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Performance Evaluation Parameters Analysis for Trellis Coded Modulation Systems

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Abstract—The newer applications of Trellis Coded Modulation (TCM) beyond 5G has triggered the research activities for further inventions in this area. As a result, the inventions and research analysis on different aspects of the technique is in boom. This paper analyzes various performance evaluation parameters as applicable to coded modulation schemes which can be structured as a finite state machine. The significances of the parameters are discussed and the measurements through simulation are evaluated. Tradeoff between the decoding computational complexity and the coding-gain factor improvement has been analyzed.

***Index Terms*—Performance-Parameter, minimum-distance, Spectrum, Error-Event, Bit-Error.**

I. INTRODUCTION

Trellis-Codes (TC) are the structured codes where redundancy is added to a set of data bits before modulation. The redundant coded bits-set decide the set of symbols for transmission, which is non-binary. Only a specific pattern of the symbols-sequences is permitted to transmit. The dependency among the symbols in the sequence of transmission aids the performance improvement in the decoding process. The features of TCM technique are: (1) Coding and Modulation processes are Integrated; (2) The Euclidean Distance (ED) between the symbols is increased to the maximum possible extent before transmission [1]. The newer applications of the Trellis Coded Modulation (TCM) beyond 5G has triggered the research activities for further inventions in this arena [6-7]. In this paper various parameters of significance for the performance evaluation of the coded modulation schemes are discussed and the measurements through simulation are evaluated. Tradeoff between the computational complexity and the coding gain improvement for ISI free condition has been analyzed. Channel with ISI effect would follow a different approach for evaluation [3-5].

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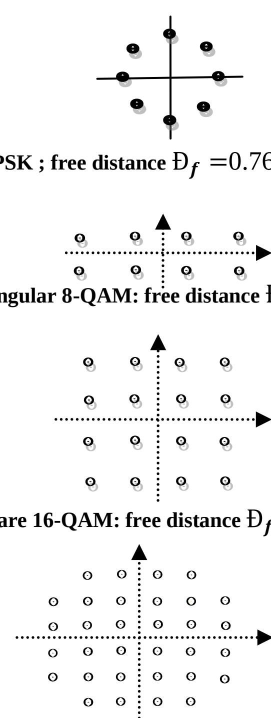
This paper has been organized as follows: The section I is the introduction to TCM and its features, Section II contain various signal constellations and the ED measured. In Section III, the concept of code generation has been given. In section IV, evaluation parameters are explained, and the section V contain the results and Discussions. Section VI contains conclusions.

II. SIGNAL CONSTELLATION AND THE EUCLIDEAN DISTANCE

The following are the examples of various signal constellations in the Two-Dimensional (2D) signal space, as shown in the Fig. 1(a-d). The ED is the minimum distance between the adjacent signal points of the constellation. The signal points distribution in the Fig.1(a) has the ED of $0.765\sqrt{\mathcal{E}}$ where $\mathcal{E}$ is the symbol energy. The constellations in the Fig.1(b-d) have the ED of value $2\sqrt{\mathcal{E}}$ [8].



a). 8-PSK ; free distance $Đ_f = 0.765\sqrt{\mathcal{E}}$

b). Rectangular 8-QAM: free distance $Đ_f = 2\sqrt{\mathcal{E}}$

c). Square 16-QAM: free distance $Đ_f = 2\sqrt{\mathcal{E}}$

d). 32-QAM: free distance $Đ_f = 2\sqrt{\mathcal{E}}$

Fig.1. Signal Space Diagram

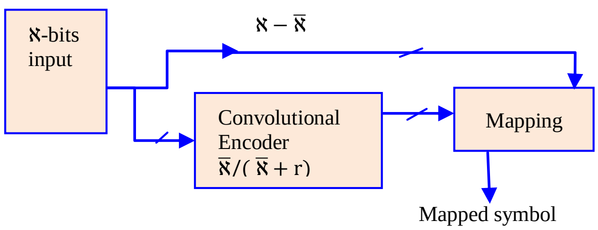


Fig.2. Trellis Code Generator/Modulator

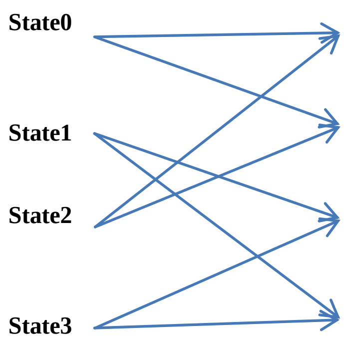


Fig.3. Trellis Structure for 4-State Scheme

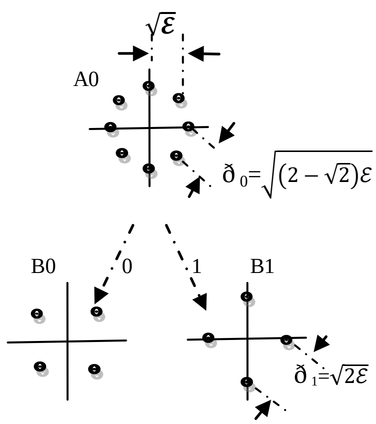


Fig.4. First Level Set-Partitioning of 8-PSK Signal Set

III. TRELLIS CODE GENERATION AND MODULATION

Considering $\aleph$ as the number of bits to be transmitted during a symbol interval T_s, and, $\mathcal{M}$ as the total number of the non-binary symbols in the signal-constellation derived from the trellis codes generated, the following definitions are applicable to the trellis encoder/modulator, as a finite state machine: 1. Number of Finite States of the Encoder: $N_{\delta} = 2^L$, where L is the encoder memory size, also called the constraint length; 2. The Trellis-Code generated: $\beta_n = \{\beta_n^0, \beta_n^1, \ldots, \beta_n^{\aleph+r-1}\}$; 3. The number of elements of the signal constellation for modulation and transmission, $\mathcal{M} = 2^{\aleph+r}$, where r is the redundancy added in the code generation [8].

The Fig.2. shows the trellis encoder as a N_{δ}-state machine, and, the Fig.3. presents its state diagram. The state transitions are presented along with the possible symbol transmission. It is also called as trellis diagram.

Labeling with the coded bits is an important process in the selection of the signal points from the signal constellation to a sub-set, and from the sub-set to the next level of smaller sub-sets. This process is called set-partitioning.

The Fig.4 shows the first level set-partitioning of 8-PSK scheme. Here TWO sub-sets B0 and B1 ate generated from the original set A0 based on the least significant bit of the trellis code β_n^0. It will select the subset B0 for the binary value 0 and B1 for the value 1.

The partitioning is symmetric in nature for a rectangular signal constellation of size $\mathcal{M}$, and, at each level of set-partitioning the ED is increased by $\sqrt{2}$.

The coded bits associated with the signal set partitioning carry significant information. Two subsets match in the last p positions of the label will have the minimum ED ∂_p. It is the bit β_n^p which divide the subset further into two more smaller subsets at level p+1 in the partition tree [8].

IV. EVALUATION PARAMETERS

There are five parameters which influence and determine the performance of a TCM scheme over AWGN channel [2], namely: the Minimum Distance in Euclidean space: $Đ_f$, the average number of nearest neighbors: $\mathcal{N}_f$, the distance spectrum (DS), the error event probability (EEP), and, the bit error probability (BEP).

A. *Minimum Distance in Euclidean space* $Đ_f$

It is the minimum distance between the coded symbol sequences $\S_n$ in the Euclidean space considering the sequence pairs for all possible combinations. It is measured using the expression

$$Đ_f = \min_{\S_n \neq \S'_n} [\textstyle\sum_n ð^2(\S_n, \S'_n)]^{\frac{1}{2}}, \S_n, \S'_n \in C \quad (1)$$

It is called the free distance. The significance feature of this parameter is that it is used for the determination of the asymptotic performance through the measurements given below:

1. The lower bound on the probability of occurrence of the error event $\mathcal{P}_e(e)$

$$\mathcal{P}_e(e) > \frac{1}{2} erfc\left(\frac{Đ_f}{\sqrt{8\zeta}}\right) \quad (2)$$

where ζ is the noise variance of the Additive White Gaussian Noise (AWGN) channel.

2. The error event probability asymptotically tight bound:

$$\mathcal{P}_e(e) \gtrsim \frac{1}{2} \mathcal{N}_f erfc\left(\frac{Đ_f}{\sqrt{8\zeta}}\right) \quad (3)$$

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Where $\mathcal{N}_f$ is the average number of nearest neighbors.

3. The asymptotic coding gain $\mathbb{G}$ is given by

$$\mathbb{G} \triangleq 10log_{10}\left(\frac{Đ_f^2}{Đ_U^2}\right) \quad (4)$$

where $Đ_u$ is the minimum squared ED between the pairs of the signals of an un-coded scheme (also represented as ∂_0) for the same data transmission rate.

The asymptotic Coding Gain (ACG) is the ratio of the squared free-distance of the coded system to that of the un-coded system. ACG can be calculated from (4), also, it can be measured through simulation [2].

*B. The **Average Number of Nearest Neighbors** $\mathcal{N}_f$:*

The parameter $\mathcal{N}_f$ is the average multiplicity of the code sequence pairs having the free distance $Đ_f$.

C. The Distance Spectrum

The distance spectrum **(DS)** is the spectrum of the set of the pairs $\{ð_i, \boldsymbol{N_i}\}$ for all values of i from zero to infinity**.** It is for all distances $ð_i$ between the code sequences and the corresponding average multiplicities $\boldsymbol{N_i}$. The values for i=0 are: $ð_i = Đ_f$ and $\boldsymbol{N_i} = \boldsymbol{\mathcal{N}_f}$.

D. The Error Event Probability

The Probability of occurrence of deviation in the detected signal sequence path from the reference path and merging back to the reference after two or more symbol intervals is the error event.

The upper bound on EEP relating $ð_i$ and $\boldsymbol{N_i}$ is given by the following equation:

$$p_{e(e)} \leq \frac{1}{2}\sum_{ð_i=Đ_f}^{\infty} \boldsymbol{N_i} \operatorname{erfc}\left(\frac{ð_i}{\sqrt{8\zeta}}\right) \quad (5)$$

It can be computed directly from the state diagrams and also from the computational algorithms

E. The Bit Error Probability

It is the probability of occurrence of bit errors in the system. The upper bound on the bit error probability (BEP) relating $ð_i$ and $\boldsymbol{N_i}$ is given by the following equation:

$$p_{b(e)} \leq \frac{1}{2}\sum_{ð_i=Đ_f}^{\infty} \frac{1}{\aleph} \boldsymbol{N_i} \mathcal{B}_i \operatorname{erfc}\left(\frac{ð_i}{\sqrt{8\zeta}}\right) \quad (6)$$

Where $\mathcal{B}_i$ is the bit error average value of the path with distance $ð_i$

Through computational algorithms BEP can be evaluated. There are also transfer function methods to evaluate EEP and BEP.

V. RESULT AND DISCUSSIONS

Parameters of primary importance such as $Đ_f$, $\mathcal{N}_f$, DS, EEP and BEP are analyzed. The Equations (2)-(6) represent the parameters mathematically. The equation (2) says that the free Euclidean Distance $Đ_f$ can be used to know the effect of coded modulation schemes over the un-coded schemes. It is used to decide the candidate codes for a good code search. Additionally, $\mathcal{N}_f$ is a requirement for the measurement. The Table.1 shows the $\boldsymbol{Đ_f}$ and $\boldsymbol{\mathcal{N}_f}$ parameters calculated theoretically for the FOUR state 16-QAM TCM scheme.

The representation in (3) and (4) say that the DS is the spectrum which conveys the information about $ð_i$ and corresponding $\boldsymbol{N_i}$ considering all distances between the coded sequences. The DS is used to obtain an upper bound to the EEP and an upper bound on the BEP. The EEP can be computed directly from the state diagrams of the code. We have evaluated the EEP using computational algorithms developed for 16-QAM TCM schemes. The FOUR state and EIGHT state encoder designs are considered in the simulation. The TABLE 3 shows the theoretical calculation of the upper bound and the lower bound for the EEP, for comparison with the simulation result. The simulation values are within the range provided by the bounds.

The EEP results are plotted in the graph in Fig.5. The right- -most curve represented as “reference” in the graph is for the un-coded 8-QAM modulation system, treated as a reference in our analysis. The curve number 2 from the right with the representation “ISI-free.4-state.16-QAM” is for the 16-QAM coded modulation scheme for FOUR state design. The Maximum Likelihood Sequence Estimation (MLSE) is done using Soft Output Decoding Algorithm (SOVA) [8], and the ACG obtained is 3.0 dB at an error rate of 10^{-6}.

The curve number 3 from the right is for the EIGHT state 16-QAM strategy mentioned as “ISI-free.4-state.16-QAM” in the graph. The ACG obtained by MLSE strategy implemented using SOVA is 4.0 dB at an error rate of 10^{-6}.

Table 2 shows the theoretical calculation of the ACG for the FOUR state 16-QAM TCM scheme and the EIGHT state 16-QAM TCM scheme. The ACG obtained from the simulation is matching with the theoretical result.

VI. CONCLUSION

From the simulation results it has been observed that the ACG for the EIGHT state scheme is improved over the FOUR state scheme due to the reason that a greater number of states are used to represent different state conditions of the encoder. At the same time the computational complexity of the decoding algorithm has been increased due to increased number of states to be considered in the computations.

The complexity of determining EEP from the algorithms depends on the factors: encoder design and the decoder design.

As optimum MLSE is not the real time solution due to its unrealistic computational complexity, developing the decoding algorithms with lesser complexity and achieving the near optimum performance has become an active research area in this field.

When the channel conditions other than the AWGN is considered, the number of parameters which influence the computational complexity increases. However, a tradeoff between the complexity of evaluation and the achievable performance is to be done for the better performances.

The parameters $Ð_f$ and $\mathcal{N}_f$ are required to determine the upper bounds for the EEP and BEP. Additionally, they are required in the exhaustive search for a good code. There are transfer function methods exist to evaluate EEP and BEP. These parameters can also be determined from the trellis structures of the encoder defined in full form. Then, computational algorithms are required, and, the complexity of such algorithms depends on the type of the codes, efficiency of the exhaust search algorithms, the complexity of the trellis structure and other parameters defining the channel condition if the channel is other than AWGN. Under different communication channel conditions minimizing the computational complexity is a challenging task to any researcher. Developing upper bounds will become more challenging when the effects like interference and degradation are considered in addition to the implementation of advanced technologies.

TABLE I
Distance Spectrum of trellis coded modulation for FOUR STATES, 16-QAM

Error Events						$ð^2_{free}$	N_{free}
Length	3	4	5	6	7		
$ð^2$	20	24	24	28	28	16	2
N_d	4	12	12	36	36		

TABLE II
Asymptotic Coding Gain for coded 16-QAM, for FOUR States

TCM Scheme	Distance parameters			ACG
	Coded		Un-coded	
	$ð^2_{free}$	$N_{ð,free}$	$ð^2_{min}$	
4-State 16-QAM	16	2	8	3
8-State 16-QAM	20	8	8	4

TABLE III
Error Event Performance of coded 16-QAM for FOUR States

SNR dB	Lower Bond	Upper Bound	Simulation
10	2.34E-03	6.18E-02	0.009894
11	7.53E-04	1.33E-02	0.002952
12	1.85E-04	2.07E-03	0.000356
13	3.23E-05	2.22E-04	0.000062
14	3.68E-06	1.59E-05	0.000006
15	2.46E-07	7.36E-07	0.000003

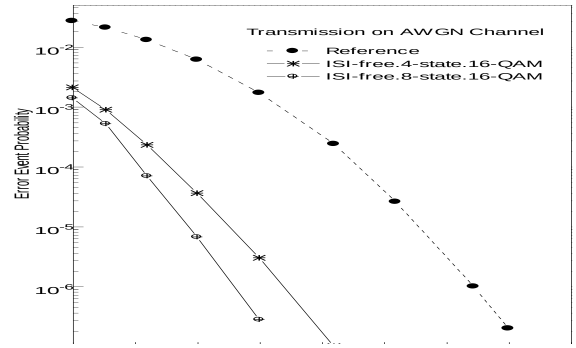

Fig.5. Graph of EEP Vs SNR

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Vanaja Shivakumar obtained her B.E. degree in Electrical and Electronics Engineering in the year 1987 from Govt. BDT College of Engineering, Davangere, under Mysore University, Karnataka-state, India. She obtained her Master Degree in Digital Electronics from SDMCE, Dharwad, under Karnatak University, Karnataka-state, India, in 1994. She completed her Ph.D. in the year 2012, from Kuvempu University, Karnataka-state, India, in the area Electronics and Communication Engineering. She served at various positions in various engineering colleges and university. Currently, she is working as Professor, and, Head for Innovation and Entrepreneurship cell at CMR Institute of Technology, Bengaluru, Karnataka-State, India. Her Research interest is: Information Theory and Coding, Signal Processing and Embedded Applications.