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Face Recognition using Discrete Haar Wavelet Transforms

Meenu Kumari, Anil Kumar, Manish Saxena

Abstract: ***Wavelet transform is being applied as an important analyzing tool in signal and image processing because it provides better signal representations with less computation time because of property of multiresolution analysis. An analogue image is transformed into a discrete image in a 2D space through a sampling process. The wavelet transforms and analysis of discrete image are performed using Haar wavelet, level-2. Through analyzing and comparing different patterns, one or more persons are identified in the face recognition process. It is accomplished by using correlation coefficient and Euclidean distance of scaling coefficients of wavelet transformed images. In the face recognition the typically facial features are extracted and compared to a database for finding the best match. Matching of features is performed between test image and stored database images.***

Keywords: ***Approximation, correlation, Euclidean, face, image, wavelet.***

I. INTRODUCTION

Wavelet theory is a new concept for analyzing one dimensional as well as two dimensional signals. Wavelet transforms provide better results over Fourier transforms (in which any stationary signal or function is represented as a sum of sin or cosine terms), when functions having discontinuities and sharp peaks are studied. This is also useful for analyzing finite, non-stationary and short duration (transient) signals. Frequency localization property is found in both Fourier and wavelet transforms. But an additional time localization property is found in the wavelet transforms than that of Fourier transforms. Due to this, many signals which are non-sparse in the Fourier domain may be very sparse in the wavelet domain [1]. A wavelet is a small wave or mathematical function by which a continuous time signal or a given function is divided into different scale or frequency components. Each scale or frequency component is assigned a particular frequency range. The processing of a signal or function with help of wavelets is called wavelet transform. The dilated and translated forms of mother wavelet are called daughter wavelets. Both continuous and discrete wavelet transforms are used to represent and analyze continuous time signals.

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* Correspondence Author
Meenu Kumari*, Department of Physics, IFTM University, Moradabad-244001, India, E-mail: meenu127.mbd@gmail.com
Anil Kumar*, Department of Physics, Hindu College, Moradabad-244001, India,Email:akumarmbd@gmail.com
Manish Saxena, Department of Physics, Moradabad Institute of Technology, Moradabad -244001, India, E-mail:manishsaxena67@yahoo.co.in

Continuous wavelet transforms are operated on all possible dilation and translation while discrete wavelet transforms are used in particular subset of dilation and translation values [2]. Wavelets provide better signal representations because of multi-resolution analysis and additional time localized properties.
That is why, the wavelet transforms have become important tool now for many applications in place of the Fourier transforms in Physics and Engineering. It is also frequently being used in signal and image processing, climate dynamics, speech recognition, signature identifying, computer graphics and , multi-fractal analysis.
In face recognition or identification [3] facial features in terms of discrete wavelet coefficients are extracted and compared with database images. face recognition is an important part of many biometric, security and surveillance systems as well as image and video indexing systems. Therefore, it has become a popular area of research in many branches of Physics and Engineering.

II. DISCRETE WAVELET TRANSFORMS

Wavelet is a small wave having oscillatory behaviour for a short time interval and then disappeared [4]. Generally, wavelets are intentionally generated to have specific and desirable properties that make them useful for signal processing.. With help of wavelet transforms, the spectral analysis of any signal is performed. A signal is broken into two orthogonal subspaces up to different levels using adaptive wavelet. By wavelet transforms of a signal, we find the localized time and frequency information of a signal or function.

A. *Haar wavelet*

The Haar wavelet is discrete in nature and identical with a step function [5]. The mother wavelet function ψ(t) is defined as:-

$$\psi(t) = \left.\begin{matrix} 1 \text{ for } 0 < t < 1/2 \\ -1 \text{ for } 1/2 < t < 1 \\ 0 \text{ elsewhere} \end{matrix}\right\} \quad (1)$$

The scaling function $\phi(t)$ of Haar wavelet is defined as:-

$$\phi(t) = \left.\begin{matrix} 1 \text{ for } 0 < t < 1 \\ 0 \text{ otherwise} \end{matrix}\right\} \quad (2)$$

Its wavelet and scaling function are shown in figure 1.

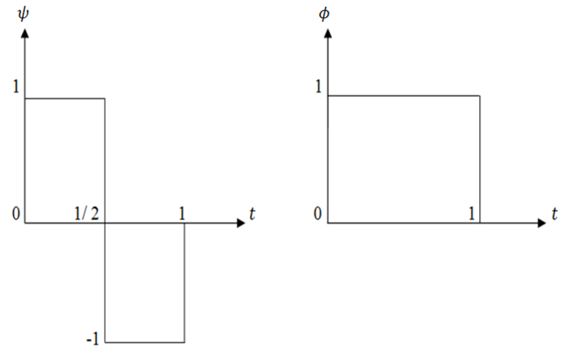

Fig. 1: Haar wavelet and scaling function

Haar wavelet conserves the energy of signal and its compaction. Haar wavelet transform provides a simple and computationally efficient framework for analysing the local aspects of a signal. That is why, it is being used mainly in image processing and computer graphics. Haar wavelet is orthogonal, bi-orthogonal and compact supported [6].

A wavelet function is defined as:-

$$\psi_{a,b} = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right) \quad (3)$$

where a and b are two real numbers. Let us choose $a = 2^{-j}$ and $b/a = k$, we obtain the discrete wavelet,

$$\psi_{j,k}\,(x) \;=\; 2^{-j\,/\,2}\psi\,(2^{-j}\,x \;-\; k) \quad (4)$$

where $j, k \in \mathbb{Z}$.

B. ***Multiresolution analysis***

A multiresolution analysis (MRA) is a new method of discrete wavelet analysis in which the resolution scale is changed at per requirement. An MRA introduced by Mallat [7] is consisting of a sequence $V_j : j \in \mathbb{Z}$ of closed subspaces of square integrable functions called Lebesgue space $L^2(\mathbb{R})$, satisisfy the following properties:-

(1) $V_{j+1} \subset V_j : j \in \mathbb{Z}$

(2) $\cap_{j\in\mathbb{Z}} V_j = \{0\}$, $\cup_{j\in\mathbb{Z}} W_j = L^2(\mathbb{R})$,

(3) For every, $L^2(\mathbb{R}), f\,(x) \in V_j \Rightarrow f\left(\frac{x}{2}\right) \in V_{j+1}, \;\forall\; j \in \mathbb{Z}$

(4) There exists a function $\phi(x) \in V_j$ such that $\{\phi(x-k): k \in \mathbb{Z}\}$ is orthonormal basis of V_j.

Here function $\phi\,(x)$ is known as a scaling function of given MRA and with help of property 3) it can be expressed as follows:-

$$\phi(x) = \sqrt{2}\,\textstyle\sum_{k\in\mathbb{Z}} h_k\;\phi(2x-k) \quad (5)$$

where h_k is low pass filter and is defined as:-

$$h_k = \int_{-\infty}^{\infty} \phi\,(x)\phi\,(2x-k)\,dx \quad (6)$$

The wavelet function ψ is expressed as:-

$$\psi(x) = \sqrt{2}\,\textstyle\sum_{k\in\mathbb{Z}} g_k\;\phi(2x-k) \quad (7)$$

where, $g_k = (-1)^{k+1}h_{1-k}$ Any space V_j can be expressed as a combination of two subspace,

$$V_j \;=\; V_{j+1} \oplus W_{j+1},\;\forall j \in \mathbb{Z} \quad (8)$$

where W_{j+1} is the orthogonal complement of V_{j+1} in V_j ($V_{j+1} \perp W_{j+1}$). By iteration, we can write,

$$V_j \;=\; W_{j+1} \oplus W_{j+2} \oplus W_{j+2} \ldots\ldots\ldots$$

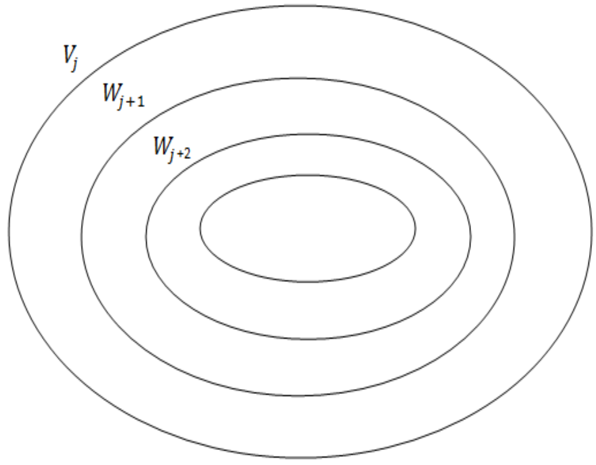

Fig. 2: Decomposition of signal space

A signal is recursively decomposed into mutually orthogonal set of many subspaces. With help of MRA, a signal can be analyzed and synthesized with less computation time. An image signal is passed through an analysis filter bank through a decimation process [8]. The analysis filter bank consists of a low pass and a high pass filter at each decomposition level. The low pass filter corresponds to an approximation operation, extracts the coarse information and the high pass filter corresponds to a differential operation, extracts the detail information of the signal. Thereafter, the filtering output is passed through decimation process.

III. WAVELET ANALYSIS OF SIGNAL

We can approximate a discrete signal in space of square summable sequences $\ell^2(\mathbb{Z})$ as follows:

$$f[n] = \frac{1}{\sqrt{M}}\textstyle\sum_k a[j_0,k]\phi_{j_0,k}\,[n] + \frac{1}{\sqrt{M}}\sum_{j=j_0}^{\infty}\sum_k d[j,k]\psi_{j,k}\,[n] \quad (9)$$

Here $f[n]$, $\phi_{j_0,k}[n]$ and $\psi_{j,k}[n]$ are discrete functions defined in range $[0, M-1]$ having total M points. Here the sets $\{\phi_{j_0,k}[n]\}_{k\epsilon\mathbb{Z}}$ and $\{\psi_{j,k}[n]\}_{j,k\epsilon\mathbb{Z},j\geq j_0}$ are mutually orthogonal. The wavelet coefficients are calculated by taking the inner product,

$$a[j_0,k] = \frac{1}{\sqrt{M}}\textstyle\sum_n f[n]\;\phi_{j_0,k}\,[n]$$

$$d[j,k] = \frac{1}{\sqrt{M}}\textstyle\sum_n f[n]\;\psi_{j,k}\,[n]$$

with $j \geq j_0$, where $a[j_0,k]$ and $d[j,k]$ are called approximation and detailed coefficients respectively. The approximation, or scaling, coefficients show the low pass representation and the detail coefficients show the high pass representation of the signal [9]. At each subsequent stage, the approximation coefficients can be further divided into an approximation and a detail parts.

A. ***Wavelet analysis of image***

In digitization, an analogue image $f(x,y)$ in a two dimensional continuous space is transformed into a digital image $f[m,n]$ through a sampling process. The $2D$ analogue image is divided into a matrix of M rows and N columns [10].

The intersecting area of a row and a column is named as a pixel. The values assigned to the integer coordinates $[m, n]$ are $\{m = 0, 1, 2, \ldots, M-1\}$ and $\{n = 0, 1, 2, \ldots, N-1\}$. A wavelet orthonormal basis in $L^2(\mathbb{R}^2)$ is built up from (tensor) products involving a scale function ϕ associated to a multiresolution $\{V_j\}_{j\in\mathbb{Z}}$ of $L^2(\mathbb{R})$ and a wavelet ψ whose dilated-translated $2^{-j/2}\psi(2^j x - n)$ form an orthonormal basis of $L^2(\mathbb{R}) = \oplus_j W_j$. In the $2D$ Fourier transforms, the basis used are $\exp j(\omega_1 x + \omega_2 y)$ in place of $\exp j(\omega x)$. In 2D wavelet transforms, let $\phi(x, y)$ and $\psi(x, y)$ are the he scaling and wavelet function with two variables x and y. The scaling and wavelet basis functions can be defined as,

$$\phi_{j,m,n}(x, y) = 2^{-j/2}\phi(2^{-j}x - m, 2^{-j}y - n)$$
$$\psi^i_{j,m,n}(x, y) = 2^{-j/2}\psi^i\phi(2^{-j}x - m, 2^{-j}y - n)$$

where, $i = \{V, F, D\}$ (10)

There are three different wavelet functions $\psi^V(x, y)$, $\psi^H(x, y)$ and $\psi^D(x, y)$ corresponding to vertical, horizontal and diagonal functions. Since, the wavelet functions are separable, i.e. $f(x, y) = f_1(x)f_2(y)$. The scaling and wavelet functions can be easily described as,

$$\phi(x, y) = \phi(x)\phi(y)$$
$$\psi^V(x, y) = \psi(x)\phi(y)$$
$$\psi^H(x, y) = \phi(x)\psi(y)$$
$$\psi^D(x, y) = \psi(x)\psi(y)$$

Since scaling and wavelet 2D functions are separable, it is easier to analyze the $2D$ function focussing on the design of $1D$ scaling and wavelet and functions [11]. The approximation and detail coefficients of 2D image are modified to,

$$a[j_0, m, n] = \frac{1}{\sqrt{MN}}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x, y)\, \phi_{j_0,m,n}(x, y)$$
$$d^i(j, m, n) = \frac{1}{\sqrt{MN}}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x, y)\, \psi^i_{j,m,n}(x, y);$$
$$i = \{V, H, D\} \quad (11)$$

$$f(x, y) = \frac{1}{\sqrt{MN}}\sum_m\sum_n a[j_0, m, n]\, \phi_{j_0,m,n}(x, y) + \frac{1}{\sqrt{MN}}\sum_{i=V,H,D}\sum_{j=j_0}^{\infty}\sum_m\sum_n d^i(j, m, n)\, \psi^i_{j,m,n}(x, y) \quad (12)$$

The functions $\{\psi^V_{j,n}, \psi^H_{j,n}, \psi^D_{j,n}\}$ form an orthonormal basis of the subspace of details,

$W_j^2 = (V_j \otimes W_j)\,(W_j \otimes V_j\ ) \oplus (W_j \otimes W_j\ )$

at scale j, i.e.,

$$L^2(\mathbb{R}^2) = \oplus_j W_j^2 \quad (13)$$

B. *Images, Euclidean Distance and Correlation*

The bases e_1, e_2 ,….., e_{MN} form a coordinate system of the image space of dimension MN having $M \times N$ size images. The Euclidean distance [12] of images is determined in terms of metric coefficients of the basis given. The metric coefficients g_{ij} $i, j = 1, 2, \ldots\ldots, MN$, of an image are defined as,

$$g_{ij} = \langle e_i, e_j\rangle = \sqrt{\langle e_i, e_i\rangle}\sqrt{\langle e_j, e_j\rangle}.\cos\theta_{ij}$$

where <, > represents to the inner product, and θ_{ij} to the angle between e_i and e_j. If,

$$\langle e_i, e_i\rangle = \langle e_j, e_j\rangle = \ldots,$$

we can say that all the basis vectors have the equal length, then g_{ij} is the function of angle θ_{ij} only. The Euclidean distance of two images X and Y in terms of metric coefficients is described as:-

$$d_E^2(X, Y) = \sum_{i,j=1}^{MN} g_{ij}(X^i - Y^i)\,(X^j - Y^j)$$
$$= (X - Y)^T G(X - Y) \quad (14)$$

where the symmetric matrix G = $\left(g_{ij}\right)_{MN}$ is also named as metric matrix. MNth order of an image of size MxN and definite symmetric identity matrix G decides a Euclidean distance. The Euclidean distance of two images can be written as,

$$d_E^2(X, Y) = \sum_{k=1}^{MN}(X^k - Y^k)^2 \quad (15)$$

Correlation describes a measure of similarity relation between two signals, data or functions [13]. Correlation coefficient between two images X and Y is described as follows:-

$$r = \frac{\sum_m\sum_n(X_{mn}-\bar{X})(Y_{mn}-\bar{Y})}{\sqrt{(\sum_m\sum_n(X_{mn}-\bar{X})^2)(\sum_m\sum_n(Y_{mn}-\bar{Y})^2)}} \quad (16)$$

where $\bar{X}$ and $\bar{Y}$ are the average of observations of variables X_{mn} and Y_{mn} respectively

IV. METHODOLOGY

The wavelet analysis for 2D image is performed through generalizing 1D wavelet analysis. The wavelet basis functions for 2D image are of three kinds: $\psi_{j,m}(x), \phi_{k,m}(y)$, $\phi_{j,m}(x), \psi_{k,m}(y)$ and $\psi_{j,m}(x), \psi_{k,m}(y)$. In terms of these three kinds wavelet basis functions, a function or 2D image $F(x, y)$ can be expressed as,

$$F(x, y) = \sum_{i=H,V,D}\sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} g_{j,k}\, \psi^i_{j,k}(x, y) \quad (17)$$

where,

$$g_{j,k} = \int_{\mathbb{R}^2} F(x, y)\, \psi^i_{j,k}(x, y)dx\, dy.$$

Here $\psi^i_{j,k}(x, y)$ stands for any one of the three types of basis functions. A scaling series using $\{\phi_{j,M}(x), \phi_{k,M}(y)\}$ also exists the basis. A discrete wavelet transform of an image of size $M \times N$, where both M and N are even, is performed in two steps. It produces the following first-level transform.

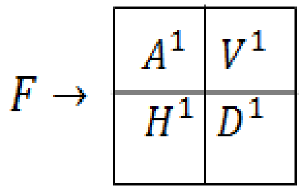

Fig. 3: Image processing at level-1

where A^1, V^1, H^1 and D^1 are each $M/2 \times N/2$ size submatrices. The scaling coefficients A^1 corresponds th the trend whereas the wavelet coefficients V^1, H^1 and D^1 correspond to the fluctuations consisting of each of the three kinds of wavelet basis functions. The trend consisting of scaling coefficients for the scaling basis $\{\phi_{j,M-1}(x), \phi_{k,M-1}(y)\}$ occupies the upper left quadrant of the transforms, and represents the lower resolution version of the original image. The vertical fluctuation V^1 consisting of wavelet coefficients for the basis elements $\psi_{j,M-1}(x), \phi_{k,M-1}(y)$, occupies the upper right quadrant of the transforms. Similarly the H^1 occupies the lower left quadrant of the transforms. The horizontal fluctuation and vertical fluctuation are identical but their roles are reversed. There is the diagonal fluctuation D^1 consisting of the wavelet coefficients for the basis elements $\psi_{j,M-1}(x), \psi_{k,M-1}(y)$, occupies the lower right quadrant of the transforms.

In the same manner, the second order wavelet transform is performed as:-

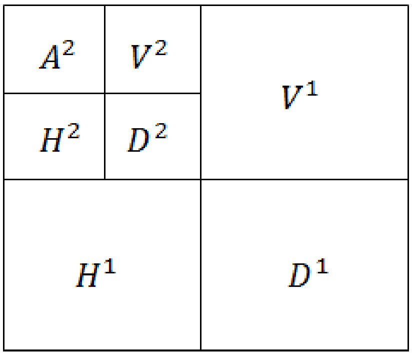

Fig. 4: Image processing at level-2

We have developed a face recognition system, in which we are using 15 images out of which two images are of same person having different pose. We assign one image as test image (T1) and compare it with 15 data based images (D1-D15). The steps of proposed work are as follows:-

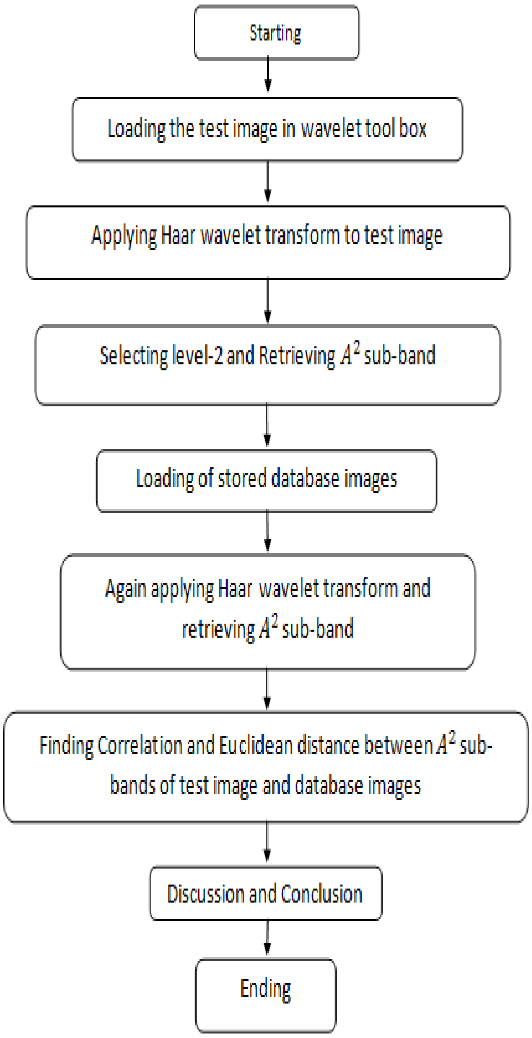

Fig. 5: Flow-chart of face recognition using approximation sub-bands

V. RESULTS AND DISCUSSION

We consider the test image T1, as follows:-

Fig. 6: Test image (T1)

The 15 database images are as follows:-

Fig.7: Database images (D1-D15)

We select original image T1 as a test image for experiment analysis. We take Haar wavelet transform at level 2 and retrieve A^2 sub-band. In Table 1, correlation coefficient and Euclidean distance between test image and database images for level 2 are shown as follows:-

Table- I: Comparison of approximation sub-bands of test and database images

Test & Database Image	Correlation Coefficient	Euclidean distance
T1-D1	0.799007652	4.2969×10^3
T1-D2	0.696404307	6.1474×10^3
T1-D3.	0.54714295	7.0418×10^3
T1-D4.	0.58940092	6.2038×10^3
T1-D5	0.463466281	6.8772×10^3
T1-D6	0.607258073	8.5751×10^3
T1-D7	0.585050747	8.4941×10^3
T1-D8	0.38781801	9.5820×10^3
T1-D9	0.343902601	9.1579×10^3
T1-D10	0.565973211	1.1767×10^4

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T1-D11	0.55449083	9.3658×10^{3}
T1-D12	0.540936532	1.0193×10^{4}
T1-D13	0.506927815	1.1087×10^{4}
T1-D14	0.48197378	1.4404×10^{4}
T1-D15	0.536904387	1.1671×10^{4}

From table- I, it is clear that correlation coefficients between test image (T1) and database images (D1 and D2) are greater compared to the correlation coefficients between A^2 sub-bands of test image and rest database images for Haar wavelet, level 2. Also the Euclidean distances between A^2 sub-bands of test image (T1) and database images (D1 and D2) are smaller compared to the Euclidean distances between A^2 sub-bands of test image and rest database images (D3-D15). Here D1 and D2 are different pose of a single person; hence the correlation coefficients and Euclidean distances for them are extremum.

VI. CONCLUSION

The wavelet transforms perform averaging and differencing between pixel values to form the approximation and detail coefficients of a signal (image). Most of the energy of the signal is compacted into the approximation sub signal. The low pass filter corresponds to approximation operation, and extracts the coarse information of the signal, while the high pass filter corresponds to detail operation, and extracts the fine information of the signal. The trend of the image is the lower resolution version of the original image and occupies the upper left quadrant of the transforms. The face recognition is addressed in wavelet domain using sub-band image representing the approximation of the image and is used to extract the face features. The wavelet and statistical analysis of trend of images taking second order approximation coefficients using Haar wavelet, level-2 is performed. This method is useful in the face recognition even in the condition of different poses or face expressions of a person.

Taking into account these results, it is concluded that the wavelet analysis of 2D signal (image) provides a simple and accurate framework for extracting the face and its recognition.

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AUTHORS PROFILE

Meenu Kumari (Date of Birth: July 12, 1991) secured her graduation in 2009 and post-graduation in Physics in 2011. She is presently pursuing her Ph. D. from A. K.T. University, Lucknow, Uttar Pradesh. She has been working as Assistant Professor in School of Science, IFTM University, Moradabad, Uttar Pradesh, since 2011.

Dr. Anil Kumar (Date of Birth: June 12, 1972) did his M.Sc. (Physics) in 1996 and Ph.D. on *Study of Wavelets and Signal Processing*, in 2008 from Mahatma Jyotiba Phule Rohilkhand University, Bareilly, Uttar Pradesh. His major fields of study are Material Science, Environmental Physics and Signal Processing. Dr. Kumar is presently working as an Associate Professor in the Department of Physics, Hindu College, Moradabad, INDIA and teaching Physics there since Oct. 27, 1999. He has published 26 research papers in National/International Research Journals. Dr. Kumar is life member of Indian Physics Association, Indian Association of Physics Teachers, and Indian Science Congress Association.

Dr. Manish Saxena secured his Graduation and Post Graduation degrees in Physics in 1985 and 1987. He secured his M. Phil. Degree in 1989 and was awarded Ph.D. in 1995 from University of Delhi, New Delhi. He also secured DIM, PGDBM, PGDFM, MBA (Finance) from IGNOU, New Delhi. Dr. Manish Saxena has been working in Moradabad Institute of Technology, Moradabad since its inception in 1996. Dr. Saxena had also taught to M.Sc. (Electronics) and M. Tech. (Microwave Electronics) students in Delhi University from 1991-1993. Apart from regular teaching assignment, Dr. Saxena is also continuously involved in Research activities. He is life member of various Internationally acclaimed scientific societies like ISCA, MRSI, SSI and ISTE. He has more than 100 quality publications to his credit in various National / International Journals and Conferences. Seven of his research scholars have been awarded Ph.D. degrees under his supervision and at present three research scholars are pursuing for Ph.D. degree. Dr. Saxena has also reviewed a number of research papers as referee for reputed International Journals associated with renowned publishing greats like Springer.