OPEN ACCESS
International Journal of Recent Technology and Engineering (IJRTE)
ISSN: 2277-3878, Volume-8 Issue-3, September 2019

Bipolar Fuzzy Soft Digraph

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***Abstract:** Fuzzy sets and soft sets are two different tools for representing uncertainty and vagueness. In this paper , we combine the concepts of bipolar fuzzy soft sets and digraph theory and we introduce the notions of Bipolar fuzzy soft digraphs, Bipolar fuzzy soft subgraph in digraph and some operations in Bipolar fuzzy soft digraph.*

***Keywords:** Bipolar fuzzy soft digraphs Bipolar fuzzy soft subgraph in digraph, Strong Bipolar fuzzy soft digraph, complete Bipolar fuzzy soft digraph and Cartesian product of Bipolar fuzzy soft digraph.*

I. INTRODUCTION

The Concept of soft set theory was initiated by Molodtsov [1] for dealing uncertainties. A Rosenfeld [2] developed the theory of fuzzy graphs in 1975 by considering fuzzy relation on fuzzy set, which was developed by Zadeh [3] in the year 1965.Some operations on fuzzy graphs are studied by Mordeson a C.S. Peng [4] .Later Ali et al. discussed about fuzzy sets and fuzzy soft sets induced by soft sets. M.Akram and S Nawaz [5] introduced fuzzy soft graphs in the year 2015. Sumitmohinta and T K samanta [6] also introduced fuzzy soft graphs independently.

The concept of bipolar fuzzy sets which is a generalization of fuzzy sets [3] was initiated by Zhang [11]. Bipolar fuzzy sets whose range of membership degree is [-1,1]. If the membership degree of an element is 0, then the element is irrelevant to the corresponding property. If the membership degree of an element is within (0,1], then the element some what satisfies the implicit counter property. If the membership degree of an element is within [-1,0), then the element some what satisfies the implicit counter property. It is emphasized that positive information gives us what is granted to be possible, while negative information gives us what is considered to be impossible.

In this paper, we apply concept of bipolar fuzzy soft sets to digraph structure. We introduce notations of bipolar fuzzy soft digraph ,strong bipolar fuzzy soft digraph and complete bipolar fuzzy soft digraph. We also present cartesian product on bipolar fuzzy soft digraphs and investigate some properties of them.

II. PRELIMINARIES

We now review some elementary concepts of digraph and fuzzy soft graph

Manuscript published on 30 September 2019
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Definition: 2.1

Let U be an initial universe set and E be the set of parameters. Let A $\subset E$, A pair (F,A) is called ***fuzzy soft set*** over U where F is a mapping given by
F : A → I^U, where I^U denotes the collection of all fuzzy subsets of U.

Definition: 2.2

Let V be a nonempty finite set and $\sigma : V \to [0,1]$. Again , let $\mu : VXV \to [0,1]$such that $\mu(x,y) \le \sigma(x) \wedge \sigma(y)$for all $(x,y) \in VXV$. Then the pair $G = (\sigma, \mu)$ is called ***a fuzzy graph over the set V***. Here $\sigma and \mu$ are respectively called *fuzzy vertex and fuzzy edge* of the fuzzy graph G = (σ, μ)

Definition: 2.3

A fuzzy digraph $G_D = (\sigma_D, \mu_D)$ is a pair of function $\sigma_D : V \to [0,1]$ and $\mu_D : VXV \to [0,1]$
Where $\mu_D(x,y) \le \sigma_D(x) \wedge \sigma_D(y)$ for all $(x,y) \in VXV$ and μ_D is a set of fuzzy directed edges are called *fuzzy arcs.*

Definition: 2.4

The ***degree of any vertex***$\sigma(x_i)$ of a fuzzy graph is sum of degree of membership of all those edges which are incident on a vertex $\sigma(x_i)$ and is denoted by $deg(\sigma(x_i))$.

Definition: 2.5

In a fuzzy digraph the number of arcs directed away from the vertex$\sigma(x)$is called the ***outdegree of vertex***, it is denoted by $od(\sigma(x))$. The number of arcs directed to the vertex $\sigma(x)$ is called ***indegree of vertex,*** it is denoted by $id(\sigma(x))$.

The ***degree of vertex$\sigma(x)$ in a fuzzy digraph*** is defined to be $deg(\sigma(x)) = id(\sigma(x)) + od(\sigma(x))$.

Definition: 2.6

Let $G = (\sigma, \mu)$ be a fuzzy graph. The ***Order of G*** $= (\sigma, \mu)$ is defined as

$$O(G) = \sum_{u \in V} \sigma(u)$$

and***the size of G*** $= (\sigma, \mu)$ is defined as

$$S(G) = \sum_{u,v \in V} \mu(u,v).$$

Definition: 2.7

The ***Cartesian product*** of a family of digraphs $\{D_1, D_2, D_3, \dots, D_n\}$ denoted by $D_1 X D_2 X D_3 X \dots X D_n$

or $\prod_{i=1}^{n} D_i$, where $n \ge 2$ is the digraph D having $V(D) = V(D_1) X V(D_2) X \dots V(D_n)$

$$= \{(w_1, w_2, \dots w_n) : w_i \in V(D_i), i = 1,2 \dots n\}$$

and a vertex $(u_1, u_2, \dots u_n)$ dominates a vertex $(v_1, v_2, \dots, v_n)$ of D iff there exists an $r \in \{1,2, \dots n\}$ such that

$u_r v_r \in A(D_r)$ and $u_i = v_i$ for all $i \in \{1,2,\ldots.n\} - \{r\}$.

$\mu_{\widetilde{D}}$	(x_1, x_2)	(x_2, x_3)	(x_3, x_4)	(x_4, x_1)	(x_3, x_1)
e_1	0.6,-0.2	0.7,-0.1	0	0	0
e_2	0.3,-0.1	0.1,-0.4	0	0	0.4,-0.1
e_3	0.5,-0.1	0.4,-0.3	0.4,-0.2	0.3,-0.1	0

Definition: 2.8

A digraph D is ***strongly connected (or just , Strong)*** if , for every pair x,y of distinct vertices in D , There exist an (x,y) – path and a (y,x) path . In Other words, D is strong if every vertex of D is reachable from every other vertex of D.

III. BIPOLAR FUZZY SOFT DIGRAPH

We introduce some concept of Bipolar fuzzy soft digraph in this section

Definition: 3.1

Let $V = \{x_1, x_2, x_3, \ldots., x_n\}$ non empty set, $E($ parameter set) and $A \subseteq E$, also

(i) $(\rho_{\widetilde{D}}, A)$is a bipolar fuzzy soft vertex

(ii) $(\mu_{\widetilde{D}}, A)$is a bipolar fuzzy soft directed edges are called fuzzy arcs.

(iii) $(\rho_{\widetilde{D}}^{(e)}, \mu_{\widetilde{D}}^{(e)})$is a bipolar fuzzy graph over all $e \in A$.

i.e.,$\mu^{+}_{\mu_{\widetilde{D}}^{(e)}}(x,y) \leq \min\left(\mu^{+}_{\rho_{\widetilde{D}}^{(e)}}(x), \mu^{+}_{\rho_{\widetilde{D}}^{(e)}}(y)\right)$

and$\mu^{-}_{\mu_{\widetilde{D}}^{(e)}}(x,y) \geq max\left(\mu^{-}_{\rho_{\widetilde{D}}^{(e)}}(x), \mu^{-}_{\rho_{\widetilde{D}}^{(e)}}(y)\right)$ for all $\{x,y\} \in (\mu_{\widetilde{D}}, A)$

Through out this paper , we denote

$\widetilde{H}(e) = (\rho_{\widetilde{D}}^{(e)}, \mu_{\widetilde{D}}^{(e)})$is a bipolar fuzzy digraph and

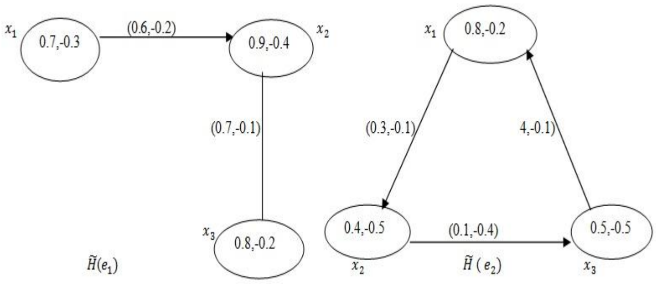
$\widetilde{D}_{A,V} = ((\rho_{\widetilde{D}}, A), (\mu_{\widetilde{D}}, A))$is a ***bipolar fuzzy soft digraph***

Example:3 .2

Consider a bipolar fuzzy soft digraph $\widetilde{D}_{A,V}$where V = { x_1, x_2, x_3, x_4 } $andA = \{e_1, e_2, e_3\}$ $\widetilde{D}_{A,V}$Described by Table 3.1 and $\mu_{\widetilde{D}}^{e}(x_i, x_j) = 0$ $forall(x_i, x_j) \in VXV$ / $\{(x_1, x_2), (x_2, x_3),$ $(x_3, x_4), (x_4, x_1), (x_3, x_1)$ } and for all e ∈ E

Table 3.1

$\rho_{\widetilde{D}}$	x_1	x_2	x_3	x_4
e_1	0.7,-0.3	0.9, -0.4	0.8,-0.2	0
e_2	0.8,-0.2	0.4,-0.5	0.5,-0.5	0
e_3	0.5,-0.2	0.7,-0.3	0.4,-0.3	0.7,-0.3



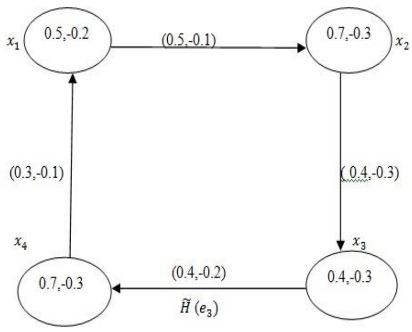


Fig 3.2 $\widetilde{D}_{A,V} = \left(\widetilde{H}(e_1), \widetilde{H}(e_2), \widetilde{H}(e_3)\right)$

Bipolar Fuzzy soft Digraph

Definition 3.3

Let $\widetilde{D}^1 = (V_1, A)$, $\widetilde{D}^2 = (V_2, B)$ be two bipolar fuzzy soft digraphs. Then $\widetilde{D}^1$ is called bipolar fuzzy soft subgraph of digraph of $\widetilde{D}^2$ if

(i) $A \subseteq B$

(ii) $\widetilde{H}_1(e) = (\rho^{(e)}_{\widetilde{D}^1}, \mu^{(e)}_{\widetilde{D}^1})$ is a ***bipolar fuzzy soft subgraph*** of digraph of

$\widetilde{H}_2(e) = (\rho^{(e)}_{\widetilde{D}^2}, \mu^{(e)}_{\widetilde{D}^2})\ \forall e \in A.$

Example 3 .4

Consider the bipolar fuzzy soft graph $\widetilde{D}_{A,V}$ as taken in Example 3.2. and also Consider a bipolar fuzzy soft digraph $\widetilde{D}^2_{A,V}$ where $V_2 = \{x_1, x_2, x_3, x_4\}$ *and*

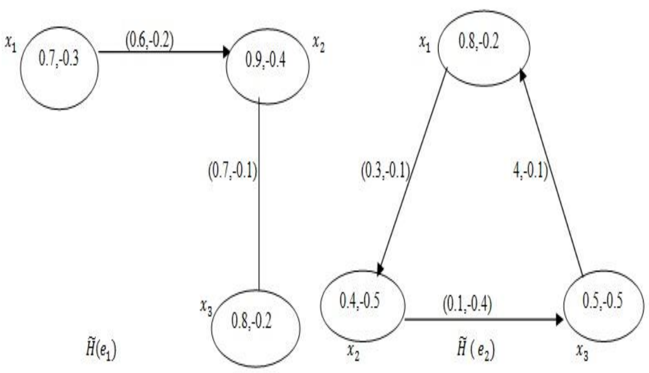
$$B = \{e_1, e_2, e_3\}$$

$\widetilde{D}^2_{B,V}$ described by Table 3.2 and $\mu^e_{\widetilde{D}^2}(x_i, x_j) = 0$ $forall\ (x_i, x_j) \in V_2 X V_2 \ / \ \{(x_1, x_2), (x_2, x_3),$ $(x_3, x_1)\}$ and for all e $\in B$

Table 3.1

$\rho_{\widetilde{D}}$	x_1	x_2	x_3	x_4
e_1	0.7,-0.3	0.9, -0.4	0.8,-0.2	0
e_2	0.8,-0.2	0.4,-0.5	0.5,-0.5	0
e_3	0.5,-0.2	0.7,-0.3	0.4,-0.3	0.7,-0.3

$\mu_{\widetilde{D}}$	(x_1, x_2)	(x_2, x_3)	(x_3, x_4)	(x_4, x_1)	(x_3, x_1)
e_1	0.6,-0.2	0.7,-0.1	0	0	0
e_2	0.3,-0.1	0.1,-0.4	0	0	0.4,-0.1
e_3	0.5,-0.1	0.4,-0.3	0.4,-0.2	0.3,-0.1	0



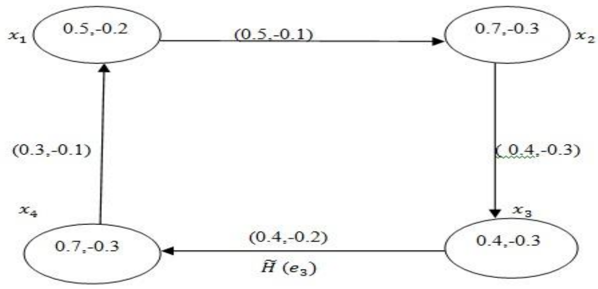


Fig 3.1 $\widetilde{D}_{A,V} = \left(\widetilde{H}(e_1), \widetilde{H}(e_2), \widetilde{H}(e_3)\right)$
Bipolar Fuzzy soft Digraph

Definition 3.3

Let $\widetilde{D}^1 = (V_1, A)$, $\widetilde{D}^2 = (V_2, B)$ be two bipolar fuzzy soft digraphs. Then $\widetilde{D}^1$ is called bipolar fuzzy soft subgraph of digraph of $\widetilde{D}^2$ if

(i) $A \subseteq B$

(ii) $\widetilde{H}_1(e) = (\rho^{(e)}_{\widetilde{D}^1}, \mu^{(e)}_{\widetilde{D}^1})$ is a ***bipolar fuzzy soft subgraph*** of digraph of $\widetilde{H}_2(e) = \left(\rho^{(e)}_{\widetilde{D}^2}, \mu^{(e)}_{\widetilde{D}^2}\right) \ \forall e \in A.$

Example 3 .4

Consider the bipolar fuzzy soft graph$\widetilde{D}_{A,V}$ as taken in Example 3.2. and also Consider a bipolar fuzzy soft digraph $\widetilde{D}^2_{A,V}$where $V_2 = \{ x_1, x_2, x_3, x_4 \}$ *and*

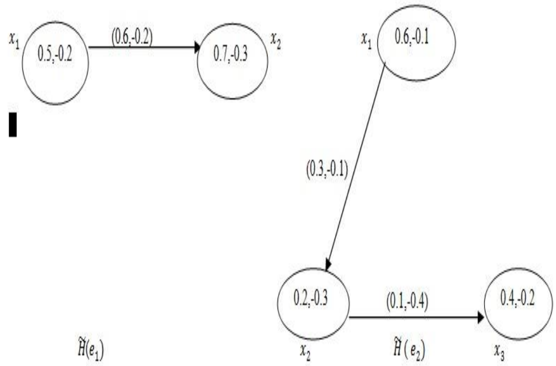
$$B = \{e_1, e_2, e_3\}$$

$\widetilde{D}^2_{B,V}$ described by Table 3.2 and $\mu^e_{\widetilde{D}^2}(x_i, x_j) = 0$ *for all* $(x_i, x_j) \in V_2 X V_2 / \{(x_1, x_2), (x_2, x_3), (x_3, x_1)\}$ and for all e $\in B$

Table 3.2

$\rho_{\widetilde{D}^2}$	x_1	x_2	x_3
e_1	0.5,-0.2	0.7,-0.3	0
e_2	0.6,-0.1	0.2,-0.3	0.4,-0.2
e_3	0.3,-0.1	0.5,-0.2	0.2,-0.1

$\mu_{\widetilde{D}^2}$	(x_1, x_2)	(x_2, x_3)	(x_3, x_1)
e_1	0.4,-0.2	0	0
e_2	0.1,-0.1	0.2,-0.1	0
e_3	0.2,-0.1	0.1,-0.1	0



International Journal of Recent Technology and Engineering (IJRTE)
ISSN: 2277-3878, Volume-8 Issue-3, September 2019

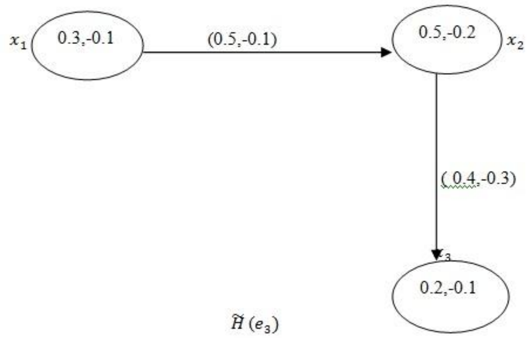


Fig 3.2 $\widetilde{D}^2_{B,V} = \left(\widetilde{H}(e_1), \widetilde{H}(e_2), \widetilde{H}(e_3)\right)$
Bipolar Fuzzy soft in Digraph

Hence $\widetilde{D}^2_{B,V}$ is a bipolar fuzzy soft subgraph of digraph $\widetilde{D}_{A,V}$

Definition 3.5

A bipolar fuzzy soft digraph $\widetilde{D}_{A,V} = (\rho_{\widetilde{D}}, \mu_{\widetilde{D}}, A)$ is a **strong bipolar fuzzy soft digraph** if it has only one disconnected and also if

$$\mu^+_{\mu^{(e)}_{\widetilde{D}}}(x,y) = \min\left(\mu^+_{\rho^{(e)}_{\widetilde{D}}}(x), \mu^+_{\rho^{(e)}_{\widetilde{D}}}(y)\right)$$

$\mu^-_{\mu^{(e)}_{\widetilde{D}}}(x,y) = \max\left(\mu^-_{\rho^{(e)}_{\widetilde{D}}}(x), \mu^-_{\rho^{(e)}_{\widetilde{D}}}(y)\right)$ and also

$\mu^+_{\mu^{(e)}_{\widetilde{D}}}(x,y) = \mu^+_{\mu^{(e)}_{\widetilde{D}}}(y,x)$ & $\mu^-_{\mu^{(e)}_{\widetilde{D}}}(x,y) = \mu^-_{\mu^{(e)}_{\widetilde{D}}}(y,x)$ for all $e \in A$ *and* (x,y)& $(y,x) \in E$.

Example:3 .6

Consider a bipolar fuzzy soft digraph $\widetilde{D}_{A,V}$ where V = { $x_1,\ x_2,\ x_3,\ x_4$ } *and* $E = \{e_1,\ e_2\}$

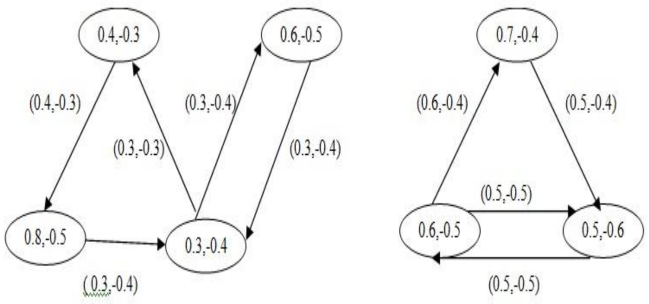
$\mu_{\widetilde{D}}$	(x_1, x_2)	(x_2, x_3)	(x_3, x_2)	(x_1, x_3)	(x_3, x_1)	(x_2, x_1)
e_1	0.4,-0.2	0	0	0	0	0.4,-0.2
e_2	0.4,-0.2	0.6,-0.2	0.6,-0.2	0.4,-0.3	0.4,-0.3	0.4,-0.2

$\widetilde{D}_{A,V}$ Described by Table 3.3 and $\mu^e_{\widetilde{D}}(x_i, x_j) = 0$ *for all* $(x_i, x_j) \in V\,X\,V$ / $\{(x_1, x_2), (x_2, x_3),$ $(x_3, x_4), (x_4, x_3), (x_3, x_1)$ } and for all e ∈ E

Table 3.3

$\rho_{\widetilde{D}}$	x_1	x_2	x_3	x_4
e_1	0.4,-0.3	0.8,-0.5	0.3,-0.4	0.6,-0.5
e_2	0.7,-0.4	0.5,-0.6	0.6,-0.5	0

$\mu_{\widetilde{D}}$	(x_1, x_2)	(x_2, x_3)	(x_3, x_4)	(x_4, x_3)	(x_3, x_1)	(x_3, x_2)
e_1	0.4,-0.3	0.3,-0.4	0.3,-0.4	0.3,-0.4	0.3,-0.3	0
e_2	0.5,-0.4	0.5,-0.5	0	0	0.6,-0.4	0.5,-0.5



$\widetilde{H}(e_1)$ $\widetilde{H}(e_2)$

Fig 3.3 $\widetilde{D}_{A,V} = \left(\widetilde{H}(e_1), \widetilde{H}(e_2)\right)$

Strong Bipolar Fuzzy soft Digraph

Definition 3.7

A Bipolar fuzzy soft digraph $\widetilde{D}_{A,V} = (\rho_{\widetilde{D}}, \mu_{\widetilde{D}}, A)$ is called ***complete*** if

$$\mu^{+}_{\mu^{(e)}_{\widetilde{D}}}(x,y) = \min\left(\mu^{+}_{\rho^{(e)}_{\widetilde{D}}}(x), \mu^{+}_{\rho^{(e)}_{\widetilde{D}}}(y)\right)$$

$$\mu^{-}_{\mu^{(e)}_{\widetilde{D}}}(x,y) = \max\left(\mu^{-}_{\rho^{(e)}_{\widetilde{D}}}(x), \mu^{-}_{\rho^{(e)}_{\widetilde{D}}}(y)\right),$$

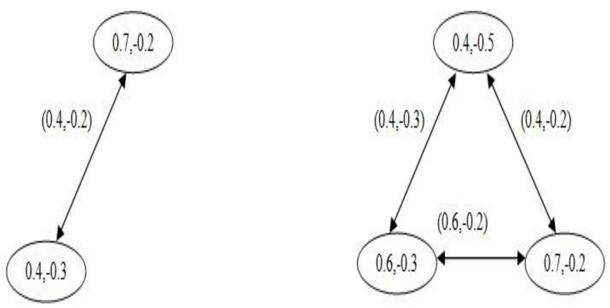
for all $e \in A$ & $x, y \in V$

Example:3 .8

Consider a bipolar fuzzy soft digraph $\widetilde{D}_{A,V}$ where V = $\{x_1, x_2, x_3\}$ $and\ E = \{e_1, e_2\}$ $\widetilde{D}_{A,V}$ Described by Table 3.4 and $\mu^{e}_{\widetilde{D}}(x_i, x_j) = 0\ for\ all\ (x_i, x_j) \in V\ X\ V\ /\ \{(x_1, x_2), (x_2, x_3), (x_3, x_2), (x_1, x_3), (x_3, x_1), (x_2, x_1)\}$ and for all e $\in E$

Table 3.4

$\rho_{\widetilde{D}}$	x_1	x_2	x_3	x_4
e_1	0.4,-0.3	0.8,-0.5	0.3,-0.4	0.6,-0.5
e_2	0.7,-0.4	0.5,-0.6	0.6,-0.5	0



$\widetilde{H}(e_1)$ $\widetilde{H}(e_2)$

Fig 3.4 $\widetilde{D}_{A,V} = \left(\widetilde{H}(e_1), \widetilde{H}(e_2)\right)$ Complete Bipolar Fuzzy soft Digraph

Definition 3.9

Let $\widetilde{D}^{1}_{A,V_1} = \left(\rho_{\widetilde{D}^1}, \mu_{\widetilde{D}^1}, A\right)$ $and\ \widetilde{D}^{2}_{B,V_2} = (\rho_{\widetilde{D}^2}, \mu_{\widetilde{D}^2}, B)$ be two bipolar fuzzy soft digraphs of simple digraphs

$D_1^* = (V_1, E_1)\ and\ D_2^* = (V_2, E_2)\ respectively.$

The **Cartesian product** of $\widetilde{D}^1_{A,V_1}$ *and* $\widetilde{D}^2_{B,V_2}$ is denoted by $\widetilde{D}^1 X\, \widetilde{D}^2 = (\rho_D, \mu_D, A\, XB)$ and is defined by

$$\left(\mu^+_{\rho^{(e)}_{\widetilde{D}^1}} X\, \mu^+_{\rho^{(e)}_{\widetilde{D}^2}}\right)(x_1, x_2) = \min\left(\mu^+_{\rho^{(e)}_{\widetilde{D}^1}}(x_1)\ , \mu^+_{\rho^{(e)}_{\widetilde{D}^2}}(x_2)\right)$$

$$\left(\mu^-_{\rho^{(e)}_{\widetilde{D}^1}} X\, \mu^-_{\rho^{(e)}_{\widetilde{D}^2}}\right)(x_1, x_2) = \max\left(\mu^-_{\rho^{(e)}_{\widetilde{D}^1}}(x_1)\ , \mu^-_{\rho^{(e)}_{\widetilde{D}^2}}(x_2)\right), \qquad \forall\, (x_1, x_2) \in V_1 X V_2.$$

$$\left(\mu^+_{\mu^{(e)}_{\widetilde{D}^1}} X\, \mu^+_{\mu^{(e)}_{\widetilde{D}^2}}\right)\left((x, x_2)(x, y_2)\right) = \min\left(\mu^+_{\rho^{(e)}_{\widetilde{D}^1}}(x)\ , \mu^+_{\mu^{(e)}_{\widetilde{D}^2}}(x_2, y_2)\right)$$

$$\left(\mu^-_{\mu^{(e)}_{\widetilde{D}^1}} X\, \mu^-_{\mu^{(e)}_{\widetilde{D}^2}}\right)\left((x, x_2)(x, y_2)\right) = \max\left(\mu^-_{\rho^{(e)}_{\widetilde{D}^1}}(x)\ , \mu^-_{\mu^{(e)}_{\widetilde{D}^2}}(x_2, y_2)\right) \qquad \forall\, x \in V_1 \text{ and } (x_2, y_2) \in E_2$$

$\mu_{\widetilde{D}^2}$	(y_1, y_2)	(y_2, y_3)	(y_3, y_1)
e_1	0.3,-0.1	0	0
e_2	0.3,-0.2	0.1,-0.1	0.2,-0.2

$$\left(\mu^+_{\mu^{(e)}_{\widetilde{D}^1}} X\, \mu^+_{\mu^{(e)}_{\widetilde{D}^2}}\right)(x_1, z)(y_1, z) = \min\left(\mu^+_{\mu^{(e)}_{\widetilde{D}^1}}(x_1, y_1), \mu^+_{\rho^{(e)}_{\widetilde{D}^2}}(z)\right)$$

$$\left(\mu^-_{\mu^{(e)}_{\widetilde{D}^1}} X\, \mu^-_{\mu^{(e)}_{\widetilde{D}^2}}\right)(x_1, z)(y_1, z) = \max\left(\mu^-_{\mu^{(e)}_{\widetilde{D}^1}}(x_1, y_1), \mu^-_{\rho^{(e)}_{\widetilde{D}^2}}(z)\right) \qquad \forall\, (x_1, y_1) \in E_1, z \in V_2.$$

Example: 3.10

Consider V = { $x_1,\ x_2,\ x_3$ } *and* $E = \{e_1,\ e_2, e_3\}$.

Let V_1 = { $x_1,\ x_2,\ x_3$} , A = {$e_1,\ e_2$ },

V_2 = { $y_1,\ y_2,\ y_3$} } & $B = \{e_1,\ e_2\}$.

$\widetilde{D}^1_{A,V_1}$ is defined *by Table* 3.5 and
$\mu^e_{\widetilde{D}^1}(x_i, x_j) = 0$ for all $(x_i, x_j) \in (V_1 X\, V_1) \backslash (x_1, x_2), (x_2, x_3), \}$ and for all e ∈ A.

$\widetilde{D}^2_{B,V_2}$ is defined by Table 3.6 and
$\mu^e_{\widetilde{D}^2}(y_i, y_j) = 0$ for all $(y_i, y_j) \in (V_2\, XV_2)/$
$\{(y_1, y_2), (y_2, y_3), (y_3, y_1)\}$ and for all
e ∈ B.

Then the Cartesian product of $\widetilde{D}^1_{A,V_1}$ and $\widetilde{D}^2_{B,V_2}$ is $\widetilde{D}^3_{C,V_3}$ given by the Table 3.7 and
$\mu^e_{\widetilde{D}^3}(x_i, x_j) = 0$ for all $\left(x_i, y_j\right) \in (V_3 X\, V_3) \backslash$
$\{(x_1\ y_1, x_1\ y_2), (x_2\ y_1, x_2\ y_2), (x_2\ y_1, x_3\ y_1),$
$(x_3\ y_1, x_3\ y_2), (x_1\ y_2, x_2\ y_2), (x_2\ y_2, x_3\ y_2),$
$(x_1\ y_3, x_1\ y_1), (x_1\ y_2, x_1\ y_3), (x_2\ y_3, x_2\ y_1),$
$(x_2\ y_2, x_2\ y_3), (x_3\ y_3, x_3\ y_1), (x_3\ y_2, x_3\ y_3),$
$(x_1 y_3, x_2 y_3), (x_2 y_3, x_3 y_3), (x_1\ y_1, x_2\ y_1)\}$
and for all e ∈ C, $V_3 = V_1 X V_2$

Table 3.5

$\mu_{\widetilde{D}^1}$	(x_1, x_2)	(x_2, x_3)
e_1	0.2,-0.1	0.1,-0.1
e_2	0.4,-0.2	0

$\rho_{\widetilde{D}^1}$	x_1	x_2	x_3
e_1	0.3,-0.1	0.5,-0.2	0.2,-0.1
e_2	0.5,-0.2	0.7,-0.3	0

Table 3.6

$\rho_{\widetilde{D}^2}$	y_1	y_2	y_3
e_1	0.4,-0.4	0.5,-0.2	0
e_2	0.6,-0.3	0.4,-0.2	0.2,-0.3

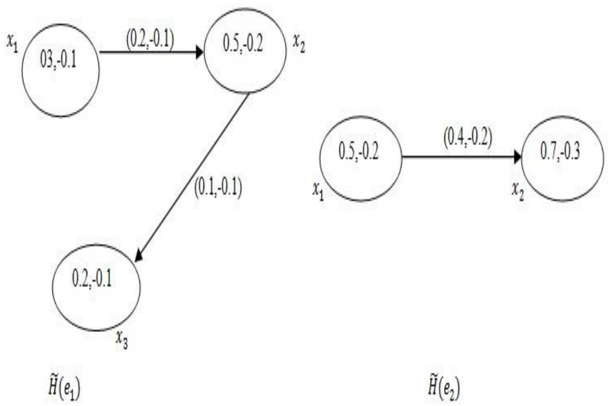


Fig 3.5 $\widetilde{D}^1_{A,V_1} = \left(\widetilde{H}(e_1), \widetilde{H}(e_2)\right)$ Bipolar Fuzzy soft Digraph

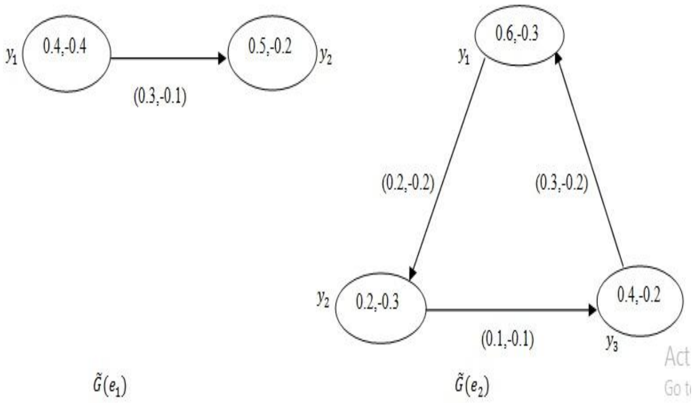


Fig 3.6 $\widetilde{D}^2_{B,V_1} = \left(\widetilde{G}(e_1), \widetilde{G}(e_2)\right)$ Bipolar Fuzzy soft Digraph

ISSN: 2277-3878, Volume-8 Issue-3, September 2019

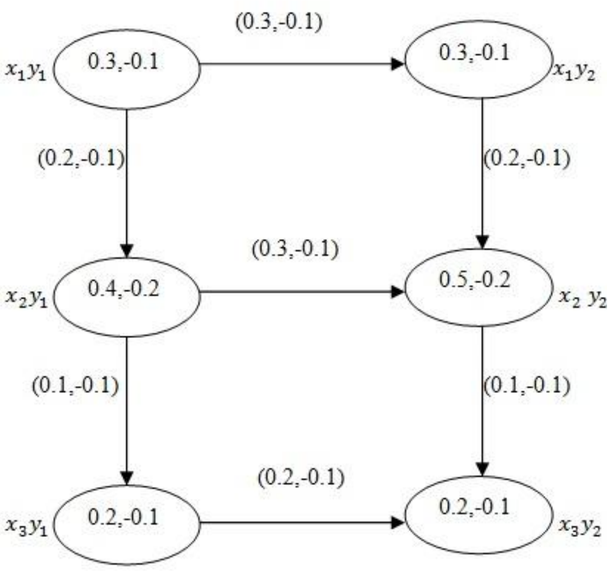
Table 3.7

$\rho_{\widetilde{D}^3}$ (H X G)	$x_1\ y_1$	$x_1\ y_2$	$x_1\ y_3$	$x_2\ y_1$	$x_2\ y_2$	$x_2\ y_3$	$x_3\ y_1$	$x_3\ y_2$	$x_3\ y_3$
$e_1\ X\ e_1$	0.3,-0.1	0.3,-0.1	0	0.4,-0.2	0.5,-0.2	0	0.2,-0.1	0.2,-0.1	0
$e_1\ X\ e_2$	0.3,-0.1	0.3,-0.1	0.2,-0.1	0.5,-0.2	0.4,-0.2	0.2,-0.2	0.2,-0.1	0.2,-0.1	0.2,-0.1
$e_2\ X\ e_1$	0.4,-0.2	0.5,-0.2	0	0.4,-0.3	0.5,-0.2	0	0	0	0
$e_2\ X\ e_2$	0.5,-0.2	0.4,-0.2	0.2,-0.2	0.6,-0.3	0.4,-0.2	0.2,-0.3	0	0	0

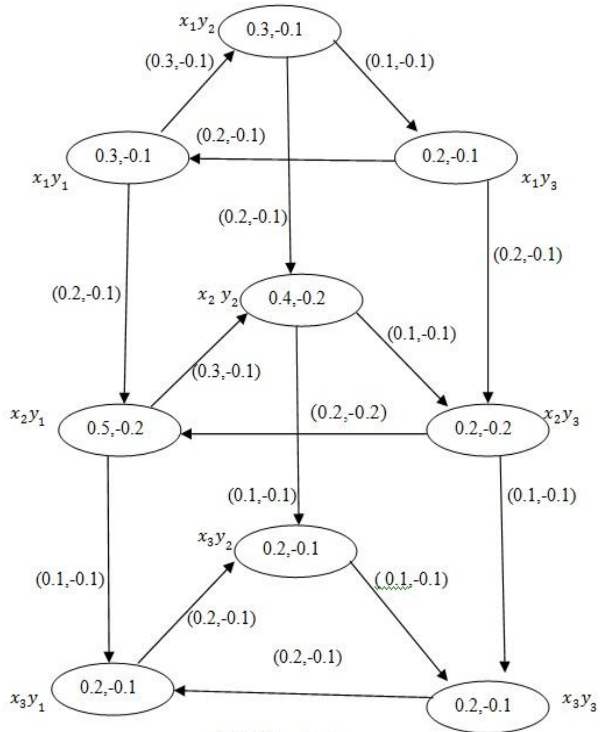
$\mu_{\widetilde{D}^3}$ (H X G)	$(x_1\ y_1\ ,x_1\ y_2)$	$(x_1\ y_1\ ,x_2\ y_1)$	$(x_2\ y_1\ ,x_2\ y_2\ )$	$(x_2\ y_1\ ,x_3\ y_1\ )$	$(x_3\ y_1\ ,x_3\ y_2\ )$	$(x_1\ y_2\ ,x_2\ y_2\ )$	$(x_2\ y_2\ ,x_3\ y_2\ )$
$e_1\ X\ e_1$	0.3,-0.1	0.2,-0.1	0.3,-0.1	0.1,-0.1	0.2,-0.1	0.2,-0.1	0.1,-0.1
$e_1\ X\ e_2$	0.3,-0.1	0.2,-0.1	0.3,-0.1	0.1,-0.1	0.2,-0.1	0.2,-0.1	0.1,-0.1
$e_2\ X\ e_1$	0.3,-0.1	0.4,-0.2	0.3,-0.1	0	0	0.4,-0.2	0
$e_2\ X\ e_2$	0.3,-0.2	0.4,-0.2	0.3,-0.2	0	0	0.4,-0.2	0

$\mu_{\widetilde{D}^3}$ (H X G)	$(x_1\ y_3\ ,x_1\ y_1\ )$	$(x_1\ y_2\ ,x_1\ y_3\ )$	$(x_2\ y_3\ ,x_2\ y_1\ )$	$(x_2\ y_2\ ,x_2\ y_3\ )$	$(x_3\ y_3,x_3\ y_1\ )$	$(x_3\ y_2\ ,x_3\ y_3\ )$	(x_1y_3,x_2y_3)	(x_2y_3,x_3y_3)
$e_1\ X\ e_1$	0	0	0	0	0	0	0	0
$e_1\ X\ e_2$	0.2,,-0.1	0.1,-0.1	0.2,-0.2	0.1,-0.1	0.2,-0.1	0.1,-0.1	0.2,-0.1	0.1,-0.1
$e_2\ X\ e_1$	0	0	0	0	0	0	0	0
$e_2\ X\ e_2$	0.2,-0.2	0.1,-0.1	0.2,-0.2	0.1,-0.1	0	0	0.2,-0.2	0

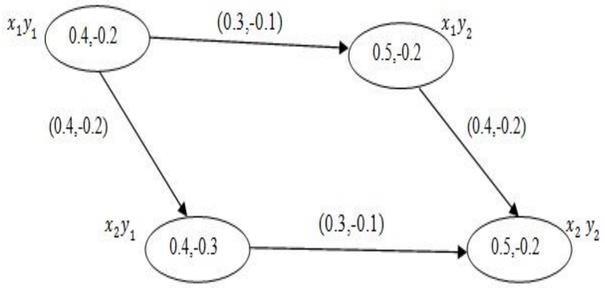
The Cartesian Product of $\widetilde{D}^1_{A,V_1}\ X\ \widetilde{D}^2_{B,V_2}$ is



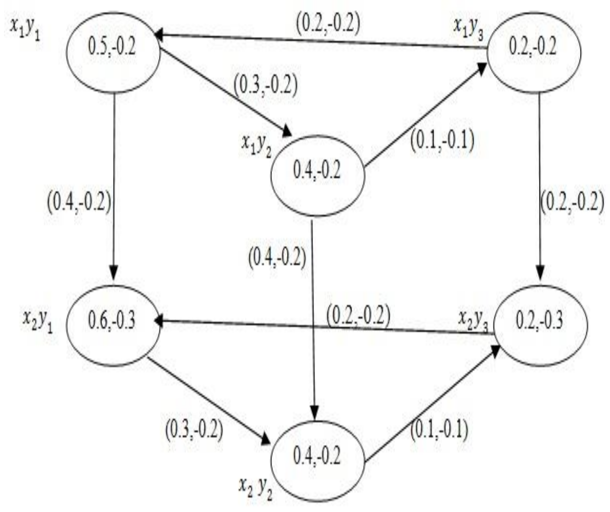
$(\widetilde{H}X\widetilde{G})(e_1\ X\ e_1)$



$(\tilde{H} X \tilde{G})(e_1\, X\, e_2)$



$(\tilde{H} X \tilde{G})(e_2\, X\, e_1)$



$(\tilde{H} X \tilde{G})(e_2 \, X \, e_2)$

Fig 3.7 $\widetilde{D}^3_{C,V_3}$ Cartesian product of Bipolar Fuzzy soft Digraph

Theorem : 3.7

If $\widetilde{D}^1_{A,V_1}$ *and* $\widetilde{D}^2_{B,V_2}$ are two bipolar fuzzy soft digraph , then $\widetilde{D}^1 \; X \; \widetilde{D}^2$ is also bipolar fuzzy soft digraph.

Proof:

Let $\widetilde{D}^1 = \left(\rho_{\widetilde{D}^1}, \mu_{\widetilde{D}^1}, A \right)$ *and*

$\widetilde{D}^2 = (\rho_{\widetilde{D}^2}, \mu_{\widetilde{D}^2}, B)$ be two bipolar fuzzy soft digraphs of simple digraphs $D_1^* = (V_1, E_1)$ *and* $D_2^* = (V_2, E_2)$ respectively.

From Definition,

If $e_1 \in A$ *and* $e_2 \in B$ *, then*

Case (i)
If $x_1 \in V_1$ *and* $x_2 \in V_2$ *then*

$$\left(\mu^+_{\mu_{\widetilde{D}^1}}(e_1) X \, \mu^+_{\mu_{\widetilde{D}^2}}(e_2) \right) (x_1, x_2) = \min \left(\mu^+_{\rho_{\widetilde{D}^1}}(e_1)(x_1), \mu^+_{\rho_{\widetilde{D}^2}}(e_2)(x_2) \right)$$

$$= \min \left(\left(\mu^+_{\rho_{\widetilde{D}^1}}(e_1) X \, \mu^+_{\rho_{\widetilde{D}^2}}(e_2) \right)(x_1), \left(\mu^+_{\rho_{\widetilde{D}^1}}(e_1) X \, \mu^+_{\rho_{\widetilde{D}^2}}(e_2) \right)(x_2) \right)$$

$$\left(\mu^-_{\mu_{\widetilde{D}^1}}(e_1) X \, \mu^-_{\mu_{\widetilde{D}^2}}(e_2) \right) (x_1, x_2) = \min \left(\mu^-_{\rho_{\widetilde{D}^1}}(e_1)(x_1), \mu^-_{\rho_{\widetilde{D}^2}}(e_2)(x_2) \right)$$

$$= \min \left(\left(\mu^-_{\rho_{\widetilde{D}^1}}(e_1) X \, \mu^-_{\rho_{\widetilde{D}^2}}(e_2) \right)(x_1), \left(\mu^-_{\rho_{\widetilde{D}^1}}(e_1) X \, \mu^-_{\rho_{\widetilde{D}^2}}(e_2) \right)(x_2) \right)$$

Case (ii)
If $x \in V_1$ *and* $(x_2, y_2) \in E_2$, then

$$\left(\mu^+_{\mu_{\widetilde{D}^1}}(e_1) X \, \mu^+_{\mu_{\widetilde{D}^2}}(e_2) \right) \left((x, x_2)(x, y_2) \right)$$

$$= \min \left(\mu^+_{\rho_{\widetilde{D}^1}}(e_1)(x), \mu^+_{\mu_{\widetilde{D}^2}}(e_2)(x_2, y_2) \right)$$

$$\leq \min\left(\mu^{+}_{\rho^{(e_1)}_{\tilde{D}^1}}(x), \min\left(\mu^{+}_{\rho^{(e_2)}_{\tilde{D}^2}}(x_2), \mu^{+}_{\rho^{(e_2)}_{\tilde{D}^2}}(y_2)\right)\right)$$

$$= \min\left[\min\left(\mu^{+}_{\rho^{(e_1)}_{\tilde{D}^1}}(x), \mu^{+}_{\rho^{(e_2)}_{\tilde{D}^2}}(x_2)\right), \min\left(\mu^{+}_{\rho^{(e_1)}_{\tilde{D}^1}}(x), \mu^{+}_{\rho^{(e_2)}_{\tilde{D}^2}}(y_2)\right)\right]$$

$$= \min\left[\left(\mu^{+}_{\rho^{(e_1)}_{\tilde{D}^1}} X\, \mu^{+}_{\rho^{(e_2)}_{\tilde{D}^2}}\right)(x, x_2), \left(\mu^{+}_{\rho^{(e_1)}_{\tilde{D}^1}} X\, \mu^{+}_{\rho^{(e_2)}_{\tilde{D}^2}}\right)(x, y_2)\right]$$

$$\left(\mu^{-}_{\mu^{(e_1)}_{\tilde{D}^1}} X\, \mu^{-}_{\mu^{(e_2)}_{\tilde{D}^2}}\right)((x, x_2)(x, y_2))$$

$$= \max\left(\mu^{-}_{\rho^{(e_1)}_{\tilde{D}^1}}(x), \mu^{-}_{\mu^{(e_2)}_{\tilde{D}^2}}(x_2, y_2)\right)$$

$$\geq \max\left(\mu^{-}_{\rho^{(e_1)}_{\tilde{D}^1}}(x), \max\left(\mu^{-}_{\rho^{(e_2)}_{\tilde{D}^2}}(x_2), \mu^{-}_{\rho^{(e_2)}_{\tilde{D}^2}}(y_2)\right)\right)$$

$$= \max\left[\max\left(\mu^{-}_{\rho^{(e_1)}_{\tilde{D}^1}}(x), \mu^{-}_{\rho^{(e_2)}_{\tilde{D}^2}}(x_2)\right), \max\left(\mu^{-}_{\rho^{(e_1)}_{\tilde{D}^1}}(x), \mu^{-}_{\rho^{(e_2)}_{\tilde{D}^2}}(y_2)\right)\right]$$

$$= \max\left[\left(\mu^{-}_{\rho^{(e_1)}_{\tilde{D}^1}} X\, \mu^{-}_{\rho^{(e_2)}_{\tilde{D}^2}}\right)(x, x_2), \left(\mu^{-}_{\rho^{(e_1)}_{\tilde{D}^1}} X\, \mu^{-}_{\rho^{(e_2)}_{\tilde{D}^2}}\right)(x, y_2)\right]$$

Case (iii)

If $z \in V_2$ *and* $(x_1, y_1 \in E_1)$, *then*

$$\left(\mu^{+}_{\mu^{(e_1)}_{\tilde{D}^1}} X\, \mu^{+}_{\mu^{(e_2)}_{\tilde{D}^2}}\right)((x_1, z)(y_1, z)) = \min\left(\mu^{+}_{\mu^{(e_1)}_{\tilde{D}^1}}(x_1, y_1), \mu^{+}_{\mu^{(e_2)}_{\tilde{D}^2}}(z)\right)$$

$$\leq \min\left[\min\left(\mu^{+}_{\rho^{(e_1)}_{\tilde{D}^1}}(x_1), \mu^{+}_{\rho^{(e_1)}_{\tilde{D}^1}}(y_1)\right), \mu^{+}_{\rho^{(e_2)}_{\tilde{D}^2}}(z)\right]$$

$$= \min\left[\min\left(\mu^{+}_{\rho^{(e_1)}_{\tilde{D}^1}}(x_1), \mu^{+}_{\rho^{(e_2)}_{\tilde{D}^2}}(z)\right), \min\left(\mu^{+}_{\rho^{(e_1)}_{\tilde{D}^1}}(y_1), \mu^{+}_{\rho^{(e_2)}_{\tilde{D}^2}}(z)\right)\right]$$

$$= \min\left[\left(\mu^{+}_{\rho^{(e_1)}_{\tilde{D}^1}} X\, \mu^{+}_{\rho^{(e_2)}_{\tilde{D}^2}}\right)(x_1, z), \left(\mu^{+}_{\rho^{(e_1)}_{\tilde{D}^1}} X\, \mu^{+}_{\rho^{(e_2)}_{\tilde{D}^2}}\right)(y_1, z)\right]$$

$$\left(\mu^{-}_{\mu^{(e_1)}_{\tilde{D}^1}} X\, \mu^{-}_{\mu^{(e_2)}_{\tilde{D}^2}}\right)((x_1, z)(y_1, z)) = \max\left(\mu^{-}_{\mu^{(e_1)}_{\tilde{D}^1}}(x_1, y_1), \mu^{-}_{\mu^{(e_2)}_{\tilde{D}^2}}(z)\right)$$

$$\geq \max\left[\max\left(\mu^{-}_{\rho^{(e_1)}_{\tilde{D}^1}}(x_1), \mu^{-}_{\rho^{(e_1)}_{\tilde{D}^1}}(y_1)\right), \mu^{-}_{\rho^{(e_2)}_{\tilde{D}^2}}(z)\right]$$

$$= \max\left[\max\left(\mu^{-}_{\rho^{(e_1)}_{\tilde{D}^1}}(x_1), \mu^{-}_{\rho^{(e_2)}_{\tilde{D}^2}}(z)\right), \max\left(\mu^{-}_{\rho^{(e_1)}_{\tilde{D}^1}}(y_1), \mu^{-}_{\rho^{(e_2)}_{\tilde{D}^2}}(z)\right)\right]$$

$$= \max\left[\left(\mu^{-}_{\rho^{(e_1)}_{\tilde{D}^1}} X\, \mu^{-}_{\rho^{(e_2)}_{\tilde{D}^2}}\right)(x_1, z), \left(\mu^{-}_{\rho^{(e_1)}_{\tilde{D}^1}} X\, \mu^{-}_{\rho^{(e_2)}_{\tilde{D}^2}}\right)(y_1, z)\right]$$

IV CONCLUSIONS

Graph theory is an extremely useful tool in solving the combinatorial problems in different areas including geometry, algebra, number theory, topology, operations research, optimization and computer science. The bipolar fuzzy sets constitute a generalization of Zadeh's fuzzy set theory. The bipolar fuzzy models give more precision, flexibility and compatibility to the systems as compared to the classical and fuzzy models. We have introduced some concept of bipolar fuzzy soft digraphs in this paper.

The concept of bipolar fuzzy soft digraphs can be applied in various areas of engineering, computer science: database theory, expert systems, neural networks, artificial intelligence, signal processing, robotics, computer networks, and medical diagnosis.

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