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'River Water DO Mechanics & New DO-Sag Equation'

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Nomenclature-

Following terms have been used to define DO-Sag Equation by alternative algorithm

a) DO_{Ref} – Reference DO Model made by Streeter Phelps equation
b) DO_{New}- DO Model made using new equation (quadratic polynomial)
c) De-oxidation Coefficient – k_1
d) Re-oxygenation Coefficient – k_2
e) A- Re-aeration Coefficient in DO_{New} Model

Abstract: ***DO modeling by Streeter Phelps equation [1] is most popular method to determine the water quality of a River. To compute DO by Streeter Phelps equation River coefficients k_1 and k_2 (de-oxidation and re-oxygenation) are required. Determination of these coefficients is tedious because it requires field observation of river velocity and depth over a long period of time at river site. To avoid maximum field work in calculating DO of River water DO Modeling approach is developed by combining Lab analysis of water samples DO with field data, e.g. river velocity and depth. Streeter Phelps (1925) developed the 1st important water quality model describing the BOD-DO relationship in a stream. In their pioneering work the simplest system was considered, in which biodegradable waste is discharged to the stream and consumes oxygen, atmospheric re-aeration being the only source of oxygen. The model is based on complicated solution of differential equation for above process. The equation is derived assuming River coefficients k_1 and k_2 as exponential function of time variation. The authors have simplified the derivation of DO-Sag equation [4] by replacing the exponential function with a quadratic polynomial. To explain the use of new equation, authors have defined the geometry of DO curve known as 'River Water DO Mechanics'. Also in this paper, new equation is applied to make 'Shivnath River water DO Model' with data taken by the author as part of his Ph. D. research work. The results justify the acceptance of new modified equation for River Water Quality Assessment.***

Key words- DO Modeling, River coefficients of new DO-Sag equation: De-oxidation (k_1), Re-oxygenation (k_2) & Super Aeration (A), River Water DO Mechanics

I. INTRODUCTION

A. DO mechanics defined from Streeter Phelps equation (1925):

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Basics of river water DO mechanics [4] can be defined from DO-Sag curve introduced by Streeter and Phelps in 1925. Appendix- Fig.

(1) shows the geometry of DO-Sag curve with Cartesian coordinates marked as (Day, DO : for x, y). The curve shows initial DO-point 'B' (0, 6), DO deficit point 'C' (2, 2.4) and DO after re-aeration point 'D' (5, 5.5), over particular distance of river reach about 100 km. Let

DO-Sag curve coordinates be expressed as:

X- Axis represents No. of days or distances

Y- Axis represents DO values in mg/L

Coordinates of points on ideal DO-Sag curve are expressed as follows:

i) Point O- Origin (0, 0) ➔ (d=0.DO=0)

ii) Point A- Coordinates (0, 10) ➔ fully DO saturated river water in contact with atmosphere

iii) Point B- Coordinates (0, 6) ➔ DO of river surface water before waste water stream joins

iv) Point C- Coordinates (2, 2.4) ➔ DO minimum level due to de-oxidation process and become dangerous for aquatic life. The process takes 2 days time for completion. Thereafter re-oxygenation begins.

v) Point D- Coordinates (5, 5.5) ➔ Re-aeration continues and DO becomes equal to point C.

vi) Point E- Coordinates (10, 10) ➔ Re-aeration continues to wards super saturation zone and DO finally after 10 days becomes 8.8 mg/L.

The DO-Sag curve obtained by above coordinates is known as ***Reference DO Model with ideal River data.***

II. 2. Formulation of DO-Sag Equation by alternative algorithm-

Close examination of DO-Sag curve of Reference DO model with ideal River data (Fig.(1) shows that the path described by unit polluted mass rolling can be divided into three branches viz:

i) Stage –I: The decaying vector BC where Oxygen of rolling ball is reduced as it mix-up with the main river stream. This stage continues for 1st 3 days.

ii) Stage- II: Re-aeration vector CD is the stage when the rolling ball acquires Oxygen from Main River. This stage is known as re-oxygenation and it ends in 5 days. The path traced out by the rolling ball is resembles to a ***quadratic polynomial given by:***

$DO = k_1 d^2 + k_2 d + C$ -----------------(1)

Where k_1 - de-oxidation coefficient

k_2 - re-oxygenation coefficient)

d - is time period (in days) between test of two water samples

C- is constant for sample temp (20^0C)

By experimentation it is found that $k_1 = 2$ v, (river mean velocity-v) and $k_2 =$ h, depth of river bed. Therefore DO of River Water at minimum 3 locations are required to determine k_1, k_2 and C of Equation (1).

iii) Stage- III: The rolling ball move ahead from Re-aeration stage-II to DO saturation stage.

After 5 days, DO of rolling ball moves from aeration zone to DO saturation zone. It is given by the exponential curve Eq.(2)

$Cs = DO_5 + vA^d$ -----------------(2)

Where v : is mean velocity of river water

d : is time period between two samples

DO_5: DO value of water sample on 5^{th} day of lab test

A : Aeration Coefficient of the River

DO Model resulted by Eqn. (1) and (2) when combined gives ***New DO–Sag equation***

3. **DO_{Ref} Model -** from DO_{Ref} Model Fig. (1.1), DO coordinates are shown in Table (1),

Table (1)- DO coordinate of DO_{Ref} Model Fig. (1)

Days	DO (mg/L)	$DO_{saturated}$
0	6	10
2	2.4	10
5	5.4	10

Therefore equations of DO_{New} Model De-oxidation and Re-aeration coefficients k_1 and k_2, can be expressed from eqn. (1) as:

$DO_n = k_1 d^2 + k_2 d + DOs$ -------(3)

Where, n – number of days 1 to 5

Substituting d as 0, 2, 5 we have following equations from (3)

$k_1(0)^2 + k_2(0) + DOs = 6$ }Therefore initial Coordinate (0, 6) i.e. (d, DO)

$k_1(2)^2 + k_2(2) + 6 = 2.4$ } ----------- (4)

$k_1(5)^2 + k_2(5) + 6 = 6$}

Solving set of equations (5) we have $k_1 = 0.6$, $k_2 = - 3$

Since $k_1 = 2$ v, and $k_2 = -$ h, the river parameter we have v = 0.3 m/s and h = -3 m

Note- (-)ve sign of h show that the river depth is measured from the River Surface to its bed.

Hence DO Model equation of Ideal River is:

$DO_n = 0.6 d^2 – 3 d + 6$ ------(5)

Therefore DO_1, DO_2 etc. can be computed from eqn. (5). It takes 5 days to complete de-oxidation and re-aeration process and DO mass balance is achieved. Thereafter DO is further computed from DO_5 to super saturation level given by equation-

$DO = DO_5 + k_1A^d$ ----------(6)

On substitution A = 1.36

Where A = Super aeration coefficient

Therefore DO_6, DO_7 etc. can be computed from eqn. (6)

The combined effect Eqn. (5) & (6) gives ***DO_{New} Model***

III. DONew Model by new Equation

The main advantage of DO_{New} Model equation is that there is no need to make field observation at river site to ***determine mean velocity of river flow and its depth***. Determination of River coefficients k_1, k_2 and A of DO_{New} Model equation takes care of velocity and depth components.

DO_{Ref} model is made using Streeter Phelps eqn. Fig.(1) at initial point 'B', Deficit Point 'C' and Re-aeration point 'D' have been used for determine ***Coefficients*** k_1 and k_2 as explained below to determine of hydraulic parameters (v-velocity & depth-h) at site, is not required. The computation is explained below:

5 days DO coordinates are tabulated in Table (2) from Fig.(1) DO_{Ref} Model

Table (2)- DO coordinate of DO_{Ref} Model Fig. (1.1)

Days	DO (mg/L)	$DO_{saturated}$
0	6.00	8.8
1	3.00	8.8
2	2.20	8.8
3	3.40	8.8
4	3.90	8.8
5	5.40	8.8

Excel Worksheet – ***computing template*** for coefficients k_1, k_2 and A have been prepared for eqn. (1) stated above. It resulted in set of 5- linear equations in the form stated below:

$DO_n = k_1 d^2 + k_2 d + DOs$ -------(7)

5. Comparison of Results- DO_{Ref} Model & DO_{New} Model

a) DO_{Ref} Model Coefficients- $k_1 = 0.12$, $k_2 = 0.41$

b) DO_{New} Model Coefficients- $k_1 = 0.0.62$, $k_2 = 3.12$ A = 1.1.38

DO_{New} Model Equation – $DO_n = 0.62 d^2 – 3.12 d + 6$

DO_{New} Model in Super aeration zone – equation: $DO_A = DO_5 + 0.62 \times 1.38^d$

Additional Features of DO-Sag new equation-

i) River coefficients k_1, k_2 determination in case of DO_{Ref} model by Streeter Phelps equation and new DO_{New} equation are represented by bar graph in Fig.(3).

ii) In case of DO_{New} Model coefficients k_1, k_2 and A, ***(de-oxidation/ Re-oxygenation, Super- aeration -A)*** are determined from lab tests. Solution of quadratic equation using the results gives coefficient k_1 and k_2. It is found that k_1= 2 v (mean velocity of River) and k_2 = mean depth (depth-h). Hence there is no need to determine them at river site.

iii) ***Error comparison of DO_{New} Model with DO_{Ref} Model-***

DO values of DO_{Ref} Model and DO_{New} Model are compared by determination of mean square deviation from true values tabulated in Table (3).

Table (3)- Mean square error of New DO Eqn. Vs. Reference DO Model

Days	DO-New Eqn.	DO Ref. Model	$Error^2$	DO Saturated
0	6.00	6.00	0.00	8.80
1	3.50	3.20	0.09	8.80
2	2.24	2.40	0.03	8.80
3	2.22	3.00	0.61	8.80
4	3.44	4.20	0.58	8.80
5	5.90	5.50	0.16	8.80
6	6.74	6.10	0.41	8.80
7	7.05	7.00	0.00	8.80
8	7.46	7.25	0.05	8.80
9	8.03	8.00	0.00	8.80
10	8.80	8.20	0.36	8.80
			2.29	
		m.s.e.=	0.32	

It is apparent from above analysis that mean square error of DO_{New} with respect to DO_{Ref} Model = 0.32. It implies that:

Error in DO_{New} Model (made by quadratic equation) may be = ± 0.32 mg/L. Thus deviation from true values is very small.

Therefore DO_{Ref} Model and DO_{New} Model are in close resemblance obvious from Fig.(2). Hence ***new equation*** is acceptable for study of River DO Model for water quality assessment.

IV. 6 Season-wise comparison of Results-

DO_{Ref} Model Vs Shivnath River DO_{New} Model

Following DO model equation season have been resulted while processing Shivnath River Data observed during Jan. to Dec. 2017. The processing was done in Excel Worksheets.

i) DO equation for ***winter season***:

$DO_n = 0.36\ d^2 - 2.25\ d + 6.76$

DO variation in super-aeration zone:

$DO = DO_5 + k_1\ (1.64)^d$

ii) DO equation for ***Summer season***:

$DO_n = 0.22\ d^2 - 1.86\ d + 7.35$

DO variation in super-aeration zone:

$DO = DO_5 + k_1\ (1.89)^d$

iii) DO equation for Rainy season:

$DO_n = 0.31\ d^2 - 2.04\ d + 7.67$

DO variation in super-aeration zone:

$DO = DO_5 + k_1\ (1.63)^d$

iv) DO equation for Autumn season:

$DO_n = 0.42\ d^2 - 2.69\ d + 7.82$

DO variation in super-aeration zone:

$DO = DO_5 + k_1\ (1.52)^d$

DO values computed by new equation in ***Excel Worksheet. The extract of*** results of worksheet are shown in Table (4). Season-wise curves are compared with DO_{Ref} model in Fig. (4). Results show that excepting summer, the DO variation in all seasons resemble to DO_{Ref} model curve.

Table-(4): Summary of DO_{New} Model of all season Vs DO_{Ref} Model

Days	DO Ref. Model	DO_{New} Winter	DO_{New} Summer	DO_{New} Rainy	DO_{New} Autumn
0	6.00	6.76	7.35	7.67	7.83
1	3.20	4.87	5.71	5.94	6.10
2	2.40	3.70	4.51	4.83	4.99
3	3.00	3.25	3.75	4.34	4.50
4	4.20	3.52	3.43	4.47	4.63
5	5.50	4.51	3.55	5.22	5.38
6	6.10	5.10	3.96	5.73	5.86
7	7.00	5.48	4.33	6.04	6.19
8	7.25	6.10	5.03	6.57	6.70
9	8.00	7.12	6.33	7.41	7.47
10	8.20	8.80	8.80	8.80	8.64

V. 7. Results & Discussion-

i) Results of Shivnath River coefficients k_1, k_2 and A are analyzed below to produce the bar chart. ***Season-wise variation of River Coefficients k_1, k_2 and A* of New DO equation** of Shivnath River have been tabulated in Table No.(5).

Table No.(5)- Seasonal variation of Coefficients (k_1, k_2, A) of DO_{New} Model

No.	Season	K1	K2	A
1	Winter	0.36	2.25	1.640
2	Summer	0.22	1.86	1.890
3	Rainy	0.31	2.04	1.630
4	Autumn	0.42	2.69	1.520

Coefficients of Shivnath River for ***four seasons*** (Jan. to Dec. 2017). Bar Graph is given in Fig. (5). It predicts DO values for Winter, Summer, Rainy and Autumn season of year 2017.

ii) Coefficients k_2 ***needs to be modified*** to determine sector area covered in each season.Thus data is arranged season-wise as shown in Table (6) to depict the seasonal variation of DO model. The Table (6) also gives processed data for making the Pie chart Fig.(5).

Table No.(6)- Seasonal variation of DO_{New} Model Coefficients (k_1, k_2 , A)

Pie Chart	Arranging River Coefficients Season wise						Sector
No.	Season	% Area	K1	K2	A	Km	Area
1	Winter	27.21	0.36	2.25	1.640	2.66	0.479
2	Summer	14.58	0.22	1.86	1.890	2.33	0.257
3	Rainy	21.56	0.31	2.04	1.630	2.45	0.379
4	Autumn	36.64	0.42	2.69	1.520	3.07	0.645
		100.00					1.759

3. ***System Performance/ Efficiency Indicator*** - It is accurate method to compute central angles of each season wise sector area from Coefficients ***k_1 & km, determined for New DO-Sag equation.***

Test criteria- As there are 4 seasons in which central angle of each sector lies therefore:

I) Central angle of each area vector Δ is nearly 90^0.

Ii) Let each pair of coefficients k_1 and k_m be considered season wise

Iii) Estimated angle value (Acute & Obtuse) and in each set there are 4 options.

Iv) Compare the central angle (90^0) with above options and ***select*** the value depending upon $k_m > k_1$, in each sector as $>45^0$ & $<90^0$, or $>90^0$ & $<135^0$.

V) The computation of efficiency index is illustrated in ***Excel worksheet*** with grouping of coefficients in Table (6) to process data and determine DO model efficiency index shown in Fig.(7).

Table No.(7)- Seasonal Efficiency Index of Shivnath River DO model

System Performance/ Efficiency Test of DO model					
Shivnath River coefficients k1 & km seasonwise					
Season	k1	km	Included∠	Area	P.I.
Winter	0.36	2.66	95.49	0.477	28.62
Summer	0.22	2.33	41.17	0.169	10.13
Rainy	0.31	2.45	129.87	0.291	17.50
Autumn	0.42	3.07	84.45	0.642	38.54
Deficit	0.36	3.07	9.02	0.087	5.20
			360	1.665	100.00

It ranges from 10.13% in Summer to 38.54% in Autumn. The designed model efficiency is 95%.

VI. CONCLUSION-

Streeter-Phelps (1925) developed DO Model to determine ***River coefficients k_1 & k_2*** by using lab test results of water samples for DO presence and DO deficit and hydraulic parameters like river velocity, depth etc. Computation of ***k_1 & k_2 are performed*** in Excel worksheet. The computation results are presented in Table No. (4), (5), (6) and (7) for months Jan. To Dec. 2017. Finally variation of ***k_1 & k_2*** for Shivnath is presented by means of following ***Statistical Analysis tools (see Appendix- Figures & Graphs)***:

A) Bar graph- to represent variation of k_1 and k_2 for months Jan. To Dec. 2017.

B) Pie chart and

C) Shivnath DO model season-wise study by ***Statistical efficiency index system***

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4. DO Sag definitions Curve © *McGraw Hill* Series in Water Resources and Environmental Engineering, Publication

Appendix- Figs. & Graphs

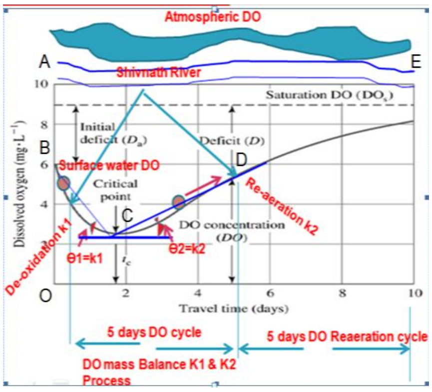


Fig.(1)- DO Mechanics to derive new DO-Sag equation as quadratic polynomial

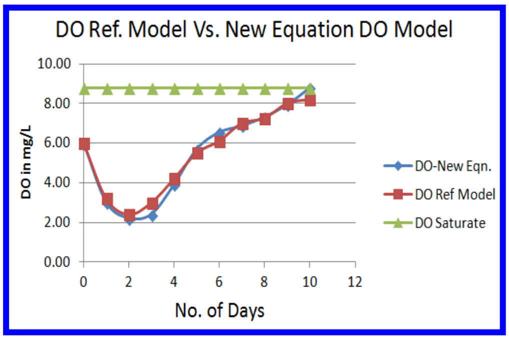


Fig.(2)– Reference DO Model Vs New Equation DO

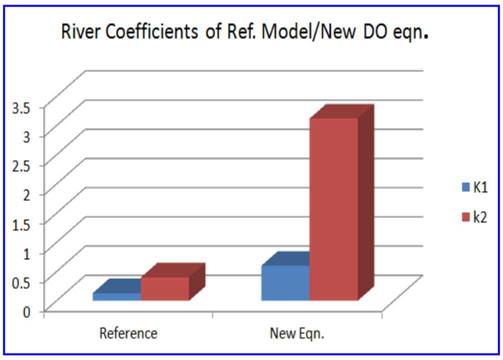


Fig.(3)- Variation of Coefficients

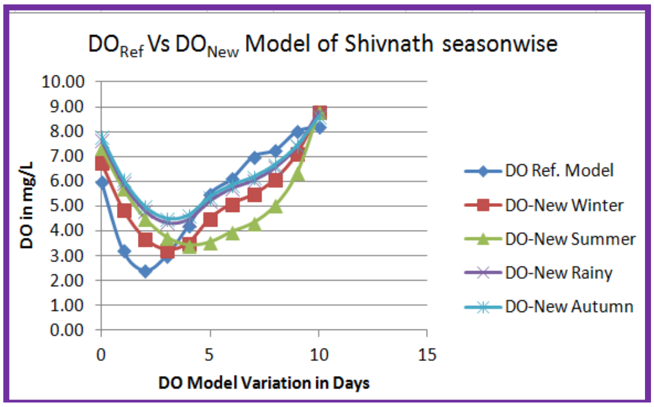


Fig. (4) – DO_{New} Shivnath River season-wise Vs DO_{Ref} Model

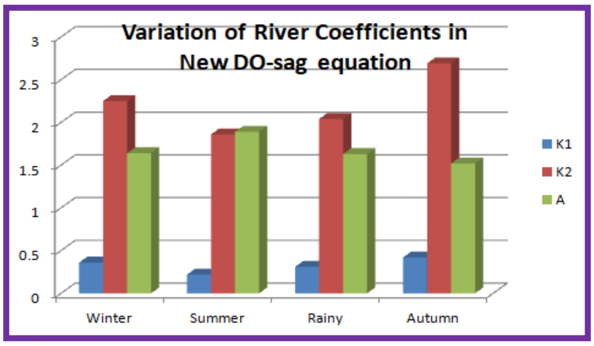


Fig. (5) – Bar Chart Season-wise

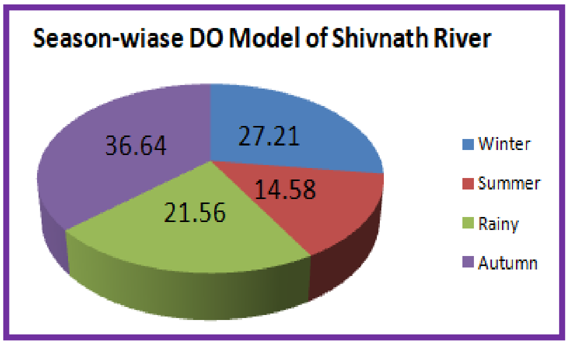
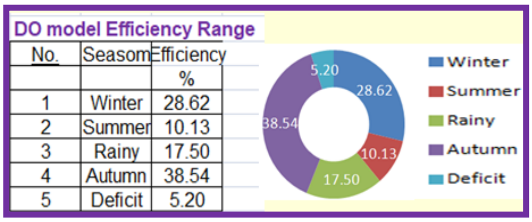


Fig. (6) – Pie Chart Season-wise Variation coefficients (k_1, k_2 & A) of coefficients (k_1, & k_m)



DO model Efficiency Range

No.	Season	Efficiency %
1	Winter	28.62
2	Summer	10.13
3	Rainy	17.50
4	Autumn	38.54
5	Deficit	5.20

Fig. (7) – Efficiency of Shivnath River DO Model = 95%
(10.13% in Summer to 38.54% in Autumn season)

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