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Half Companion Sequences of Special Dio 3-Tuples Involving Centered Square Numbers

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Abstract: *In this paper, we construct a sequence of Special Dio 3-tuples for centered square numbers involving half companion sequences under 3 cases with the properties D(-2), D(-11) & D(-26).*

Keywords *: Diophantine Triples, special dio-tuples,Centered square Number, Integer Sequences.*

I. INTRODUCTION

In Number theory, a Diophantine equation is a polynomial equation, usually in two or more unknowns, with the end goal that solitary the integer solutions are looked for or contemplated [1-4]. The word Diophantine alludes to the Greek mathematician of the third century, Diophantus of Alexandria, who made an investigation of such conditions and was one of the primary mathematician to bring symbolism into variable based mathematics.

Various mathematicians considered the problem of the occurrence of Dio triples and quadruples with the property D(n) for any integer n and besides for any linear polynomial in n[5-7]. In this unique circumstance, one may refer for a comprehensive review of different problems on Diophantine triples [8-10]. In [11], half companion sequence of diophantine triples were investigated. These results motivated us to examine for non-extendable special dio triples with elements involving centered square numbers. This paper aims at constructing a sequence of Special Dio 3-tuples for centered square numbers involving half companion sequences under 3 cases with the properties D(-2), D(-11) & D(-26).

II. NOTATIONS

CS_n : Centered Square number of rank n.

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III. BASIC DEFINITION

A set of three different polynomials with integer coefficients (a_1, a_2, a_3) is said to be a special dio 3-tuple with property $D(n)$ if $a_i * a_j + (a_i + a_j) + n$ is a perfect square for all $1 \le i < j \le 3$, where n may be non-zero integer or polynomial with integer coefficients.

IV. METHOD OF ANALYSIS:

Case 1:

Forming sequence of Special dio 3-tuples for centered square numbers of consecutive ranks n and $n+1$

Let $a_1 = CS_n = n^2 + (n-1)^2$ & $a_2 = CS_{n+1} = (n+1)^2 + n^2$ be Centered Square numbers of rank n and $n+1$ respectively such that $a_1 a_2 + a_1 + a_2 - 2$ is a perfect square say α^2.

Let p_3 be any non-zero whole number such that

$$a_1 a_3 + a_1 + a_3 - 2 = \beta_1^2 \quad (1)$$

$$a_2 a_3 + a_2 + a_3 - 2 = \gamma_1^2 \quad (2)$$

Assume $\beta_1 = x_1 + a_1 y_1$ an $\gamma_1 = x_1 + a_2 y_1$,

it becomes $x_1^2 = (a_1+1)(a_2+1) y_1^2 - 3 \quad (3)$

Therefore, $\beta_1 = 4n^2 - 2n + 3$

with the initial solution $x_1 = 2n^2 + 1, \quad y_1 = 1$

On substitution of the values of a_1 and β_1 in equation (1), we get

$$a_3 = 8n^2 + 5 = 2(CS_n + CS_{n+1}) + 1$$

Hence, The triple $(CS_n, CS_{n+1}, 2(CS_n + CS_{n+1}) + 1)$ is a special dio 3-tuple with property $D(-2)$.

Now let $'a_4'$ be any non-zero whole number such that

$$a_2 a_4 + a_2 + a_4 - 2 = \beta_2^2 \quad (4)$$

$$a_3 a_4 + a_3 + a_4 - 2 = \gamma_2^2 \quad (5)$$

Introducing the linear transformations,

$$\beta_2 = x_2 + a_2 y_2 \ , \ \gamma_2 = x_2 + a_3 y_2 \quad (6)$$

Using some algebra from (4), (5) and (6), we have

$$x_2 = 4n^2 + 2n + 3, \quad y_2 = 1$$

Therefore, $\beta_2 = 12n^2 + 2n + 9$

Substituting β_2 in (4) we have,

$$a_4 = 18n^2 + 6n + 13 = \mathrm{CS}_{3n+1} + 12$$

The triple $\left(\mathrm{CS}_{n+1}, 2(\mathrm{CS}_n + \mathrm{CS}_{n+1}) + 1, \mathrm{CS}_{3n+1} + 12\right)$ is a special dio 3-tuple with property $D(-2)$.

Now let $'a_5'$ be any non-zero whole number such that

$$a_3a_5 + a_3 + a_5 - 2 = \beta_3^2 \quad (7)$$

$$a_4a_5 + a_4 + a_5 - 2 = \gamma_3^2 \quad (8)$$

Introducing the linear transformations,

$$\beta_3 = x_3 + a_3y_3 \ , \gamma_3 = x_3 + a_4y_3 \quad (9)$$

Using some algebra from (7), (8) and (9), we have

$$x_3 = 12n^2 + 2n + 9$$

Therefore, $\beta_3 = 30n^2 + 18n + 23$

Substituting β_3 in (7) we've

$$a_5 = 50n^2 + 10n + 37 = \mathrm{CS}_{5n+1} + 36$$

Hence, $\left(2(\mathrm{CS}_n + \mathrm{CS}_{n+1}) + 1, \mathrm{CS}_{3n+1} + 12, \mathrm{CS}_{5n+1} + 36\right)$ is a special dio 3-tuple with property $D(-2)$.

Hence, $\{a_1,a_2,a_3\},\{a_2,a_3,a_4\},\{a_3,a_4,a_5\}\ldots$ forms a sequence of special dio 3-tuples with property $D(-2)$.

Some numerical illustrations satisfying the property are listed in the following table1:

TABLE 1

n	(a_1, a_2, a_3)	(a_2, a_3, a_4)	(a_3, a_4, a_5)	D(-2)
3	(13,25,77)	(25,77,193)	(77,193,517)	-2
4	(25,41,133)	(41,133,325)	(133,325,877)	-2
5	(41,61,205)	(61,205,493)	(205,493,1337)	-2
6	(61,85,293)	(85,293,697)	(293,697,1897)	-2
7	(85,113,397)	(113,397,937)	(397,937,2557)	-2

Case 2:

Forming sequence of Special dio 3-tuples for centered square numbers of rank n and $n+2$

Let $a_1 = \mathrm{CS}_n$

$a_2 = \mathrm{CS}_{n+2}$

Proceeding as in case 1, with property $D(-11)$, we get,

$$a_3 = 8n^2 + 8n + 7 = 2(\mathrm{CS}_n + \mathrm{CS}_{n+2}) - 5$$

$$a_4 = 18n^2 + 30n + 25 = \mathrm{CS}_{3n+3} + 12$$

$$a_5 = 50n^2 + 70n + 61 = \mathrm{CS}_{5n+4} + 36$$

$\therefore \{a_1,a_2,a_3\},\{a_2,a_3,a_4\},\{a_3,a_4,a_5\}\ldots$ forms half companion sequences of special dio 3-tuples with the property $D(-11)$.

Some numerical illustrations satisfying the property are listed in the following table 2:

TABLE 2

n	(a_1, a_2, a_3)	(a_2, a_3, a_4)	(a_3, a_4, a_5)	D(-11)
3	(13,41, 103)	(41, 103,277)	(103,277,721)	-11
4	(25,61,167)	(61,167,433)	(167,433,1141)	-11
5	(41,85,247)	(85,247,625)	(247,625,1661)	-11
6	(61,113,343)	(113,343,853)	(343,853,2281)	-11
7	(85,145,455)	(145,455,1117)	(455,1117,3001)	-11

Case 3:

Forming sequence of Special dio 3-tuples for centered square numbers of rank n and $n+3$

Let $a_1 = \mathrm{CS}_n$

$a_2 = \mathrm{CS}_{n+3}$

Proceeding as in earlier cases, we get

$$a_3 = 8n^2 + 16n + 13 = 2(\mathrm{CS}_n + \mathrm{CS}_{n+3}) - 15$$

$$a_4 = 18n^2 + 54n + 53 = \mathrm{CS}_{3n+5} + 12$$

$$a_5 = 50n^2 + 130n + 121 = \mathrm{CS}_{5n+7} + 36$$

$\therefore \{a_1,a_2,a_3\},\{a_2,a_3,a_4\},\{a_3,a_4,a_5\}\ldots$ forms half companion sequences of special dio 3-tuples with the property $D(-26)$.

Some numerical illustrations satisfying the property are listed in the following table 3:

TABLE 3

n	(a_1, a_2, a_3)	(a_2, a_3, a_4)	(a_3, a_4, a_5)	D(-26)
3	(13,61,133)	(61,133,377)	(133,377,961)	-26
4	(25,85,205)	(85,205,557)	(205,557,1441)	-26
5	(41,113,293)	(113,293,773)	(293,773,2021)	-26
6	(61,145,397)	(145,397,1025)	(397,1025,2701)	-26
7	(85,181,517)	(181,517,1313)	(517,1313,3481)	-26

V. CONCLUSION:

In this paper, we have introduced a few examples of developing a sequence of special dio-tuples for centered square numbers of different ranks with reasonable properties. To finish up one may look for other sequence of triples or quadruples for different numbers with their relating properties.

REFERENCES

1. R.D.Carmichael, "History of Theory of numbers and Diophantine Analysis", Dover Publication, New york, 1959.
2. L.J.Mordell, "Diophantine equations", Academic press, London, 1969.
3. T.Nagell, "Introduction to Number theory", Chelsea publishing company, New york, 1981.
4. L.K.Hua, "Introduction to the Theory of Numbers", Springer-Verlag, Berlin-New york, 1982.
5. Oistein Ore, "Number theory and its History", Dover publications, New york, 1988.
6. Y.Fujita , "The extendability of Diphantine pairs $\{k-1, k+1\}$", Journal of Number Theory, 128, 322-353, 2008.

7. M.A. Gopalan and V.Pandichelvi , "On the extendability of the Diophantine triple involving Jacobsthal numbers $(J_{2n-1}, J_{2n+1}-3, 2J_{2n}+J_{2n-1}+J_{2n+1}-3)$", International Journal of Mathematics & Applications, 2(1), 1-3, 2009.
8. G.Janaki and C.Saranya, "Special Dio 3-tuples for pentatope number", Journal of Mathematics and Informatics, vol.11, Special issue, 119-123, Dec 2017.
9. G.Janaki and C.Saranya, "Construction of the Diophantine Triple involving Pentatope Number", International Journal for Research in Applied Science & Engineering Technology, vol.6, issue III, 2317-2319, March 2018.
10. C.Saranya and G.Janaki , "Integer triples comprising of Jarasandha numbers in Arithmetic progression & Geometric progression", International Journal of Research and Analytical Reviews, vol.6, issue 1, 751-752, Jan 2019.
11. P.Saranya and G.Janaki, "Half companion sequences of dio 3-tuples from $\frac{CC_n}{Gno_n}$", IJSART, Vol. 4, Issue 4, pp.3013-3015, April 2018.

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