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Experimental Characterization of Tensairity Beams

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Abstract*: Tensairity is a new structural concept towards a sustainable architecture, combined from struts, cables, and an airbeam. In Tensairity, each bearing component is working in the most favorable conditions: struts are only compressed, cables are only tensioned and the airbeam stabilizes the system. Thus, Tensairity beam will be much lighter than a conventional beam. The bearing capacity of this type of beam depends not only on the material property but also on the inflation pressure in the airbeam and the tension in the cable. And this tension strongly depends on the cable anchoring method. This paper presents an experimental investigation aimed to the bending of a Tensairity beam submitted to a homogeneous distributed load. Prototypes in full-scale tests of Tensairity beams will be manufactured. Effect of inflation pressure, airbeam size, and cable anchoring method to structural stiffness will be studied experimentally. The obtained experimental results are in high agreement with those of an independent theory.

***Keywords*: Tensairity, lightweight structures, inflatable structures, full-scale experimental tests, bending load.**

I. INTRODUCTION

Inflatable structures made of technical fabrics, formed into an airtight tube and inflated by air pressure have been widely used in construction. In comparison with conventional structures, the inflatable structures may present some outstanding advantages such as their lightness, foldability as well as being easy to transport and deploy in some specific cases. Because of superior features, these structures are often used in temporary structures such as temporary exhibitions, temporary medical centers in disadvantaged places.

The earliest analytical expressions for the load-deflection response and the collapse load of an inflatable cantilever was found in Comer and Levy's paper, who investigated isotropic beams using Euler–Bernoulli's kinematics. Fichter [2] improved the previous theories by using Timoshenko's kinematics and minimizing the total potential energy. This result was greatly influenced on many of the subsequent works on inflatable structures. Nguyen and Le [3,4], Le van and Wielgosz [5] and Apedo et al. [6], improved Fichter's theory by using the virtual power principle in the context of the total Lagrangian formulation. The problem was formulated in finite deformations in order to account for all the nonlinear terms in the kinematic and equilibrium equations. However, the poor load-bearing capacity of airbeam has drastically limited their application potential. This deficiency can be overcome by the structural concept Tensairity, where the airbeam is combined with struts and cables [7]. In Tensairity, the compression and tension are ensured by struts and cables respectively, while the airbeam makes arm of couple between the compression and tension and gives the bearing capacity of the structure. Moreover, due to the airbeam stabilizes the struts, it is possible to minimize the strut section. The first applications of Tensairity in the field of civil engineering are roof structures and bridges [8].

This study is devoted to the experimental study of the bending load-bearing capacity of Tensairity beam. The prototypes of Tensairity beam will be manufactured and full-scale experiment will be performed to analyze the effect of inflation pressure, airbeam size, and cable anchoring method to structural stiffness of Tensairity beam. The experimental results will be compared with those of an independent theory.

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II. ANALYTICAL MODEL

A. Structure of Tensairity beam

As suggested by Luchsinger [7], a Tensairity beam consists of a cylindrical airbeam under low pressure, strut, and cable that tightly connected to each other. The strut is placed along the top of the airbeam, the cables are connected to each end of the strut, running with different helicity in a spiral form around the airbeam (see Fig. 1). In this composite structure, the airbeam makes arm of couple between the compression and tension and stabilizes the struts.

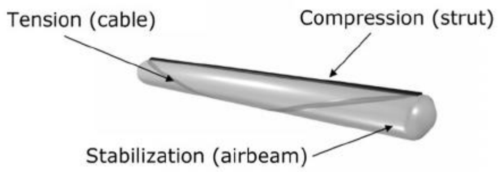


Fig. 1.The composition of Tensairity beam [7]

B. Deflection of Tensairity beam

The basic theory of Tensairity has been described elsewhere as Luchsinger [9]. When the beam is only subjected by a homogenous distributed load (see Fig. 2):

- the tension in the cable is considered constant and equal to the compression in strut;

- the spiral cable can be simplified in parabolic form.

The deflection of the Tensairity beam can be written in the following forms:

$$w(x) = \frac{1}{\lambda^4} C_0 \cosh(\lambda x) + C_1 x^4 + C_2 x^2 + C_3 \qquad (1)$$

$$C_0 = \frac{k}{EI\lambda^2 \cosh(\lambda \ell)} \left(\frac{-qH}{EIk} + \frac{2f}{\ell^2} \right); \quad C_1 = \frac{k}{24EI\lambda^2} \left(\frac{q}{H} - \frac{2f}{\ell^2} \right) \qquad (2)$$

$$C_2 = -\frac{C_0}{2\lambda^2} \cosh(\lambda \ell) - 6C_1 \ell^2; \quad C_3 = -\frac{C_0}{\lambda^4} \cosh(\lambda \ell) - C_1 \ell^4 - C_2 \ell^2 \qquad (3)$$

where: EI is the bending stiffness of the strut; q the homogenous distributed load; $k = \frac{\pi p}{2}$ the elastic stiffness of the airbeam; p is the inflation pressure in the airbeam; f is the maximum deflection of cable, in the middle of the beam, equals to the diameter of the airbeam; $\lambda = \sqrt{\left(\frac{k}{H} - \frac{H}{EI} \right)}$ and H is the horizontal forces (compression in the strut and tension in the cable), consists of two components [8]:

- H_0 is the pretension in the cable when the airbeam is inflated;
- H_1 appears when the beam is subjected to bending load.

$$H = H_0 + H_1 = \frac{2p\delta\ell^2}{\pi R} \left(1 - \frac{\delta}{R} \right) + \frac{q\ell^2}{2f} \qquad (4)$$

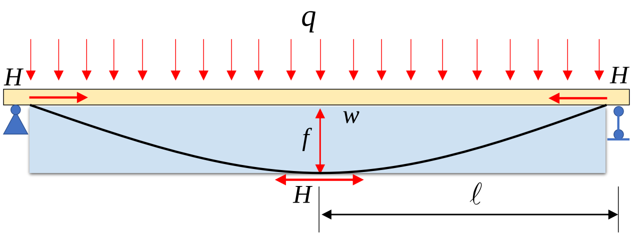


Fig. 2.Sketch of the components of the analytical model [8]

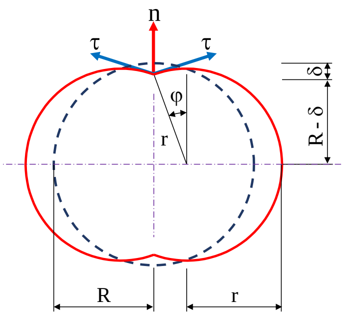


Fig. 3. Membrane deflection due to the cable [7]

In equation (4), R is the inflated radius of the airbeam; $\delta = R - R_0$ the difference between the inflated radius and natural radius of the airbeam. These values will be described more clearly in section C.

C. The geometry of the inflated beam

The theory of the inflation based upon the former work of Nguyen et al. [10] is summarized here:

- In the natural state, the thickness, radius, length of the inflated beam and the material orientation are denoted : H_0, R_0, L_0, α ., respectively (see Fig. 4).
- The notations of these quantities in the inflated state are then H, R, L and the rotation of the cross-section around the beam axis is β .

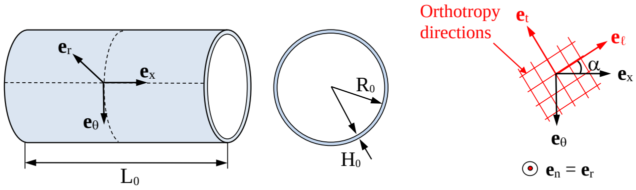


Fig. 4.The geometry of the beam in the natural state

Let $k_\theta \equiv \frac{R}{R_0}$, $k_x = \frac{dL}{dL_0}$, $k_\beta = \frac{\beta(x)}{x}$ be the ratio of the geometrical dimensions of the beam in the inflated state and the natural state, respectively. It is assumed that the constitutive law of the orthotropic membrane, related to the natural state, is hyperelastic, of the St Venant–Kirchhoff type:

$$E = C : \Sigma \qquad (5)$$

where E , Σ and C denote the Green strain tensor, the second Piola–Kirchhoff stress tensor and the compliance tensor, respectively.

A set of three nonlinear equations was established for three unknowns k_θ, k_x, k_β :

$$\begin{aligned} k_\theta^2 - 1 &= \frac{pR_0}{k_x}\left[C_{\theta\theta\theta\theta}(R_0^2 k_\theta^2 k_\beta^2 + 2k_x^2) + C_{\theta\theta xx} k_\theta^2 - C_{\theta\theta x\theta} R_0 k_\theta^2 k_\beta \right] \\ k_x^2 + R_0^2 k_\theta^2 k_\beta^2 - 1 &= \frac{pR_0}{k_x}\left[C_{xx\theta\theta}(R_0^2 k_\theta^2 k_\beta^2 + 2k_x^2) + C_{xxxx} k_\theta^2 - C_{xxx\theta} R_0 k_\theta^2 k_\beta \right] \\ 2R_0 k_\theta^2 k_\beta &= \frac{pR_0}{k_x}\left[C_{\theta x\theta\theta}(R_0^2 k_\theta^2 k_\beta^2 + 2k_x^2) + C_{\theta xxx} k_\theta^2 - C_{\theta xx\theta} R_0 k_\theta^2 k_\beta \right] \end{aligned} \qquad (6)$$

The parameters $C_{....}$ in these equations relate to the material coefficients in the natural state E_ℓ , E_t , $\nu_{t\ell}$, $G_{\ell t}$.

$$\begin{aligned} C_{\theta\theta\theta\theta} &= \frac{1}{E_\ell} s^4 + \frac{1}{E_t} c^4 + \left(\frac{1}{G_{\ell t}} - 2\frac{\nu_{\ell t}}{E_\ell} \right) s^2 c^2 \\ C_{\theta\theta xx} &= \left(\frac{1}{E_\ell} + \frac{1}{E_t} - \frac{1}{G_{\ell t}} \right) s^2 c^2 - \frac{\nu_{\ell t}}{E_\ell} (s^4 + c^4) \\ C_{\theta\theta x\theta} &= -2\left(\frac{1}{E_\ell} s^3 c - \frac{1}{E_t} c^3 s \right) - 2\frac{\nu_{\ell t}}{E_\ell} (s^3 c - c^3 s) - \frac{1}{G_{\ell t}} (c^3 s - s^3 c) \\ C_{xxxx} &= \frac{1}{E_\ell} c^4 + \frac{1}{E_t} s^4 + \left(\frac{1}{G_{\ell t}} - 2\frac{\nu_{\ell t}}{E_\ell} \right) s^2 c^2 \\ C_{xxx\theta} &= -2\left(\frac{1}{E_\ell} c^3 s - \frac{1}{E_t} s^3 c \right) + 2\frac{\nu_{\ell t}}{E_\ell} (s^3 c - c^3 s) + \frac{1}{G_{\ell t}} (c^3 s - s^3 c) \\ C_{\theta xx\theta} &= 4\left(\frac{1}{E_\ell} + \frac{1}{E_t} + 2\frac{\nu_{\ell t}}{E_\ell} \right) s^2 c^2 + \frac{1}{G_{\ell t}} (c^2 - s^2)^2 \end{aligned}$$

Given the initial radius R_0 , the orientation angle α of the membrane and the internal pressure p , the couple (k_θ, k_x) can be easily determined. The final geometry of the inflated beam is then known: $L = k_x L_0$, $R = k_\theta R_0$.

III. EXPERIMENTAL MODEL

A. Design of the Tensairity beam

The strut member is a rectangular, hollow steel section of $60mm\times 30mm\times 1mm$, is placed along the top of the airbeam. The *2mm* diameter cables are connected at both ends with the strut element and run with different helicity in a spiral form around the airbeam (see **Error! Reference source not found.**5). In this study, Young's modulus of the hollow stell is $E_c = 200kN/mm^2$. The cylindrical airbeam with natural length $L_0 = 3m$, natural radius $R_0 = 10cm;12.5cm$.

Fig. 5.Test rig for Tensairity beam

This airbeam is made of technical fabric Ferrari F502. The elastic properties of the fabric were measured in house with our inflation test [11] and found to be, $E_t H = E_\ell H = 300\ kN/m$, $\nu_{\ell t} = 0.197$, $G_{\ell t} = 5.255\ kN/m$. The warp direction of the fabric is oriented parallelly to the tube axis. Six spiraling cables of $2mm$ diameter were used as tension members, all of them were connected at both ends to the compression element.

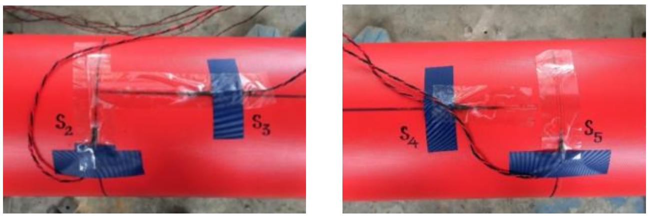
B. Instrumentation

The airbeam used for this study is a cylindrical airtight tube so the pressure in the tube is always kept stable. The pressurization equipment is air compressor Sunny Compressor 2.5Hp which is connected to a pressure gauge Flexbimec 7301 to preliminary control the pressure in the tube. The air pressure in the tube is then more accurately checked by a pressure gauge Tire Gauge 4 in 1 (see Fig. 6a). In the inflation step, the change of radius of the airbeam affects to the deflection of the cable at the middle of the Tensairity beam. This radius variation will be measured by Strain Gauge PL-60-11 having the strain limits at $\varepsilon = \pm 2\%$ (see Fig. 6b). The main objective of this study is to determine the load-displacement response of the Tensairity beam. The displacement is measured by physical indicator of $\pm 0.01mm$ precision (see Fig. 6c) placed in the middle of the Tensairity beam. The load is charged with heavy objects weighing $150N$.

a) Tire Gauge b) Strain Gauge c) Indicator

Fig. 6.Instrumentation

C. Test Rig



a) Strain gauge S2 on the left-hand side b) Strain gauge S5 on the right-hand side

Fig. 7.Positions of strain gauges

The change of radius of the airbeam is measured at the middle of the beam by using two strain gauges S2 and S5 oriented along the circumference of the beam (see Fig. 7). The results will be the average of those two obtained by these strain gauges. The variation of the radius of the airbeam is then plotted and compared with the those of the theory [11] (see **Error! Reference source not found.**).

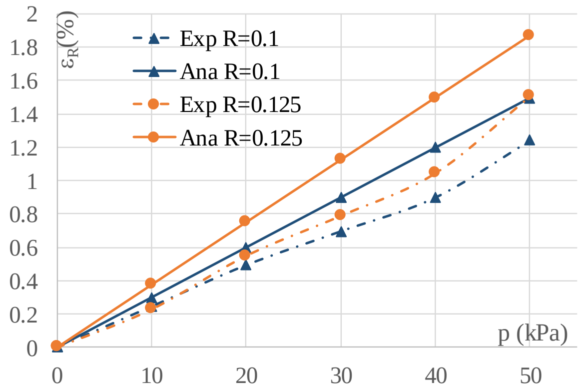


Fig. 8.Variation of the radius of airbeam

D. Measure the displacement of Tensairity beam

A Tensairity beam with span $2\ell = 3,4m$ under homogenous distributed load is investigated (see **Error! Reference source not found.**). The behavior of this type of beam strongly depends on the tension of the cable. In this study, we consider two methods of anchoring cables into strut elements:

- ***Pre-anchoring***: the airbeam will be pressurized at an extremely small pressure $p_0 = 0.01kPa$ to guarantee the cylindrical form. The tube is then connected to the struts elements through the cables system. When the tube is inflated, the deformation of the tube causes the pre-tension H_0 in the cables and increases the horizontal force H when the Tensairity beam is loaded.

- ***Post-anchoring***: the airbeam will be first pressurized at the working pressure p. The tube is then connected to the struts elements through the cables system. In this case, there is no pre-stress in the cables $H_0 = 0$, the horizontal force on the Tensairity beam is only $H = \frac{q\ell^2}{2f}$.

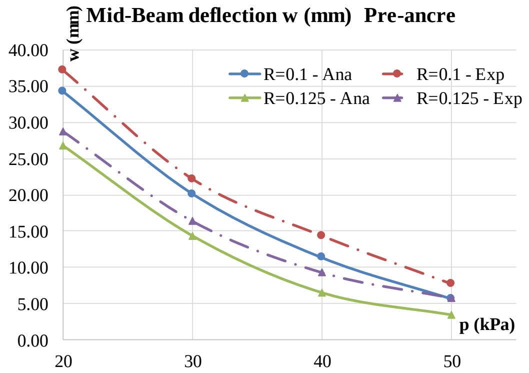
E. Discussion

❖ ***Influence of internal pressure on the bending bearing capacity of the Tensairity beam***

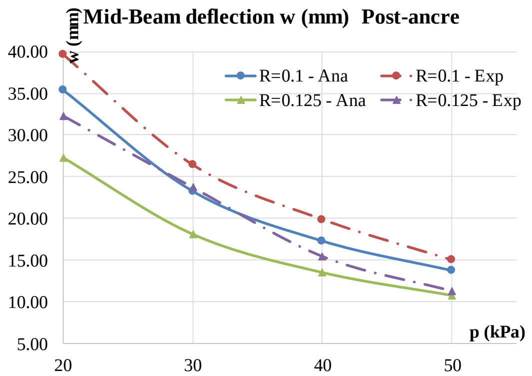
The results show the bending bearing capacity of the beam depends on the internal pressure. When the internal pressure increases from *10* to *50 kPa*, both elastic stiffness of the airbeam and tension in the cable increase thereby increasing the rigidity of the beam (see Eq.1). The mid-beam deflection decreases five times in the case of pre-anchoring and approximately three times in the case of post-anchoring.

❖ ***Influence of radius of airbeam on the bending bearing capacity of the Tensairity beam***

The radius of the airbeam plays an important role in ensuring the arm of couple between the tension in the cable and the compressive force in the strut elements. Therefore, the bending bearing capacity of the Tensairity beam increases with the radius of the airbeam. At internal pressure *p=30 kPa* for instance, the deflection of the Tensairity beam with the radius of airbeam *R=0.125m* is less than *25%* compared to *R=0.1m* Tensairity beam.



a) Pre-anchoring



b) Post-anchoring

Fig. 9.Mid-beam deflection vs internal pressure

❖ ***Influence of anchoring method on the bending bearing capacity of the Tensairity beam***

The deflection of the Tensairity beam depends strongly on the tension in the cable (see Eq.1) and therefore depends on the anchoring method (pre-anchoring / post-anchoring) (see Eq.4):

- in case of post-anchoring, the tension in the cable $H^{post-anc}$ includes only components due to bending load;
- in case of pre-anchoring, the tension $H^{pre-anc}$ is also contributed by pre-tension H_0 due to the inflation process.

The tension in the cable $H^{pre-anc}$ is always greater than or equal to that of $H^{post-anc}$, depending on the internal pressure and the radius of the airbeam. At internal pressure p = 20kPa for instance, the difference of the mid-beam deflection in case of *R =0.125m* is *12.8%*, and this value will be *94.3%* when *p = 50kPa*.

Table 1. Comparison between Experimental and Theoretical results of mid-beam deflection

R (m)	p (kPa)	Exp. results		Ana. results		Difference (%)	
		Pre-Anc	Post-Anc	Pre-Anc	Post-Anc	Pre-Anc	Post-Anc
0.1	10	>50	>50	>50	>50		
	20	37.16	39.62	34.23	35.37	7.89	10.72
	30	22.15	26.40	20.12	23.24	9.15	11.95
	40	14.31	19.85	11.33	17.27	20.82	12.98
	50	7.68	14.98	5.65	13.72	26.43	8.42
0.125	10	>50	>50	>50	>50		
	20	28.80	32.31	26.75	27.31	7.14	15.48
	30	16.41	23.74	14.33	18.13	12.68	23.61
	40	9.33	15.47	6.51	13.54	30.23	12.45
	50	5.82	11.31	3.45	10.79	40.72	4.64

❖ ***Difference between the theoretical and experimental results***

In this study, the theoretical results are calculated by analytical formulas. Concretely, inflated tube radius is calculated from equation (6); Tension in the cable is calculated from equation (4), strongly depends on the radius variation of the airbeam; the deflection of Tensairity beam is calculated from equation (1).

Difference between the theoretical and experimental results depends on the anchoring method:

- ***Post-anchoring***: ignoring the effect of pre-tension in the cable to the deflection of the Tensairity beam, so about *15%* errors can be derived from the error of the measuring device and the experimental model.
- ***Pre-anchoring***: error of the inflated radius measurement greatly affects the test results. The pre-tension H_0 in the cable strongly depends on the variation in radius. If the theoretical deflection is calculated from experimental results of radius, this difference between theoretical and experimental results decreases significantly. In the case of *R=0.125m*, *p=30kPa*, the difference decreases from *12.68%* to *4.3%*. These values will be *40.72%* and *22.5%* when *p=50 kPa*.

Moreover, the error may come from the connection of two ends of the beam, from the interaction between structural parts.

Experimental and theoretical results are summarized in Table 1, the load-displacement relation of the Tensairity beam is described in 9.

IV. CONCLUSION

In this paper, the load-bearing behavior of a Tensairity beam has been experimentally investigated. The following results have been obtained:

- Internal pressure in the airbeam contributes to bending stiffness of Tensairity beam.
- The greater the airbeam radius is, the greater torque arm between the tension and compression in the Tensairity beam is, and therefore the greater bending bearing capacity is.
- Bending stiffness of the Tensairity beam depends strongly on the anchoring method. The study shows that the pre-anchoring method is simpler for construction also increases structural stiffness compared to the post-anchoring method. Therefore, it is recommended to use this pre-anchoring method in practice.
- Differences between experimental results and theoretical results are inevitable. In further studies, it is possible to limit errors by advanced measurements, with higher accuracy.

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