Multi stage Multi-objective Transportation Problem under Uncertainty Environment

Manoranjan Mishra, Debdulal Panda

Abstract: ***In present scenario due to high competition in market, there are lots of pressures on organizations involbs with transportation industry, to provide the service in a better and effective manner. The distribution of products among the customers in systematic manner is not an easy task. Transportation models provide an effective framework to meet these challenges. If the parameters involved with multi-objective transportation model are expressed in terms of fixed parameter then it is not easy to address them in an uncertainty environment, rather it is easy to handle them when they are represented in terms of linguistic variables. It is noticed that, all the objectives of a transportation model are affected by different criteria like route of transportation, weather condition, vehicles used for transportation etc. In the present study a multi-stage transportation model with multiple numbers of objectives is developed with fuzzy relations. Minimization of both transportation cost and transportation time are considered as two different objectives of first stage which are associated with a number of different criteria like deterioration time, fixed charge and mode of transportation. In second stage, another objective i.e quantity of transported amount is considered on the performance basis of objectives of first stage. All these factors considered for this model are fuzzy parameters and are expressed in terms of linguistic variables. The fuzzy rule based transportation model is developed and the solution is obtained by Genetic Algorithm for multi-objective problems (MOGA). The model is presented with a numerical problem and optimum result is discussed.***

Key words: Multi stage multi-objective transportation problem, Genetic Algorithm

I. INTRODUCTION

In 1941, F. L. Hitchcock first developed "Transportation Problem" (TP) which involbs transfer of products from one point to different points with less transportation cost. It is a particular type of Linear programming problem and is used to find the optimal shipping patterns among a number of sources and destinations. Classical transportation models involbs transportation of products from different supply points to different destinations. Optimization of total transportation cost is considered as a common objective of a single objective transportation model. But considering the demand of the situation, the solution obtained with single objective is not acceptable because it may not approach to real optimal solution which creates an environment where decision maker is bound to consider several objectives at a time.

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Different factors involved with transportation systems like type of service, demand, supply, cost factor and vehicles used are normally defined in terms of fixed data. But with imprecise data it is easy to manage transport related decisions under uncertainty. In some instances, it is too difficult to measure the constraints as well as objectives in terms of crisp values. Traditional approaches were not effective while dealing with the problems in which the interdependence between different parameters is not well defined. In most of the real –life problems, it is too difficult to take decisions with quantitative predictions; rather it is comparatively better in case of qualitative predictions. To represent quantitative data, real numbers are used but in case of imprecise data fuzzy sets can be used.

Much of the problems related to transportation industry can be designed and solved as multi-objective problems with consideration of different conflicting objectives. It is a specific type of vector minimum problem where all the constraints are of equality type and the objectives are conflicting in nature. For example, the objectives transportation cost and transported amount of the transportation model are inversely proportional to each other whereas the total cost of transportation is directly proportional to time required for transportation of products. Some rules are manually framed. When these rules are imprecise in nature and represented by verbal words then they are called as fuzzy inferences. Using fuzzy inference systems, a few number of multi-objective transportation problems are designed and solved.

II. RELATED WORK

If all parameters involved with transportation model are represented in terms of fuzzy parameters then the model is named as fuzzy transportation model. The concept of fuzzy sets was first introduced by Zadeh (1965) and was first applied by Zimmermann to solve models with multiple numbers of objectives. A number of approaches have been introduced by many authors Diaz (1978, 1979), Aneja and Nair (1979), Isermann (1979) to solve the multi-objective transportation problem. Ringuest and Rinks (1987) also discussed the existing solution procedures for multiobjective transportation problem. Michalewicz et al (1999) applied GA (Genetic Algorithm) to solve multi-objective transportation problems. Omar and Samir et.al (2003) presented the procedure to solve transportation problems with fuzzy quantities. The fuzzy logic guided non-dominated sorting genetic algorithm is presented by Lau et al. (2009) to solve the multi-objective transportation problems. The algorithm presented by him is mainly deals with optimization of routes connecting multiple sources and multiple destinations. For the T. Leelavathy and K. Ganesan et.al (2016) applied weighted sum method for balanced fuzzy multi objective transportation problem.

Considering preference of decision makers they obtain a compromise solution. In multi criteria decision making process, the fuzzy linear programming model is claimed as an innovation by Dyson (1980). A comprehensive presentation of the fuzzy set theory can be found in Kaufman (1976) while applications of the fuzzy set theory to decision making problems can be found in Bellman and Zadeh (1970), Dubols and Prade (1980), and Zimmermann (1985). Sakawa and Yano (1990) have developed a new interactive fuzzy satisfying method for multi objective linear programming problems with fuzzy parameters. Proposed Multi stage multi-objective transportation model

2. Proposed Multi stage multi-objective transportation model

In the present study, a two stage transportation problem is developed with multiple numbers of objectives. In the first stage we consider two objectives transportation cost (Z1) and transportation time (Z2). We choose three different criteria detortion time (DT), fixed charge (FC), and mode of transportation (MT). The impacts of all criteria on different objectives are analyzed. In the second stage another objective the quantity of transported unit (Z3) is considered on the performance basis of two objectives of first stage. All these factors considered for the model are fuzzy parameters and are represented by linguistic variables. The solution of this model is obtained by using Mamdani fuzzy inference system where at a time only a singly objective is taken into consideration.

The transportation model with multiple numbers of objectives is mathematically formulated as:

$$\text{Minimize } Z_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} \ddot{c}_{ij} \ddot{x}_{ij} \quad (1)$$

$$\text{Minimize } Z_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} \ddot{t}_{ij} \quad f(\ddot{x}_{ij}) \quad (2)$$

$$\text{Maximize } Z_3 = \sum_{i=1}^{m} \sum_{j=1}^{n} \ddot{x}_{ij} \quad (3)$$

$$\text{Subject to } \sum_{j=1}^{n} \ddot{x}_{ij} \quad \ddot{a}_i \quad = \quad \text{for } i = 1, 2 \ldots m \quad (4)$$

$$\sum_{i=1}^{m} \ddot{x}_{ij} \quad \ddot{b}_j \quad = \quad \text{for } j = 1, 2 \ldots n \quad (5)$$

$$\sum_{i=1}^{m} \ddot{a}_i = \sum_{j=1}^{n} \ddot{b}_j \quad (6)$$

$$\ddot{x}_{ij} \geq 0 \quad (7)$$

$$\text{and} \quad f(\ddot{x}_{ij}) = \begin{cases} 1 & \ddot{x}_{ij} > 0 \\ 0 & \ddot{x}_{ij} = 0 \end{cases} \quad (8)$$

Where

$$f(\ddot{x}_{ij}) = \begin{cases} 1 & \ddot{x}_{ij} > 0 \\ 0 & \ddot{x}_{ij} = 0 \end{cases} \quad (8)$$

$\ddot{a}$i = available amount at origin i

$\ddot{b}$j = required amount at demand point j

$\ddot{c}_{ij}$= the cost of transportation per unit from origin i to demand point j

$\ddot{x}_{ij}$ = the shipping amount from origin i to demand point j

$\ddot{d}_{ij}$= the shipping distance from origin i to demand point j and

$\ddot{t}_{ij}$ = shipping time of the goods from origin i to demand point j.

Description of fuzzy input and output parameter

In initial stage of the present model we consider three criteria as input parameters and two objectives as output parameter. Two linguistic parameters Min and Max are used to represent the input parameter deterioration time. The input parameters fixed charge along with two output parameters cost and time of transportation have four different responses like Low, Medium, High and Very High whereas the input parameter, mode of transportation is represented in terms of Road, Train, Ship and Flight. Second stage of the model is formulated by taking previous two objectives as input parameters and another new objective quantity of transported amount as output parameter.

The linguistic terms with their ranges used for different input as well as output parameters are given below.

Table 1: Linguistic term and their range for the input parameter deterioration time (DT)

Linguistic terms	**Membership function**	**ange of parameter**
Min	Trimf	[0.0 , 0.6]
Max	Trimf	[0.4 , 1.0]

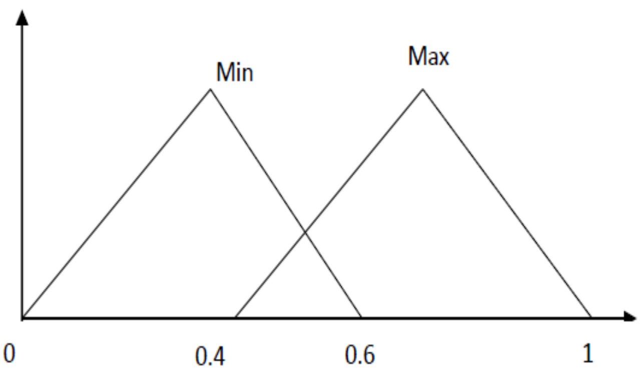

Fig 1. Membership functions of input parameter deterioration time (DT)

Table 2: The range and linguistic terms used for the input parameter fixed charge and all output parameters

Linguistic terms	**Membership function**	**ange of arameter**
Low(L)	Tri mf	[0.0 , 0.4]
Medium(M)	Tri mf	[0.2 , 0.6]
High(H)	rimf	0.4 , 0.8]
Very High(VH)	Trimf	0.6 , 1.0]

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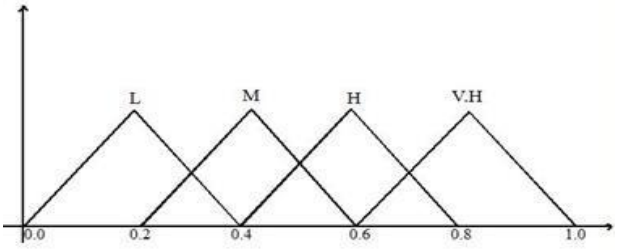

Fig 2. Membership functions of input parameter fixed charge and all output parameters

Table 3: Linguistic term and their range for the input parameter X_4= {Mode of Transportation}

Linguistic terms	Membership function	Range of parameter
Road (R)	Trimf	[0.0 , 0.4]
Train (T)	Trimf	[0.2 , 0.6]
Ship(S)	Trimf	[0.4 , 0.8]
Flight (F)	Trimf	[0.6 , 1.0]

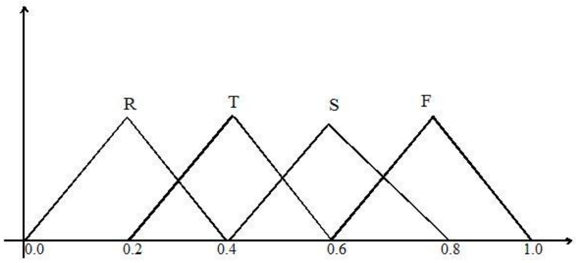

Fig 3. Membership function for input parameter mode of transportation

Determination of fuzzy rule base from input and output parameters

Traditional fuzzy logic control (FLC) system is used for all the objectives as well as for all criteria. Three different criteria are considered to analyze their effect on two objectives time of first stage. Whereas in second stage, considering previous two objectives as input parameters we analyze their impact on another objective transported amount.

During first stage of the model a set of 32 rules are framed manually to observe the impact of input parameters on two objectives cost and time of transportation times which are presented as follows.

Rule 1: if deterioration time time (DT) is **Min** with **Low** fixed charge and **Road** mode of transportation then the cost and time of transportation are **Low.**

Rule 2: if deterioration time time (DT) is **Min**, with Low fixed charge and mode of transportation is **Train** then also both the cost and time of transportation are **Low**.

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Rule 32: if deterioration time time (DT) is **Max**, with Very **High** fixed charge and **Flight** mode of transportation then the transportation cost is **Very High** and transportation time is **High.**

III. RESULT DISCUSSION

The traditional Mamdani approach is used for both the output parameters which are generated by above rules and the corresponding membership functions. All suitable combinations of input parameters are considered to observe their impact on the output parameters. During analysis it is noticed that the input parameter detortion time and fixed charge have major impact on both the objectives. Maximum deterioration time with high fixed charge increases both the cost and time of transportation time (see Fig4 and Fig5). It is also come to our notice that, high fixed charge is the prime reason to increase the cost of transportation irrespective of any mode of transportation but it has almost no impact on transportation time. The input parameters deterioration time and transportation mode are the major influencing factors for total shipping time. From the study it is also noticed that, flight mode increases transportation cost whereas it minimizes the shipping time of products irrespective of detortion time (see Fig6 and Fig7).

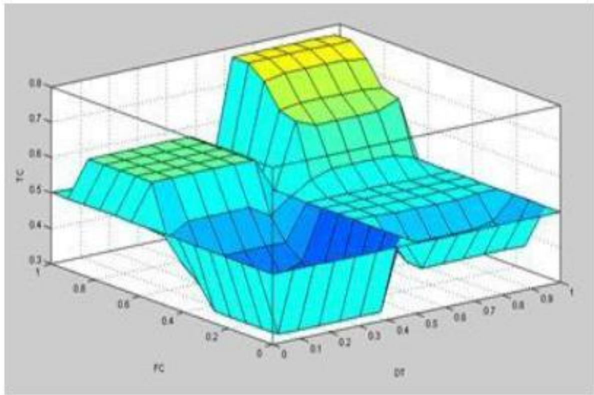

Fig 4: Detortion time and Fixed charge for

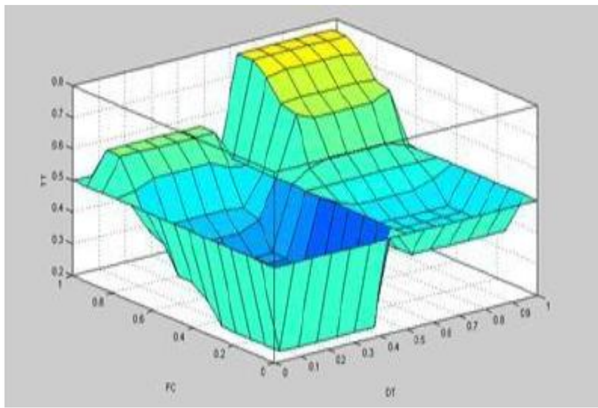

Fig 5: Detortion time and Fixed charge for Transportation cost Transportation time

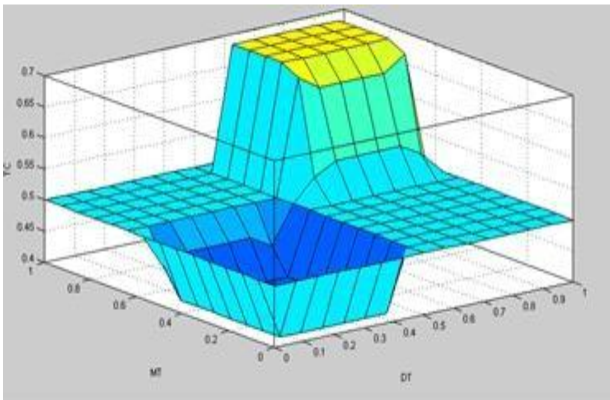

Fig 6: Deterioration time and Mode of

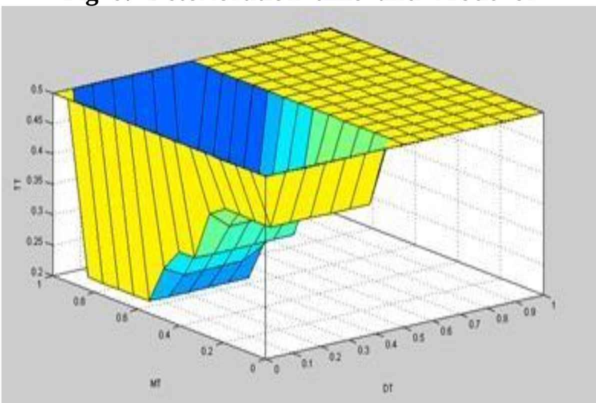

Fig 7: Deterioration time and Mode of transportation for Transportation cost transportation for Transportation time

So decision maker should choose the route where deterioration time is minimized or there is no possibility of deterioration for quick delivery of products. In addition to above criteria, focus should also be given on which mode of transportation is used to optimize both the objectives.

In second stage, we consider both the cost and time of transportation as input parameters and another set of 16 rules are framed to analyze their impact on the new objective, quantity of transported amount as follows:

Rule 1: quantity of transported amount is **Very High** in case of **Low** transportation cost and Low transportation time**.**

Rule 2: quantity of transported amount is **Very High** in case of **Low** transportation cost and **Medium** transportation time**.**

. . . .

Rule 16: quantity of transported amount is Low in case of **Very High** transportation cost and **Very High** transportation time**.**

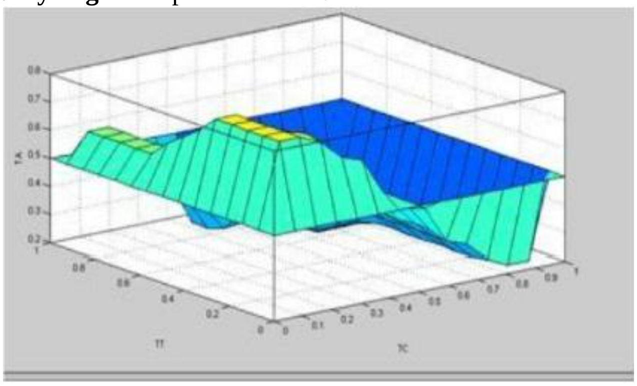

Fig 8: Transportation cost and time for transported amount

The effects of input parameters cost and time of transportation on new objective, amount of transported unit is analyzed. From the analysis it is observed that the amount of transported units is more in case of minimum transportation cost with less shipping time. It is also noticed that the shipping amount is gradually decreasing with increase of transportation cost and time (See Fig 8). Hence, both the input parameters have key role in determining the quantity of shipping amount and they are inversely proportional to both of them.

IV. GENETIC ALGORITHM FOR MULTI-OBJECTIVE PROBLEM (MOGA)

In general the objectives of multiple-objective models are conflicting in nature and are preventing simultaneous solution of each objective. In many situations due to conflicting nature of different objectives, the solution obtained with consideration of only one objective may not be acceptable for other objectives. Hence it is impossible to get an ideal solution satisfying all the constraints.

Due to this instead of finding particular solutions, focus was given to find an optimal compromise solution which satisfies each of the objectives at an acceptable level without being dominated by any other solution. Genetic Algorithm (GA) is specifically well-suited for problems of this category. Holland and his colleagues developed concept of genetic algorithm during 1960s and 1970s. Traditional genetic algorithm is designed to address multi-objective problems by using specialized fitness functions. GA is simultaneously search different solution regions to find distinct set of solutions for different models with different solution spaces. During cross over GA exploits better solutions with consideration of all objectives to get non-dominated Pareto optimalsolution.

The various steps involved with fuzzy inference module for evaluation of optimal solution of quantity of transported amount are as follows.

Step 1. Consider cost of transportation and time of transportation as input parameters and quantity of transported amount as output parameter.

Step 2. Assign the membership values to all input parameters with linguistic terms as low, medium, high and very high.

Step 3. Frame the appropriate rules and find the corresponding rule strength which is computed by min operator.

Step 4. Evaluate membership functions of each of the fuzzy variables Very Low, Low, Medium, High and Very High which are represented by the rules with non-zero strength.

Step 5. Obtain fuzzy output by applying fuzzy union operator. Step 6. Defuzzified amount is obtained by using centroid method.

V. EXAMPLE

Following example is considered to illustrate the use of Mamdani fuzzy Inference systems along with genetic algorithm (G.A) in solving multi-objective transportation problems.

A multi-objective transportation problem with three origins and three destinations is considered with total supply units are given as $a_1 = 70$, $a_2 = 75$, $a_3 = 90$ and the total requirements are $b_1 = 65$, b_2

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= 85 and b3 = 85. The input parameters cost of transportation, time of transportation and quantity of transported amount are taken in terms of triangular fuzzy number as,

Cost of transportation: Low = (4, 8, 12), Medium = (8, 12, 16), High = (12, 16, 20) and Very High = (16, 20, 24). Time of transportation: Low = (1, 7, 13), Medium = (7, 13, 19), High = (13, 19, 25) and Very High = (19, 25, 31). Transported amounts: Low = (9, 15, 22), Medium = (15, 22, 30), High= (22, 30, 35) and Very High = (30, 35, 40).

The problem is solved by using fuzzy logic with MOGA and the optimal compromise solution obtained is as follows:

Table 4:Optimal solution for transportation cost

	D1	D2	D3
S1	11.51	9.64	13.88
S2	9.46	14.26	8.19
S3	9.76	6.53	12.8

Table 5:Optimal solution for transportation time

	D1	D2	D3
S1	7.86	12.08	14.1
S2	21.39	8.68	9.73
S3	8.81	24.99	11.34

Table 6:Optimal solution for quantity of transported amount

	D1	D2	D3
S1	28	26	16
S2	19	22	31
S3	29	22	23

From the above solution matrix it is observed that transported amount does not reach at very high level because there is no cell with both low transportation cost and time. Similarly the quantity of transported amount increases when both cost and time of transportation are decreases (Cell (S2, D3)).

VI. CONCLUSION

During this study, a multi stage transportation problem is considered with multiple numbers of objectives and multiple criteria. In first stage we analyze the effect of those criteria on both the objectives transportation cost and transportation time. In next stage, we consider the two objectives of first stage as criteria and discuss their impact on quantity of transported amount, the objective of second stage. Finally a transportation model with multiple numbers of objectives is solved by combining Genetic Algorithm and Mamdani fuzzy inference rules and result is discussed.

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