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Dualities between Some Useful Integral Transforms and Sawi Transform

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Abstract: Integral transforms have a number of applications in the different fields of engineering and science to solve the problems of Newton's law of cooling, signal processing, electrical networks, bending of beams, springs, mixing problems, carbon dating problems exponential growth and decay problems. In this paper, we will discuss the dualities of some useful integral transforms namely Laplace transform, Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform, Mahgoub (Laplace-Carson) transform and Mohand transform with Sawi transform. To visualize the importance of dualities between mention integral transforms with Sawi transform, we give tabular presentation of the integral transforms (Laplace transform, Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform, Mahgoub transform and Mohand transform) of mostly used basic functions by using mention dualities relations. Results show that the mention integral transforms are strongly related to each others.

Keywords: Laplace; Kamal; Elzaki; Aboodh; Sumudu; Mahgoub (Laplace-Carson); Mohand; Sawi transforms.
AMS Subject Classification 2010: 44A05, 44A10, 44A15.

I. INTRODUCTION

Many process and phenomenon of science, engineering and real life solved by using integral transforms by expressing them into mathematical models. The problems arise in the field of heat conduction, economics, telecommunications, nuclear reactors, statistics, thermal science, space science, marine science, biology, gravitation, detection of diabetes, chemistry, stress analysis, electricity, physics, potential theory, mathematics, medicine, aerodynamics, civil engineering, control theory, cardiology, mechanics, deflection of beams, vibration of plates, defense, Brownian motion and many other fields can be easily handle with the help of integral transforms by converting them into mathematical form. In the advanced time, scholars are interested in solving the advance problems of research, science, space, engineering and real life by introducing new integral transforms. Aggarwal and Chaudhary [1] discussed Mohand and Laplace transforms comparatively by solving system of differential equations using both integral transforms.

Recently many scholars [2-7] used different integral transforms namely Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform, Mahgoub (Laplace-Carson) transform and Mohand transform for evaluating improper integrals which contains error function in the integrand. Mahgoub [8] gave Sawi transform which is a new integral transform.

The aim of this study is to establish duality relations between some useful integral transforms namely Laplace transform, Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform, Mahgoub (Laplace-Carson) transform and Mohand transform with Sawi transform.

II. LAPLACE TRANSFORM

The Laplace transform of the function $Z(\gamma), \gamma \geq 0$ is given by [1]

$$L\{Z(\gamma)\} = \int_0^\infty Z(\gamma)e^{-\epsilon\gamma}d\gamma = B(\epsilon) \quad (1)$$

III. KAMAL TRANSFORM

Kamal transform of the function $Z(\gamma), \gamma \geq 0$ is given by [2]

$$K\{Z(\gamma)\} = \int_0^\infty Z(\gamma)e^{\frac{-\gamma}{\epsilon}}d\gamma = C(\epsilon), \quad (2)$$
$$0 < k_1 \leq \epsilon \leq k_2$$

IV. ELZAKI TRANSFORM

Elzaki transform of the function $Z(\gamma), \gamma \geq 0$ is given by [3]

$$E\{Z(\gamma)\} = \epsilon \int_0^\infty Z(\gamma)e^{-\frac{\gamma}{\epsilon}}d\gamma = D(\epsilon), \quad (3)$$
$$0 < k_1 \leq \epsilon \leq k_2$$

V. ABOODH TRANSFORM

Aboodh transform of the function $Z(\gamma), \gamma \geq 0$ is given by [4]

$$A\{Z(\gamma)\} = \frac{1}{\epsilon}\int_0^\infty Z(\gamma)e^{-\epsilon\gamma}d\gamma = F(\epsilon), \quad (4)$$
$$0 < k_1 \leq \epsilon \leq k_2$$

VI. SUMUDU TRANSFORM

Sumudu transform of the function $Z(\gamma), \gamma \geq 0$ is given by [5]

$$S\{Z(\gamma)\} = \int_0^\infty Z(\epsilon\gamma)e^{-\gamma}d\gamma = G(\epsilon), \quad (5)$$
$$0 < k_1 \leq \epsilon \leq k_2$$

VII. MAHGOUB (LAPLACE – CARSON) TRANSFORM

Mahgoub (Laplace-Carson) transform of the function $Z(\gamma), \gamma \geq 0$ is given by [6]

$$M_*\{Z(\gamma)\} = \epsilon \int_0^\infty Z(\gamma)e^{-\epsilon\gamma}d\gamma = H(\epsilon), \quad (6)$$
$$0 < k_1 \leq \epsilon \leq k_2$$

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VIII. MOHAND TRANSFORM

Mohand transform of the function $Z(\gamma), \gamma \geq 0$ is given by [7]

$$M\{Z(\gamma)\} = \epsilon^2 \int_0^\infty Z(\gamma)e^{-\epsilon\gamma}d\gamma = I(\epsilon), \quad (7)$$
$$0 < k_1 \leq \epsilon \leq k_2$$

IX. SAWI TRANSFORM

Sawi transform of the function $Z(\gamma), \gamma \geq 0$ is given by [8]

$$S^*\{Z(\gamma)\} = \frac{1}{\epsilon^2}\int_0^\infty Z(\gamma)e^{\frac{-\gamma}{\epsilon}}d\gamma = J(\epsilon), \quad (8)$$
$$0 < k_1 \leq \epsilon \leq k_2$$

X. DUALITIES OF SAWI TRANSFORM WITH SOME USEFUL INTEGRAL TRANSFORMS

In this section, we define the dualities between some useful integral transforms namely Laplace transform, Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform, Mahgoub (Laplace-Carson) transform and Mohand transform with Sawi transform.

A. Laplace – Sawi Duality

If Laplace and Sawi transforms of $Z(\gamma)$ are $B(\epsilon)$ and $J(\epsilon)$ respectively then

$$B(\epsilon) = \frac{1}{\epsilon^2}J\left(\frac{1}{\epsilon}\right) \quad (9)$$

and $J(\epsilon) = \frac{1}{\epsilon^2}B\left(\frac{1}{\epsilon}\right)$ (10)

Proof: From (1),

$$B(\epsilon) = \int_0^\infty Z(\gamma)e^{-\epsilon\gamma}d\gamma$$
$$\Rightarrow B(\epsilon) = \frac{1}{\epsilon^2}\left[\epsilon^2\int_0^\infty Z(\gamma)e^{-\epsilon\gamma}d\gamma\right]$$

Now, using (8) in above Equation, we obtain

$$B(\epsilon) = \frac{1}{\epsilon^2}J\left(\frac{1}{\epsilon}\right).$$

To drive (10), we use (8)

$$J(\epsilon) = \frac{1}{\epsilon^2}\int_0^\infty Z(\gamma)e^{\frac{-\gamma}{\epsilon}}d\gamma \quad (11)$$

It is immediately concluded using (1) in (11),

$$J(\epsilon) = \frac{1}{\epsilon^2}B\left(\frac{1}{\epsilon}\right).$$

B. Kamal – Sawi Duality

If Kamal and Sawi transforms of $Z(\gamma)$ are $C(\epsilon)$ and $J(\epsilon)$ respectively then

$$C(\epsilon) = \epsilon^2 J(\epsilon) \quad (12)$$

and $J(\epsilon) = \frac{1}{\epsilon^2}C(\epsilon)$ (13)

Proof: Using (2) follows

$$C(\epsilon) = \int_0^\infty Z(\gamma)e^{\frac{-\gamma}{\epsilon}}d\gamma$$
$$\Rightarrow C(\epsilon) = \epsilon^2\left[\frac{1}{\epsilon^2}\int_0^\infty Z(\gamma)e^{\frac{-\gamma}{\epsilon}}d\gamma\right] \quad (14)$$

Now, using (8) in above equation, we obtain

$$C(\epsilon) = \epsilon^2 J(\epsilon).$$

To drive (13), we use (8)

$$J(\epsilon) = \frac{1}{\epsilon^2}\int_0^\infty Z(\gamma)e^{\frac{-\gamma}{\epsilon}}d\gamma$$

It is immediately concluded using (2) in above equation,

$$J(\epsilon) = \frac{1}{\epsilon^2}C(\epsilon).$$

C. Elzaki – Sawi Duality

If Elzaki and Sawi transforms of $Z(\gamma)$ are $D(\epsilon)$ and $J(\epsilon)$ respectively then

$$D(\epsilon) = \epsilon^3 J(\epsilon) \quad (15)$$

and $J(\epsilon) = \frac{1}{\epsilon^3}D(\epsilon)$ (16)

Proof: It is immediately concluded from (3)

$$D(\epsilon) = \epsilon\int_0^\infty Z(\gamma)e^{-\frac{\gamma}{\epsilon}}d\gamma$$
$$\Rightarrow D(\epsilon) = \epsilon^3\left[\frac{1}{\epsilon^2}\int_0^\infty Z(\gamma)e^{-\frac{\gamma}{\epsilon}}d\gamma\right]$$

Now, using (8) in above Equation, we have

$$D(\epsilon) = \epsilon^3 J(\epsilon).$$

To drive (16), we use (8)

$$J(\epsilon) = \frac{1}{\epsilon^2}\int_0^\infty Z(\gamma)e^{-\frac{\gamma}{\epsilon}}d\gamma$$
$$\Rightarrow J(\epsilon) = \frac{1}{\epsilon^3}\left[\epsilon\int_0^\infty Z(\gamma)e^{-\frac{\gamma}{\epsilon}}d\gamma\right]$$

It is immediately concluded using (3) in above equation,

$$J(\epsilon) = \frac{1}{\epsilon^3}D(\epsilon).$$

D. Aboodh – Sawi Duality

If Aboodh and Sawi transforms of $Z(\gamma)$ are $F(\epsilon)$ and $J(\epsilon)$ respectively then

$$F(\epsilon) = \frac{1}{\epsilon^3}J\left(\frac{1}{\epsilon}\right) \quad (17)$$

and $J(\epsilon) = \frac{1}{\epsilon^3}F\left(\frac{1}{\epsilon}\right)$ (18)

Proof: From (4), we have

$$F(\epsilon) = \frac{1}{\epsilon}\int_0^\infty Z(\gamma)e^{-\epsilon\gamma}d\gamma$$
$$\Rightarrow F(\epsilon) = \frac{1}{\epsilon^3}\left[\epsilon^2\int_0^\infty Z(\gamma)e^{-\epsilon\gamma}d\gamma\right]$$

Now, using (8) in above equation, we have

$$F(\epsilon) = \frac{1}{\epsilon^3}J\left(\frac{1}{\epsilon}\right).$$

To drive (18), we use (8)

$$J(\epsilon) = \frac{1}{\epsilon^2}\int_0^\infty Z(\gamma)e^{\frac{-\gamma}{\epsilon}}d\gamma$$
$$\Rightarrow J(\epsilon) = \frac{1}{\epsilon^3}\left[\epsilon\int_0^\infty Z(\gamma)e^{\frac{-\gamma}{\epsilon}}d\gamma\right]$$

It is immediately concluded using (4) in above equation,

$$J(\epsilon) = \frac{1}{\epsilon^3}F\left(\frac{1}{\epsilon}\right).$$

E. Sumudu – Sawi Duality

If Sumudu and Sawi transforms of $Z(\gamma)$ are $G(\epsilon)$ and $J(\epsilon)$ respectively then

$$G(\epsilon) = \epsilon J(\epsilon) \quad (19)$$

and $J(\epsilon) = \frac{1}{\epsilon}G(\epsilon)$ (20)

Proof: From (5), we have

$$G(\epsilon) = \int_0^\infty Z(\epsilon\gamma)e^{-\gamma}d\gamma$$

Put $\epsilon\gamma = u \Rightarrow d\gamma = \frac{du}{\epsilon}$ in above equation, we have

$$G(\epsilon)=\int_0^\infty Z(u)e^{-\frac{u}{\epsilon}}\frac{du}{\epsilon}$$
$$\Rightarrow G(\epsilon)=\frac{1}{\epsilon}\int_0^\infty Z(u)e^{-\frac{u}{\epsilon}}du$$
$$\Rightarrow G(\epsilon)=\epsilon\left[\frac{1}{\epsilon^2}\int_0^\infty Z(u)e^{-\frac{u}{\epsilon}}du\right]$$

Now, using (8) in above equation, we have

$$G(\epsilon)=\epsilon J(\epsilon).$$

To drive (20), we use (8)

$$J(\epsilon)=\frac{1}{\epsilon^2}\int_0^\infty Z(\gamma)e^{\frac{-\gamma}{\epsilon}}d\gamma$$

Put $\frac{\gamma}{\epsilon}=u \Rightarrow d\gamma=\epsilon du$ in above equation, we have

$$J(\epsilon)=\frac{1}{\epsilon^2}\int_0^\infty Z(\epsilon u)e^{-u}\epsilon du$$
$$\Rightarrow J(\epsilon)=\frac{1}{\epsilon}\left[\int_0^\infty Z(\epsilon u)e^{-u}du\right]$$

It is immediately concluded using (5) in above equation,

$$J(\epsilon)=\frac{1}{\epsilon}G(\epsilon).$$

F. Mahgoub (Laplace – Carson) – Sawi Duality

If Mahgoub and Sawi transforms of $Z(\gamma)$ are $H(\epsilon)$ and $J(\epsilon)$ respectively then

$H(\epsilon)=\frac{1}{\epsilon}J\left(\frac{1}{\epsilon}\right)$ (21)

and $J(\epsilon)=\frac{1}{\epsilon}H\left(\frac{1}{\epsilon}\right)$ (22)

Proof: From (6), we have

$$H(\epsilon)=\epsilon\int_0^\infty Z(\gamma)e^{-\epsilon\gamma}d\gamma$$
$$\Rightarrow H(\epsilon)=\frac{1}{\epsilon}\left[\epsilon^2\int_0^\infty Z(\gamma)e^{-\epsilon\gamma}d\gamma\right]$$

Now, using (8) in above equation, we have

$$H(\epsilon)=\frac{1}{\epsilon}J\left(\frac{1}{\epsilon}\right).$$

To drive (22), we use (8)

$$J(\epsilon)=\frac{1}{\epsilon^2}\int_0^\infty Z(\gamma)e^{\frac{-\gamma}{\epsilon}}d\gamma$$
$$\Rightarrow J(\epsilon)=\frac{1}{\epsilon}\left[\frac{1}{\epsilon}\int_0^\infty Z(\gamma)e^{\frac{-\gamma}{\epsilon}}d\gamma\right]$$

It is immediately concluded using (6) in above equation,

$$J(\epsilon)=\frac{1}{\epsilon}H\left(\frac{1}{\epsilon}\right).$$

G. Mohand – Sawi Duality

If Mohand and Sawi transforms of $Z(\gamma)$ are $I(\epsilon)$ and $J(\epsilon)$ respectively then

$I(\epsilon)=J\left(\frac{1}{\epsilon}\right)$ (23)

and $J(\epsilon)=I\left(\frac{1}{\epsilon}\right)$ (24)

Proof: Using (7) follows

$$I(\epsilon)=\epsilon^2\int_0^\infty Z(\gamma)e^{-\epsilon\gamma}d\gamma$$
$$\Rightarrow I(\epsilon)=\frac{1}{(1/\epsilon)^2}\int_0^\infty Z(\gamma)e^{-\frac{\gamma}{(1/\epsilon)}}d\gamma$$

Now, using (8) in above equation, we obtain

$$I(\epsilon)=J\left(\frac{1}{\epsilon}\right).$$

To drive (24), we use (8)

$$J(\epsilon)=\frac{1}{\epsilon^2}\int_0^\infty Z(\gamma)e^{-\frac{\gamma}{\epsilon}}d\gamma$$
$$\Rightarrow J(\epsilon)=\left[\left(\frac{1}{\epsilon}\right)^2\int_0^\infty Z(\gamma)e^{-\left(\frac{1}{\epsilon}\right)\gamma}d\gamma\right]$$

Now, using (7) in above equation, we obtain

$$J(\epsilon)=I\left(\frac{1}{\epsilon}\right).$$

XI. APPLICATIONS OF MENTION DUALITY RELATIONS FOR FINDING INTEGRAL TRANSFORMS (LAPLACE TRANSFORM, KAMAL TRANSFORM, ELZAKI TRANSFORM, ABOODH TRANSFORM, SUMUDU TRANSFORM, MAHGOUB TRANSFORM AND MOHAND TRANSFORM) OF USEFUL BASIC FUNCTIONS

We are giving tabular presentation of the integral transforms of mostly used basic functions by using mention dualities relations to visualize the usefulness of dualities between mention integral transforms and Sawi transform in the application field.

Table-I: Laplace transform of useful basic functions with the help of Laplace – Sawi duality relation

S.N.	$Z(\gamma)$	$S^*\{Z(\gamma)\}=J(\epsilon)$	$L\{Z(\gamma)\}=B(\epsilon)$
1.	1	$\frac{1}{\epsilon}$	$\frac{1}{\epsilon}$
2.	γ	1	$\frac{1}{\epsilon^2}$
3.	γ^2	$2!\,\epsilon$	$\frac{2!}{\epsilon^3}$
4.	γ^n, $n\in N$	$n!\,\epsilon^{n-1}$	$\frac{n!}{\epsilon^{n+1}}$
5.	γ^n, $n>-1$	$\Gamma(n+1)\epsilon^{n-1}$	$\frac{\Gamma(n+1)}{\epsilon^{n+1}}$
6.	$e^{a\gamma}$	$\frac{1}{\epsilon(1-a\epsilon)}$	$\frac{1}{(\epsilon-a)}$
7.	$sina\gamma$	$\frac{a}{(1+a^2\epsilon^2)}$	$\frac{a}{(\epsilon^2+a^2)}$
8.	$cosa\gamma$	$\frac{1}{\epsilon(1+a^2\epsilon^2)}$	$\frac{\epsilon}{(\epsilon^2+a^2)}$
9.	$sinha\gamma$	$\frac{a}{(1-a^2\epsilon^2)}$	$\frac{a}{(\epsilon^2-a^2)}$
10.	$cosha\gamma$	$\frac{1}{\epsilon(1-a^2\epsilon^2)}$	$\frac{\epsilon}{(\epsilon^2-a^2)}$

Table-II: Kamal transform of useful basic functions with the help of Kamal – Sawi duality relation

S.N.	$Z(\gamma)$	$S^*\{Z(\gamma)\}=J(\epsilon)$	$K\{Z(\gamma)\}=C(\epsilon)$
1.	1	$\frac{1}{\epsilon}$	ϵ
2.	γ	1	ϵ^2
3.	γ^2	$2!\,\epsilon$	$2!\,\epsilon^3$
4.	γ^n, $n\in N$	$n!\,\epsilon^{n-1}$	$n!\,\epsilon^{n+1}$
5.	γ^n, $n>-1$	$\Gamma(n+1)\epsilon^{n-1}$	$\Gamma(n+1)\epsilon^{n+1}$

6.	$e^{a\gamma}$	$\frac{1}{\epsilon(1-a\epsilon)}$	$\frac{\epsilon}{(1-a\epsilon)}$
7.	$sina\gamma$	$\frac{a}{(1+a^2\epsilon^2)}$	$\frac{a\epsilon^2}{(1+a^2\epsilon^2)}$
8.	$cosa\gamma$	$\frac{1}{\epsilon(1+a^2\epsilon^2)}$	$\frac{\epsilon}{(1+a^2\epsilon^2)}$
9.	$sinha\gamma$	$\frac{a}{(1-a^2\epsilon^2)}$	$\frac{a\epsilon^2}{(1-a^2\epsilon^2)}$
10.	$cosha\gamma$	$\frac{1}{\epsilon(1-a^2\epsilon^2)}$	$\frac{\epsilon}{(1-a^2\epsilon^2)}$

Table-III: Elzaki transform of useful basic functions with the help of Elzaki – Sawi duality relation

S.N.	$Z(\gamma)$	$S^*\{Z(\gamma)\} = J(\epsilon)$	$E\{Z(\gamma)\} = D(\epsilon)$
1.	1	$\frac{1}{\epsilon}$	ϵ^2
2.	γ	1	ϵ^3
3.	γ^2	$2!\,\epsilon$	$2!\,\epsilon^4$
4.	$\gamma^n, n \in N$	$n!\,\epsilon^{n-1}$	$n!\,\epsilon^{n+2}$
5.	$\gamma^n, n > -1$	$\Gamma(n+1)\epsilon^{n-1}$	$\Gamma(n+1)\epsilon^{n+2}$
6.	$e^{a\gamma}$	$\frac{1}{\epsilon(1-a\epsilon)}$	$\frac{\epsilon^2}{(1-a\epsilon)}$
7.	$sina\gamma$	$\frac{a}{(1+a^2\epsilon^2)}$	$\frac{a\epsilon^3}{(1+a^2\epsilon^2)}$
8.	$cosa\gamma$	$\frac{1}{\epsilon(1+a^2\epsilon^2)}$	$\frac{\epsilon^2}{(1+a^2\epsilon^2)}$
9.	$sinha\gamma$	$\frac{a}{(1-a^2\epsilon^2)}$	$\frac{a\epsilon^3}{(1-a^2\epsilon^2)}$
10.	$cosha\gamma$	$\frac{1}{\epsilon(1-a^2\epsilon^2)}$	$\frac{\epsilon^2}{(1-a^2\epsilon^2)}$

Table-IV: Aboodh transform of useful basic functions with the help of Aboodh – Sawi duality relation

S.N.	$Z(\gamma)$	$S^*\{Z(\gamma)\} = J(\epsilon)$	$A\{Z(\gamma)\} = F(\epsilon)$
1.	1	$\frac{1}{\epsilon}$	$\frac{1}{\epsilon^2}$
2.	γ	1	$\frac{1}{\epsilon^3}$
3.	γ^2	$2!\,\epsilon$	$\frac{2!}{\epsilon^4}$
4.	$\gamma^n, n \in N$	$n!\,\epsilon^{n-1}$	$\frac{n!}{\epsilon^{n+2}}$
5.	γ^n, $n > -1$	$\Gamma(n+1)\epsilon^{n-1}$	$\frac{\Gamma(n+1)}{\epsilon^{n+2}}$
6.	$e^{a\gamma}$	$\frac{1}{\epsilon(1-a\epsilon)}$	$\frac{1}{\epsilon(\epsilon-a)}$

7.	$sina\gamma$	$\frac{a}{(1+a^2\epsilon^2)}$	$\frac{a}{\epsilon(\epsilon^2+a^2)}$
8.	$cosa\gamma$	$\frac{1}{\epsilon(1+a^2\epsilon^2)}$	$\frac{1}{(\epsilon^2+a^2)}$
9.	$sinha\gamma$	$\frac{a}{(1-a^2\epsilon^2)}$	$\frac{a}{\epsilon(\epsilon^2-a^2)}$
10.	$cosha\gamma$	$\frac{1}{\epsilon(1-a^2\epsilon^2)}$	$\frac{1}{(\epsilon^2-a^2)}$

Table-V: Sumudu transform of useful basic functions with the help of Sumudu – Sawi duality relation

S.N.	$Z(\gamma)$	$S^*\{Z(\gamma)\} = J(\epsilon)$	$S\{Z(\gamma)\} = G(\epsilon)$
1.	1	$\frac{1}{\epsilon}$	1
2.	γ	1	ϵ
3.	γ^2	$2!\,\epsilon$	$2!\,\epsilon^2$
4.	$\gamma^n, n \in N$	$n!\,\epsilon^{n-1}$	$n!\,\epsilon^n$
5.	$\gamma^n, n > -1$	$\Gamma(n+1)\epsilon^{n-1}$	$\Gamma(n+1)\epsilon^n$
6.	$e^{a\gamma}$	$\frac{1}{\epsilon(1-a\epsilon)}$	$\frac{1}{(1-a\epsilon)}$
7.	$sina\gamma$	$\frac{a}{(1+a^2\epsilon^2)}$	$\frac{a\epsilon}{(1+a^2\epsilon^2)}$
8.	$cosa\gamma$	$\frac{1}{\epsilon(1+a^2\epsilon^2)}$	$\frac{1}{(1+a^2\epsilon^2)}$
9.	$sinha\gamma$	$\frac{a}{(1-a^2\epsilon^2)}$	$\frac{a\epsilon}{(1-a^2\epsilon^2)}$
10.	$cosha\gamma$	$\frac{1}{\epsilon(1-a^2\epsilon^2)}$	$\frac{1}{(1-a^2\epsilon^2)}$

Table-VI: Mahgoub (Laplace-Carson) transform of useful basic functions with the help of Mahgoub (Laplace-Carson) – Sawi duality relation

S.N.	$Z(\gamma)$	$S^*\{Z(\gamma)\} = J(\epsilon)$	$M_*\{Z(\gamma)\} = H(\epsilon)$
1.	1	$\frac{1}{\epsilon}$	1
2.	γ	1	$\frac{1}{\epsilon}$
3.	γ^2	$2!\,\epsilon$	$\frac{2!}{\epsilon^2}$
4.	$\gamma^n, n \in N$	$n!\,\epsilon^{n-1}$	$\frac{n!}{\epsilon^n}$
5.	$\gamma^n, n > -1$	$\Gamma(n+1)\epsilon^{n-1}$	$\frac{\Gamma(n+1)}{\epsilon^n}$
6.	$e^{a\gamma}$	$\frac{1}{\epsilon(1-a\epsilon)}$	$\frac{\epsilon}{(\epsilon-a)}$
7.	$sina\gamma$	$\frac{a}{(1+a^2\epsilon^2)}$	$\frac{a\epsilon}{(\epsilon^2+a^2)}$
8.	$cosa\gamma$	$\frac{1}{\epsilon(1+a^2\epsilon^2)}$	$\frac{\epsilon^2}{(\epsilon^2+a^2)}$

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9.	$sinha\gamma$	$\frac{a}{(1-a^2\epsilon^2)}$	$\frac{a\epsilon}{(\epsilon^2-a^2)}$
10.	$cosha\gamma$	$\frac{1}{\epsilon(1-a^2\epsilon^2)}$	$\frac{\epsilon^2}{(\epsilon^2-a^2)}$

Table-VII: Mohand transform of useful basic functions with the help of Mohand – Sawi duality relation

S.N.	$Z(\gamma)$	$S^*\{Z(\gamma)\} = J(\epsilon)$	$M\{Z(\gamma)\} = I(\epsilon)$
1.	1	$\frac{1}{\epsilon}$	ϵ
2.	γ	1	1
3.	γ^2	$2!\,\epsilon$	$\frac{2!}{\epsilon}$
4.	$\gamma^n, n \in N$	$n!\,\epsilon^{n-1}$	$\frac{n!}{\epsilon^{n-1}}$
5.	$\gamma^n, n > -1$	$\Gamma(n+1)\epsilon^{n-1}$	$\frac{\Gamma(n+1)}{\epsilon^{n-1}}$
6.	$e^{a\gamma}$	$\frac{1}{\epsilon(1-a\epsilon)}$	$\frac{\epsilon^2}{(\epsilon-a)}$
7.	$sina\gamma$	$\frac{a}{(1+a^2\epsilon^2)}$	$\frac{a\epsilon^2}{(\epsilon^2+a^2)}$
8.	$cosa\gamma$	$\frac{1}{\epsilon(1+a^2\epsilon^2)}$	$\frac{\epsilon^3}{(\epsilon^2+a^2)}$
9.	$sinha\gamma$	$\frac{a}{(1-a^2\epsilon^2)}$	$\frac{a\epsilon^2}{(\epsilon^2-a^2)}$
10.	$cosha\gamma$	$\frac{1}{\epsilon(1-a^2\epsilon^2)}$	$\frac{\epsilon^3}{(\epsilon^2-a^2)}$

XII. CONCLUSIONS

In the present paper, duality relations between some useful integral transforms namely Laplace transform, Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform, Mahgoub (Laplace-Carson) transform and Mohand transform with Sawi transform are established successfully. Tabular presentation of the integral transforms (Laplace transform, Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform, Mahgoub transform and Mohand transform) of mostly used basic functions are given with the help of mention dualities relations to visualize the importance of dualities between mention integral transforms with Sawi transform. Results show that the mention integral transforms in this paper are strongly related to each others. In future using these duality relations, we can easily solved many advanced problems of modern era such as motion of coupled harmonic oscillators, drug distribution in the body, arms race models, Brownian motion and the common health problems such as detection of diabetes and tumour growth.

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