

# Effect of Resampling on the Performance and Execution Speed of Adaptive Marginalized Particle Filter



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**Abstract:** One of the major factors that affects the performance of adaptive filters like Particle Filter (PF), Marginalized Particle Filter (MPF) and Adaptive Marginalized Particle Filter (AMPF) is sample degeneracy. Sample degeneracy occurs when the weights associated with particles converges to zero making them useless in state estimation. Resampling is the most common method used to avoid sample degeneracy problem, in which a new set of particles are generated and weights are assigned. Performance and execution time of these filter depends a lot on what type of resampling technique is employed. AMPF is the modified version of MPF which is typically faster than PF and MPF. The main aim of this paper is to find the effect of different types of resampling on the performance and execution time of AMPF. For this, a typical target tracking problem is simulated using MATLAB. AMPF with different types of resampling techniques is used for state estimation for the above-mentioned problem and the best in terms of performance and execution speed will be found out. From the simulation, it will be clear that AMPF with systematic resampling is found to be best in terms of execution speed and performance i.e. minimum Root Mean Square Error.

**Keywords:** Adaptive Marginalized Particle Filter, Resampling, State estimation, Target tracking.

## I. INTRODUCTION

Adaptive Marginalized Particle Filter (AMPF) [1] is a state estimation technique applicable when the model associated with the system contains a linear Gaussian substructure. Such type of model is very common in many applications including bearing only tracking, collision avoidance, surveillance etc. The main advantage of using

AMPF over conventional techniques like Particle Filter (PF) [2], [3] and Marginalized Particle Filter (MPF) [4]–[6] is that the former is much faster and results in estimates with less variance and performance is much better when the noise affecting the system is more. Being a modified version of MPF, just like in the case of MPF, AMPF also estimates the linear state variables using Kalman Filter (KF) [7], [8] and nonlinear variables associated with the model is estimated using Particle filter. When it comes to Particle filter, the main factor that affects the performance of the filter is sample degeneracy. Sample Degeneracy [9] generally occurs when after few iterations the weights associated with the particles becomes zero making them ineffective when it comes to state estimation. It also adds to complexity of the filter and thereby affecting the execution speed and estimation process. The complexity analysis in the case of MPF and PF is detailed in [10].

As AMPF estimates the nonlinear variables using PF, sample degeneracy is an important parameter that affects the performance of AMPF also. Resampling [11], [12] is the most common technique used to fight against degeneracy problem. There are several types of resampling techniques which have been introduced over the years. All the different types have their own advantages and disadvantages in terms of performance and complexity. Of the different types, the most commonly used resampling techniques are Multinomial, Residual, Stratified and Systematic resampling technique [9], [13].

The main objective of this paper to find the effect of these different resampling technique which when used along with AMPF on performance and complexity. The complexity is studied by evaluating the execution speed of the filter. A typical target tracking application is simulated using MATLAB using these four resampling techniques.

The paper is structured as follows, Section II Briefly explains Adaptive Marginalized Particle Filter, the different types of resampling techniques is discussed in section III, a typical target tracking example used for simulation is given in section IV, section V gives the results and discussion and finally the conclusion is given in section VI.

## II. ADAPTIVE MARGINALIZED PARTICLE FILTER

Most of the state estimation problems are associated with a dynamic state and measurement models [14], [15].

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The most general model [4] used in such state estimation problems is given below.

$$\mathbf{x}_{t+1}^n = \mathbf{f}_t^n(\mathbf{x}_t^n) + \mathbf{A}_t^n(\mathbf{x}_t^n)\mathbf{x}_t^l + \mathbf{G}_t^n(\mathbf{x}_t^n)\mathbf{v}_t^n \quad (1a)$$

$$\mathbf{x}_{t+1}^l = \mathbf{f}_t^l(\mathbf{x}_t^l) + \mathbf{A}_t^l(\mathbf{x}_t^l)\mathbf{x}_t^l + \mathbf{G}_t^l(\mathbf{x}_t^l)\mathbf{v}_t^l \quad (1b)$$

$$\mathbf{y}_t = \mathbf{h}_t(\mathbf{x}_t^n) + \mathbf{C}_t(\mathbf{x}_t^n)\mathbf{x}_t^l + \mathbf{e}_t \quad (1c)$$

here, the nonlinear and linear state variables for  $(t+1)^{th}$  and  $t^{th}$  time sample are given by  $\mathbf{x}_{t+1}^n, \mathbf{x}_{t+1}^l, \mathbf{x}_t^n, \mathbf{x}_t^l$  respectively. The measurement variable

for  $t^{th}$  time interval is denoted by  $\mathbf{y}_t$ .  $\mathbf{A}_t^n(\mathbf{x}_t^n), \mathbf{A}_t^l(\mathbf{x}_t^l), \mathbf{G}_t^n(\mathbf{x}_t^n), \mathbf{G}_t^l(\mathbf{x}_t^l), \mathbf{C}_t(\mathbf{x}_t^n)$  gives the constant matrices of the dynamic model. Linear and nonlinear function related to measurement variable is given by  $\mathbf{f}_t^n(\mathbf{x}_t^n), \mathbf{f}_t^l(\mathbf{x}_t^l), \mathbf{h}_t(\mathbf{x}_t^n)$ . The uncertainty which occurs in the process and measurement data are modelled as the process

noise and the measurement noise, denoted as  $\mathbf{v}_t, \mathbf{e}_t$ . Both process noise and measurement noise are assumed to be identical to that of Gaussian white noise. Their distribution is defined as follows

$$\mathbf{v}_t = \begin{bmatrix} \mathbf{v}_t^l \\ \mathbf{v}_t^n \end{bmatrix} \sim N(0, \mathbf{Q}_t), \quad \mathbf{Q}_t = \begin{bmatrix} \mathbf{Q}_t^l & \mathbf{Q}_t^{ln} \\ \mathbf{Q}_t^{ln^T} & \mathbf{Q}_t^n \end{bmatrix} \quad (2)$$

$$\mathbf{e}_t \sim N(0, \mathbf{R}_t) \quad (3)$$

The process and measurement noise covariance are given by  $\mathbf{R}_t$  and  $\mathbf{Q}_t$ . The initial variable is represented as  $\mathbf{x}_0^l$  and is given by

$$\mathbf{x}_0^l \sim N(\bar{x}_0, \bar{P}_0) \quad (4)$$

where  $\bar{x}_0, \bar{P}_0$ , denotes the mean and covariance of the initial state variables. The initial value of the nonlinear variable  $\mathbf{x}_0^n$  is assumed to be known and of arbitrary in nature.

In most of applications like target tracking, collision avoidance, surveillance, Ballistic missile state estimation [16]–[18], the dynamic state space models will be characterized with linear state equations and nonlinear measurement equations. This type of models is very important. This special model [4] is given as follows,

$$\mathbf{x}_{t+1}^n = \mathbf{A}_{n,t}^n \mathbf{x}_t^n + \mathbf{A}_{n,t}^l \mathbf{x}_t^l + \mathbf{G}_{n,t}^n \mathbf{v}_t^n \quad (5a)$$

$$\mathbf{x}_{t+1}^l = \mathbf{A}_{n,t}^l \mathbf{x}_t^n + \mathbf{A}_{l,t}^l \mathbf{x}_t^l + \mathbf{G}_{l,t}^l \mathbf{v}_t^l \quad (5b)$$

$$\mathbf{y}_t = \mathbf{h}_t(\mathbf{x}_t^n) + \quad + \mathbf{e}_t \quad (5c)$$

where,  $\mathbf{v}_t^n \sim N(0, \mathbf{Q}_t^n)$  and  $\mathbf{v}_t^l \sim N(0, \mathbf{Q}_t^l)$  gives the process noise in which  $\mathbf{Q}_t^n, \mathbf{Q}_t^l$  are the nonlinear and linear covariance of the nonlinear and linear part of process noise. The remaining variables are similar to the one explained while defining in Eqn. (1). It is very important to note the model

given in Eqn. (5) have nonlinear measurement equations and it doesn't contain any linear information.

The main motivation behind the development of AMPF is the variable nature of measurement noise. This variable nature of noise allows AMPF to adapt itself in terms of the number of particles required for state estimation thus giving an opportunity to reduce the complexity thereby improving the execution speed. The reduction in the number of particles usually affect the performance of these types of filters, however in the case of AMPF it is not much affected mainly due to the Noise Tolerance property of MPF [19], [20].

Another important factor that improves the performance of AMPF when compared to MPF & PF is the presence of re-initialization step in AMPF algorithm. Even though, the need for re-initialization step is to create a new set of particles based on the adapted number of particles, this step also removes the accumulated noise in the state variables which further improves the performance. A general algorithm of AMPF is given below. Based on the type of the resampling technique used, different version of AMPF are formed.

Algorithm:

1. Initialization of  $N$  particles
2. Evaluate the importance weights
3. Updating the Number of Particles based on the variance of the weights
  - 3.1 Calculation of cumulative variance of weights
  - 3.2 Calculation of average cumulative variance of weights.
  - 3.3 Update the number of particles based on threshold value.
  - 3.4 Re-Initialization of particles with updated number of particles.
4. Recalculation of importance weight with update number of particles.
5. Particle filter measurement update – Resampling (Multinomial, Residual, Stratified & Systematic)
6. Particle filter and Kalman filter time update and measurement update of Kalman filter.

A detailed explanation of Adaptive Marginalized Particle Filter is given in [1].

### III. DIFFERENT TYPES OF RESAMPLING - REVIEW

Particle Degeneracy is a bigger problem when it comes to PF, MPF and AMPF as it can lead filter to diverge within no time. The problem that occurs due to particle degeneracy is that, the weight of the particles comes to situation were only few weights will have value and the value of the rest dives to zero. This condition is just opposite to that of ideal case, where typically all the weights will have equal values thereby equally contributing to the estimation problem.

In resampling, a new set of particles will be generated around the weights with larger value and thereby neglecting the weights having smaller values.

Resampling therefore in a boarder angle depends on few factors which needs to looked upon which includes, what is the type of distribution that should be used, the strategy that should be used for sampling, the number of particles that will be generated and finally the frequency in which the resampling should be employed to avoid degeneracy.

Resampling [9], [13] can be classified into different types based on certain parameters like type of distribution, whether all the particles are resampled in a similar fashion, any type of grouping of particles are done, based on whether only particles from current time sample or both current & previous time samples are involved, frequency of applying resampling and finally stochastic or deterministic resampling is done.

There are various types of resampling techniques based on the above factors like Single distribution sampling, Compound sampling – threshold/ grouping based resampling, resampling that depends on state, modified resampling, variable-size resampling [9] etc.

This paper deals with some of the traditional resampling techniques which includes multinomial, stratified, systematic and residual resampling techniques.

In the case of multinomial resampling technique N random number are generated using a uniform distribution and using this distribution the new set of particles are selected. This type of resampling technique is also known as simple random sampling as the particles are being select at random. The variance of the resampled particles reaches maximum as the upper limit being the particle not sampled and lower limit are particle sampled N times.

The second and third type of resampling technique is known as the stratified and systematic resampling. As the name indicates in stratified resampling, the entire set of particles are divided into different strata and new particles are drawn independently from these strata's. Systematic resampling technique is similar to stratified resampling technique in which the population is divided in to strata and the only difference is that in systematic resampling some particles are drawn from uniform distribution and rest deterministically. Both systematic and stratified resampling techniques are computationally less complex when compared with that of multinomial resampling technique and in fact systematic is the least complex type.

Fourth and the final type of resampling technique that will be used in this paper is known as the residual resampling. Residual resampling is also known as remainder resampling in which the entire particles consists of two stages. First stage is composed of replication of particles with weight more than 1/N deterministically and the remaining particles are random sampled hence the name remainder resampling technique. Sometimes multinomial resampling is used for random sampling of the remaining particles.

These four resampling techniques are well explained and discussed in wide variety of papers [9], [13] and hence no included in detail in this paper.

#### IV. TYPICAL TARGET TRACKING PROBLEM

Most of state estimation problems involves in the estimation of state variables using some available measurements which may or may not be directly related to the

state variables. Filters like Adaptive Marginalized Particle Filter are capable to solve such problems with ease. In order to understand how resampling affects the complexity and performance of AMPF a typical target tracking example is considered in this paper.

The target tracking problem [1], [10], [20] considered in this paper involves in the tracking of a level flighted aircraft which is modelled using a 2D constant acceleration model. The model consists of a linear Gaussian substructure which is very similar to the important model discussed in Eqn. 5. Here the state model is linear and associated noise is assumed to be Gaussian in nature. The measurement model is purely nonlinear, which means that it does not provide an information of the linear state variables.

As a level flight of aircraft is considered the height component of the dynamic state space model is neglected. The dynamic state space model of 2D constant acceleration model of the target tracking example is given below.

$$\mathbf{x}_{t+1} = \begin{bmatrix} 1 & 0 & T & 0 & \frac{T^2}{2} & 0 \\ 0 & 1 & 0 & T & 0 & \frac{T^2}{2} \\ 0 & 0 & 1 & 0 & T & 0 \\ 0 & 0 & 0 & 1 & 0 & T \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}_t + \mathbf{v}_t \quad (8a)$$

$$\mathbf{y}_t = \begin{pmatrix} r \\ \theta \end{pmatrix} = \begin{pmatrix} \sqrt{p_x^2 + p_y^2} \\ \arctan(p_x/p_y) \end{pmatrix} + \mathbf{e}_t \quad (8b)$$

State model of the example in given in Eqn. 8(a) and measurement model is mentioned in Eqn. 8(b). Position, velocity and acceleration along x and y coordinates are denoted as  $p_x, p_y, v_x, v_y, a_x, a_y$ . Here the z coordinates are neglected as the height component is discarded as mentioned earlier.  $p_x, p_y, v_x, v_y, a_x, a_y$  constitutes the state variables and the corresponding state vector is denoted by  $\mathbf{x}_t$ . T represents the sampling period which is assumed to be 1 sec. The measurement includes range and bearing angle which are denoted by  $r$  and  $\theta$  respectively. Covariance of the measurement  $\mathbf{e}_t$  and process noises  $\mathbf{v}_t$  is defined as follows.

$$\mathbf{R} = \text{cov}(\mathbf{e}) = \text{diag}(100, 0.01) \quad (9a)$$

and

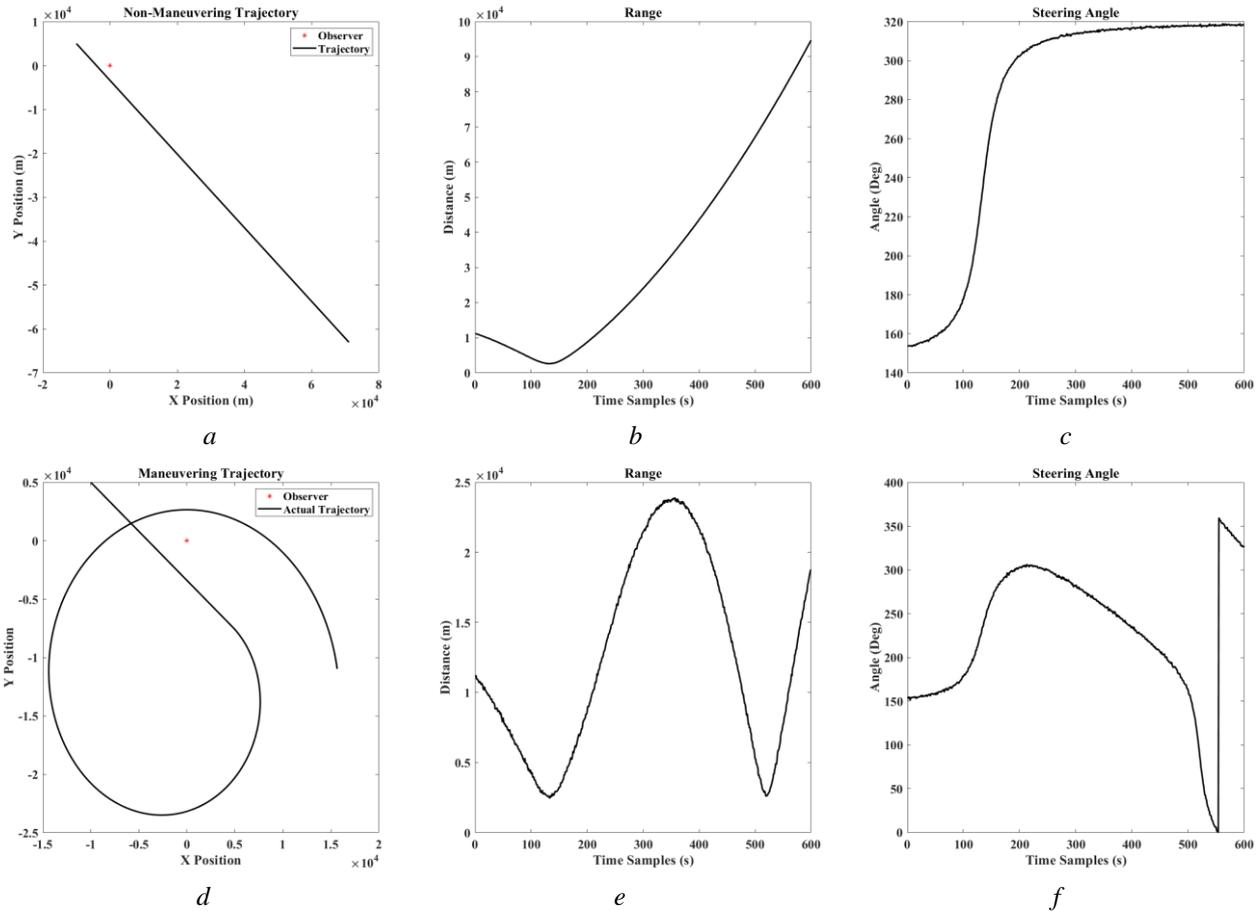
$$\mathbf{Q}^n = \text{cov}(\mathbf{v}^n) = \text{diag}(100, 100) \quad (9b)$$

$$\mathbf{Q}^l = \text{cov}(\mathbf{v}^l) = \text{diag}(2, 3, 0.09, 0.09) \quad (9c)$$

respectively.



State variables in the state vector contains linear and nonlinear parts which are represented as  $\mathbf{x}_t^n$  and  $\mathbf{x}_t^l$  example and the corresponding state variables will be estimated.



**Fig. 1.** Different target trajectories- solid line, observer at origin shown as ‘\*’ and their corresponding range and bearing angles. (a) True Non-Maneuvering Target Trajectory, (b) Range of Non-Maneuvering Target Trajectory, (c) Bearing angle of Non-Maneuvering Target Trajectory, (d) True Maneuvering Target Trajectory, (e) Range of Maneuvering Target Trajectory, (f) Bearing angle of Maneuvering Target Trajectory.

$$\mathbf{x}_t^n = \begin{bmatrix} p_x \\ p_y \end{bmatrix}, \mathbf{x}_t^l = \begin{bmatrix} v_x \\ v_y \\ a_x \\ a_y \end{bmatrix} \quad (10)$$

Comparing this model with that mentioned by Eqn 5, i.e the special model, it is clear that the term  $\mathbf{A}_{n,t}^l \mathbf{x}_t^n$  reduces to zero and

$$\mathbf{G}_t^n = \mathbf{I}_{2 \times 2}; \mathbf{G}_t^l = \mathbf{I}_{4 \times 4}$$

$$\mathbf{A}_{n,t}^n = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \mathbf{A}_{n,t}^l = \begin{pmatrix} 1 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0.5 \end{pmatrix}$$

$$\mathbf{A}_{n,t}^l = \mathbf{0}_{4 \times 2}; \mathbf{A}_{l,t}^l = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (11)$$

Adaptive Marginalized Particle Filter will be applied to this

## V. RESULTS AND DISCUSSION

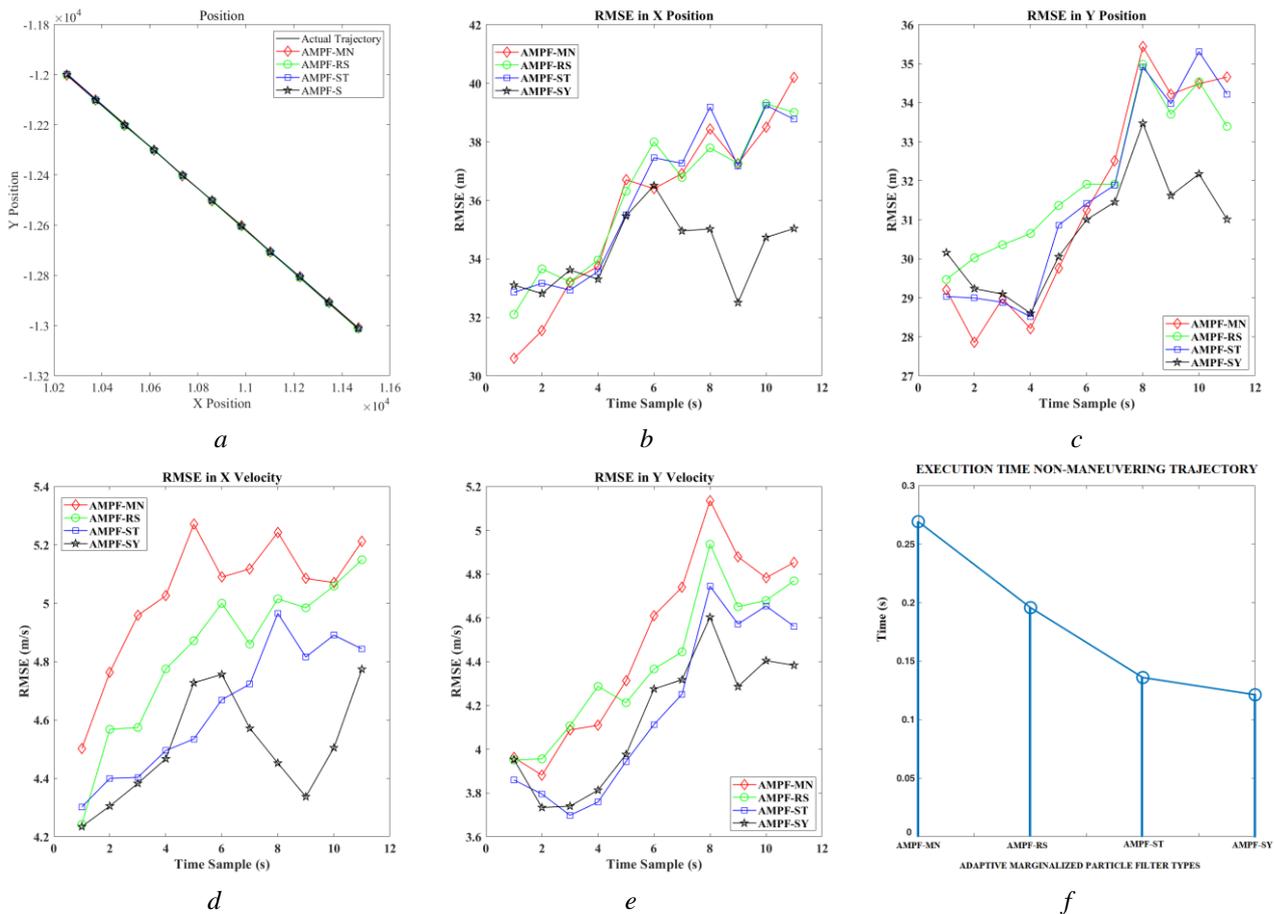
The main aim of this paper is to find out the best version of AMPF in terms of complexity and performance using different types of resampling techniques. The result is important as resampling is an unavoidable step in the case of AMPF.

The detailed algorithm of AMPF is given in [1]. Also, the noise tolerance property of MPF which is the basic idea behind in the development of AMPF is detailed in [19], [20].

In this paper, four variants of AMPF using Multinomial, Residual, Stratified, Systematic resampling namely, AMPF-Multinomial (AMPF-MN), AMPF- Residual (AMPF-RS), AMPF – Stratified (AMPF-ST) and AMPF-Systematic (AMPF-SY) Resampling techniques are stimulated and evaluated. Simulation is done using MATLAB in the case of the typical target example mentioned in section IV. 200 Monte Carlo simulations are done in order to get a stable output. The simulation done in this paper is generated using Eqn. 8 and corresponding values are generated for 600-time samples.

The parameters that are used in the simulation are given in Table 1.

is then evaluated by calculating the root mean square error (RMSE) between the actual generated values with that of the



**Fig. 2.** Performance of AMPF-MN (red line with diamond), AMPF-RS (green with circle), AMPF-ST (blue with square), AMPF-SY (black with pentagram) with Non-Maneuvering Trajectory with 10 time samples between 250<sup>th</sup> and 260<sup>th</sup> time sample (a) Estimated Maneuvering Target Trajectory, (b) RMSE of x-Position, (c) RMSE of y-Position, (d) RMSE of x-velocity, (e) RMSE of y-velocity, (f) Execution time of all variants of AMPF for the entire 600 time samples.

TABLE I: PARAMETER VALUES

Parameter	Values
Number of Monte Carlo Simulations	200
Initial Position $[p_x, p_y]$ in m	$[-1000*10, 1000*5]$
Initial Velocity $[v_x, v_y]$ in m/s	56
Acceleration $[a_x, a_y]$ in $m/s^2$	0.4
Initial state covariance $P_o$	diag(0.01,0.01,0.01, 0.01,0.01,0.01)
Measurement Noise Covariance $R$	diag(100,0.01)
Non-linear Process Noise Covariance $Q^n$	diag(200,200)
Linear Process Noise Covariance $Q^l$	diag(2,3,0.2,0.2)

Two types of aircraft trajectories and their corresponding range and bearing angle values which constitute the measurements are first generated using the state and measurement model. Then the range and bearing angle values are contaminated with Gaussian noise and then it is applied to all the four variants of AMPF. The performance of the filters

estimated ones. Root mean square error, which is the most common performance evaluating parameter is defined as

$$\left( \frac{1}{N} \sum_{t=1}^N \left( \frac{1}{N_{MC}} \sum_{j=1}^{N_{MC}} \|x_t^{true} - \hat{x}_t^{(j)}\|_2^2 \right)^{1/2} \right) \quad (12)$$

Here  $x_t^{true}$  and  $\hat{x}_t^{(j)}$  represents the actual value of the state variables and estimated values for t time sample and j<sup>th</sup> iteration of simulation. N denotes the number of particles and  $N_{MC}$  represents the number of Monte Carlo simulations.

Non-Maneuvering and Maneuvering trajectories are generated using Eqn (8) and the corresponding range and bearing angle contaminated with Gaussian noise are applied to AMPF-MN, AMPF-RS, AMPF-ST, AMPF-SY and RMSE in position and velocity state variables along x and y coordinates are evaluated. Maneuvering and Non-Maneuvering trajectories used in the simulation along with the corresponding generated range and bearing angle is shown in Fig. 1. (a), (d), Fig. 1. (b), (e) and Fig. 1. (c), (f) respectively.



The aircraft is assumed to initially at the coordinates [-1000\*10,1000\*5] having a velocity 56 m/s and with a constant acceleration of  $0.4 \text{ m/s}^2$ . Simulation are done using 1000 and 5000 particles in the case of Non-Maneuvering and Maneuvering trajectories. The performance and execution speed of all the four variants are evaluated for both Non-Maneuvering and Maneuvering trajectories.

The performance of AMPF-MN, AMPF-RS, AMPF-ST and AMPF-SY in terms of RMSE, execution time and the estimated trajectory in the case of Non-Maneuvering trajectory is given in Fig. 2 (a)-(f). RMSE in position parameter along x and y coordinates are shown in Fig.2 (b) & (c) between 250<sup>th</sup> and 260<sup>th</sup> time samples obtained when AMPF-MN (red line with diamond), AMPF-RS (green with circle), AMPF-ST (blue with square), AMPF-SY (black with pentagram) filters are used for state estimation. From the figure, it can be noted that the RMSE in x and y position is comparatively less in the case of AMPF-SY, and highest in the case of AMPF-MN.

RMSE in velocity parameter along x and y coordinates respectively is shown in Fig. 2 (c) & (d). Just as seen in RMSE in x & y- position similar results are obtained were AMPF-SY is better than all the other three variants of AMPF. Fig. 2 (e) gives the estimated trajectory by the four variants. From the figure, it is clear that all four variants performance is very close to each other properly tracking the trajectory. All the figures are plotted between 250<sup>th</sup> and 260<sup>th</sup> times samples in order to get better visualization.

An overall performance for 600-time samples in terms of RMSE in x & y position, velocity is given in table II and from the table it can be concluded that AMPF-SY is able to find the

TABLE- II: SIMULATION RESULT OF AMPF-MN, AMPF-RS, AMPF-ST, AMPF-SY IN THE CASE OF NON-MANEUVERING TRAJECTORY WITH 1000 PARTICLES

Parameter	AMPF -MN	AMPF -RS	AMPF -ST	AMPF -SY
Execution Time	0.2691	0.1956	0.1359	0.1212
RMSE X Position	85.197	71.986	66.933	66.161
RMSE Y Position	93.426	78.690	72.957	72.092
RMSE X Velocity	7.9775	6.8371	6.5143	6.4565
RMSE Y Velocity	8.5217	7.4204	7.0115	6.9313

estimates with less variance when compared to other variants of AMPF. It is also important to note that there is not much variation in the performance between all the four types. The performance of all the four variants are close to each other however AMPF-MN seems to find the estimates with higher variance compared with other variants of AMPF.

Finally, Fig. 2 (f) shows the execution time of all the variants of AMPF for 600-time samples evaluated using 200 Monte Carlo simulations. It is clear from figure that AMPF-SY is the taking around 0.12 sec on an average to complete the estimation process whereas the AMPF-MN takes about 0.26 sec in the case of estimation of Non-Maneuvering Trajectory. It is also worth noting that the execution time of AMPF-ST is less than AMPF-RS. In other words, it can be concluded that when it comes to the complexity AMPF-SY is the least complex and AMPF-MN is

the highest complex one. This is in line with the complexity analysis of various resampling techniques given in [9].

Thus, from Fig. 2 (a) – (f), it can be concluded that AMPF-SY which is Adaptive Marginalized Particle Filter using Systematic resampling technique is able to produce superior performance when compared to other resampling technique in the case of Non-Maneuvering Trajectory with lesser variance and faster execution speed.

A similar simulation done in the case of Non-Maneuvering Trajectory is also performed with Maneuvering Trajectory. The basic difference with latter case is that, the number of particles is increased to 5000 and the Measurement noise covariance R is changed to diag(9000,0.9).

Figure 3 (b) & (c) indicates the RMSE in position along x & y coordinates in the case of Maneuvering Trajectory. It can be noted that AMPF-SY is slight better than other variants most of the time. The performance of AMPF-MN is the worst when compared with AMPF-RS, AMPF-ST & AMPF-SY. There is not much difference to be noted and the performance of all the four variants is close to each other. The RMSE in velocity parameter in the case of Maneuvering trajectory is shown in Figure 3 (d) & (e). From figure it can be concluded similar to the one obtained in the case of Non-Maneuvering trajectory performance of all the four variants are close to each other with AMPF-SY slight better.

Estimated Maneuvering trajectory using AMPF-MN, AMPF-RS, AMPF-ST and AMPF-SY from 250<sup>th</sup> time sample to 260<sup>th</sup> time sample is shown in Fig. 3 (a). The overall performance of estimation in terms of RMSE in x & y position, velocity is shown in table III. From the table also, the performance of four filter is much clearer. It can be noted that performance of AMPF-SY is better than other filters and the worst filter performance is observed in the case of AMPF-MN and the performance of AMPF-RS & AMPF-ST are very close to each other. Another point worth noting is that from the performance point of view there is not much difference among all the four variants of AMPF, this is in line with the result obtained from Fig. 3. Similar results are seen in the case of Non-Maneuvering trajectory also, which indicates the credibility of the findings.

Execution time of AMPF-MN, AMPF-RS, AMPF-ST & AMPF-SY is shown in Fig. 3(f) in the case of Maneuvering trajectory. It can be concluded that AMPF-SY is faster compared to other variants of AMPF with 0.5899 sec when compared to the slowest 3.9491 sec that of AMPF-MN. The execution time is also mentioned in table 3 along the overall performance of the filters in terms of RMSE.

TABLE- III: SIMULATION RESULT OF AMPF-MN, AMPF-RS, AMPF-ST, AMPF-SY IN THE CASE OF MANEUVERING TRAJECTORY WITH 5000 PARTICLES

Parameter	AMPF -MN	AMPF -RS	AMPF -ST	AMPF -SY
Execution Time	3.9491	2.0328	0.6134	0.5866
RMSE X Position	97.383	95.670	95.443	94.846
RMSE Y Position	72.045	71.302	70.952	70.323

RMSE X Velocity	12.145	11.697	11.654	11.627
RMSE Y Velocity	10.042	9.9012	9.7985	9.7774

Thus, from the simulations done in the case of typical target tracking example using AMPF-MN, AMPF-RS, AMPF-ST & AMPF-SY the following observations and conclusion can be drawn. From Fig. 2, 3 and Tables 2, 3, it can be concluded that from the performance point of view all the four variants have RMSE very close to each other while Adaptive Marginalized Particle Filter with systematic resampling is found to be slightly better than AMPF-MN, AMPF-RS and AMPF-ST.

Efficiency of filters is also determined by the execution time, i.e. how fast the state variables can be estimated. Execution time indirectly shows the complexity of the filter, i.e. it can be concluded that if the execution time is less then complexity is less and when the execution time is more which clear indicates that the complexity of the algorithm is more.

In the second analysis, the execution time of AMPF-MN, AMPF-RS, AMPF-ST and AMPF-SY was evaluated. Fig. 2 (f), Fig. 3 (f) and Tables 2 & 3 indicates the execution time of AMPF using different resampling techniques. Thus, it is clear that, AMPF-SY is the fastest among the different types and AMPF-MN is the slowest. Execution time of AMPF-RS and AMPF-ST lies in between that of AMPF-SY and AMPF-MN. Thus, it can be concluded that the complexity of AMPF-SY is small compared to that of AMPF-MN.

Thus, from the simulation it can be clearly stated that the resampling does have an effect on the performance and execution time of Adaptive Marginalized Particle Filter.

## VI. CONCLUSION

Resampling is an inevitable step in the case of state estimation filters like Particle Filter, Marginalized Particle Filter and Adaptive Marginalized Particle Filter. Adaptive Marginalized Particle Filter is known for its superior execution speed. In this paper the dependence of resampling on the performance and execution time of AMPF is studied. For the simulation point of view a typical target example was considered and the performance and execution time of four variants of AMPF was evaluated and plotted. From the simulation it can be concluded that AMPF with Systematic resampling techniques tends to outperform other forms of AMPF both in the case of performance and execution speed. It can be also worth noting that, by reducing the complexity of the resampling step the overall execution speed of state estimation filters like Particle Filter, Marginalized Particle Filter and Adaptive Marginalized Particle Filter can be improved.

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