, \dots, V_n\}$ be the vertex set and $E = \{E_1, E_2, \dots, E_n\}$ be the edge set. Let $P_1 < P_2 < P_3 < \dots < P_n$ for $n \geq 5$ be consecutive primes each ≥ 13 . Attach the primes $P_1 < P_2 < P_3 < \dots < P_n$ for $n \geq 5$ with the vertices V_1, V_2, \dots, V_n of the semigraph SG called as prime weight of the vertices. Considering W_1, W_2, \dots, W_n for the edges E_1, E_2, \dots, E_n , where the weights are defined as sum of two or more primes vertices.

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Semigraphs and Goldbach Conjecture

We construct a new graph with a set containing only prime vertices, VSGP and the labelled/numbered/weighted edge sets ESGP and the semigraph drawn this way is called as non composite or prime point/vertex line/edge labelled semigraph shown in figure 1.

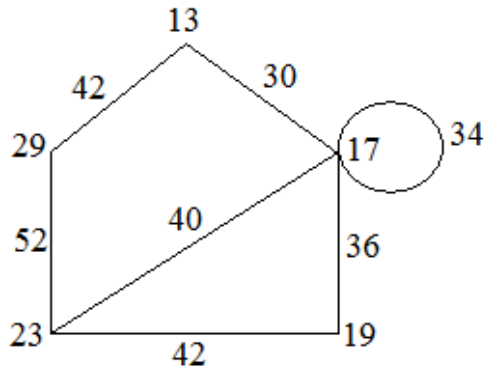


Fig.1

$V = \{13, 17, 19, 23, 29\}$
 $ESGP = \{30 \text{ to } 114\}$

Table-I:

Sum of primes	Total
13+17	30
17+17	34
17+19	36
17+23	40
19+23	42
13+17+19	49
13+17+23	63
13+23+29	75
17+17+23	57
17+17+29	63
17+19+29	65
13+17+19+23	72
13+17+17+19	66
17+17+19+23	76
17+19+23+29	82
17+17+23+29	86
17+17+19+23+29	105
13+17+17+19+23+29	114

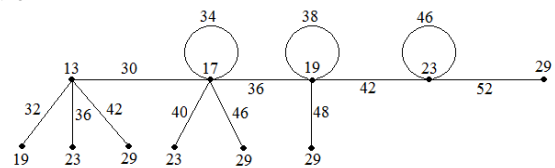
We have Goldbach conjecture for the even numbers 30,34,36,40,42,52 through 2 edge semigraphs. For 3,4,5 edge semigraphs we have some composite numbers from 49 to 114 as sum of 3 or 4 or 5 primes. We express r even numbers between 30 and 52 as a sum of 2 primes and also composite numbers between 30 and 114 as a sum of 3 or more primes through the construction of new semigraphs.

B. Unique type non composite point and line labelled semigraph

Let $SG(V,E)$ be a semigraph with loops which are self and points that are pendant. We obtain a graph PVEWSG from $SG(V,E)$ called as the Special Type Prime Vertex Edge Weighted Semigraph for the non composites $P_1 < P_2 < P_3 < P_4 < P_5$ for $n \geq 5$ consecutive primes with the following conditions

1. The graph has self loops that are $n-2$ in number at its centre vertices for $n \geq 5$ successive non composites implying that there are labelled edges that are $n-2$ in number obtained from the total of non composite weight.
2. There are $N-(n-1)$ vertices that are pendant in number in which N is the number of points of the graph obtained from the join of non composite vertices equals the point set V of the graph. The graph is denoted by STPVEWSG/UTPVEWSG.

In figure 2, with 2 edge semigraphs, all even numbers between 30 and 52 except 44,50 are expressed as sum of 2 primes and with 3 or more edge semigraphs, some composite numbers as sum of three or more primes
 Figure 2



C. Complete Prime Vertex Edge Weighted Semigraph:

A PVEWSG is called as CPVEWSG if it can be constructed from a complete semigraph $SG(V,E)$. It is written as CPVEWSG.

For the consecutive primes $P_1 < P_2 < P_3 < P_4 < P_5$, each ≥ 13 and $n \geq 5$, there exist a Complete Prime Vertex Edge Weighted Semigraph

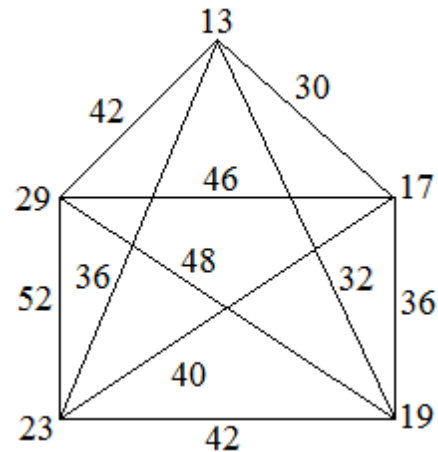
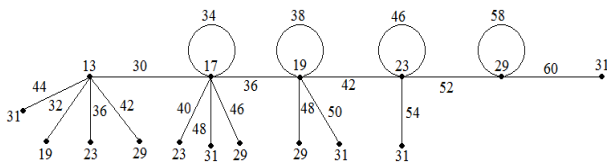


Fig.2

The weighted edges which lie in STPVEWSG, but does not lie in the CPVEWSG, denotes the set of all consecutive weighted edge which are not odd are called as exception type weighted edges which are not odd and this collection is written as ETEWE. The elements of the set ETEWE can be obtained from STPVEWSG/UTPVEWSG.



in the above graph we have exception type even edges 44 and 50 as sum of two primes.

We have Goldbach conjecture for all even numbers between 30 and 52 as sum of two primes and all composite numbers between 30 and 114 can be expressed as sum of three or more primes.

Thus we conclude that all consecutive weighted edges equals consecutive even numbers and all composite numbers ≥ 30 can be obtained from union of even weighted edges of the CPVEWSG and the elements of the set ETEWE.

IV. CONCLUSION:

The study done in this paper will help us to work on an important application of Goldbach conjecture that is the banking system. We can also explore number theory concepts like definitions, theorems, conjecture through different forms of graphs. We can make a comparative study of number theory through various forms of graphs. We can work on Goldbach partitions and sequences.[4]

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AUTHORS PROFILE



Hanumesh A.G is working as an Assistant Professor in SJC Institute of Technology, Chikkaballapur. He has done M.Sc., M.Phil and is pursuing Ph.D. He has published 3 papers on semigraphs in the reputed journals. He is having 24 years of teaching experience.



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