

# Effect of Viscous Dissipation on MHD Williamson Nanofluid Flow in a Porous Medium

N. Manjula, K. Govardhan, M. N. Rajashekar

Abstract: The main objective of this paper is to focus on a numerical study of viscous dissipation effect on the steady state flow of MHD Williamson nanofluid. A mathematical modeled which resembles the physical flow problem has been developed. By using an appropriate transformation, we converted the system of dimensional PDEs (nonlinear) into coupled dimensionless ODEs. The numerical solution of these modeled ordinary differential equations (ODEs) is achieved by utilizing shooting technique together with Adams-Bashforth Moulton method of order four. Finally, the results of discussed for different parameters through graphs and tables.

Keyword: Williamson Nanofluid, Adams – Moulton method; Thermophoresis; Brownian motion; viscous dissipation nanofluids.

#### I. INTRODUCTION

A substance in the gas or liquid phase is referred to as the fluid. Flow of fluid has all kinds of aspects, steady and unsteady, compressible and incompressible, viscous and inviscid, rotational and irrotational, uniform and nonuniform etc., Meir [1]. The study of fluid flow on a stretching surface is one of the important problems in the current era, as it occurs in different processes of engineering for example, extrusion, wire drawing, food manufacturing, metal spinning, manufacturing of rubber sheets and cooling of huge metallic plates such as an electrolyte, etc. Sakiadis [2] was the first who introduced the problem of boundary layer approximations over stretching surface. The flow caused by stretching sheet was investigated by Crane [3]. Recently, many researchers such as Shehzad et al. [4], Zheng et al. [5], and Gireesha et al. [6] due to wide applications as described above.

The study of magnetic properties of electrically conducting fluids is known as Magnetohydrodynamics (MHD). MHD fluid flow was first introduced by Swedish Physicist, Alfven

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[7]. In recent years, mass and heat transfer on unsteady MHD natural convention rotating flow of fluid about a porous plate with heat transfer and radiation was studied by Mbeledogu and Ogulu [8]. Kesavaiah et al. [9] investigated the unsteady MHD convective flow over a vertical porous plate. The Jeffrey fluid effect of convection in MHD flow of heat transfer about a stretching sheet is reported by Hayat et al. [10]. Mustafa et al. [11] inspected the MHD Maxwell fluid flow with convective heat transfer. MHD viscous incompressible flow has many applications in engineering for example, cooling of reactors, a power generator, MHD accelerators and design of heat exchangers, as provoked by Hari etal. [12].

. Non-Newtonian fluids are those for which the shear stress is not linearly proportional to the deformation rate. In other words, fluids that do not satisfy Newton's law of viscosity are known as non-Newtonian fluids. Blood, paints, ketchup, shampoo, mud etc., behave like non-Newtonian fluids. Williamson fluid is one of non-Newtonian fluids. The study of Williamson fluids for the boundary layer flow is of great interest because of its vast range of applications in different branches of science, technology and engineering, especially in nuclear and chemical industries, bioengineering and geophysics. Considering these applications extensive range of scientific models has been established to simulate the flow actions of these non-Newtonian fluids.

Williamson [13] discussed the flow of pseudo plastic materials and presented a model equation to discuss the pseudo plastic fluids flow and verified the results experimentally. Nadeem et al. [14] presented the Williamson fluid flow past a stretching surface and found that, by increasing values of Williamson fluid parameter, the dimensionless velocity decreases. Heat transfer characteristics on non-Newtonian nanofluid flow over a stretching sheet were presented by Nadeem and Hussain [15]. Hayat and Hina [16] studied the impact of mass and heat transfer with flexible walls on Williamson fluid flow.

Recently, flows of boundary layer of Newtonian and non-Newtonian fluids have drawn considerable attention because of their significant applications in processing of metallurgical, phenomena of chemical engineering transport, molten polymers extrusion, plastic sheets and wrapping foils fabrication etc. Species, momentum and heat transport play a major role in such processes [17]. In this article, provides the review work of research paper of M. Krishnamurthy et al. [18] and extends the work for the impact of viscous

dissipation on MHD Williamson nanofluid past the horizontally stretching sheet.



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#### II. MATHEMATICAL FORMULATIONS

Consider MHD boundary layer flow of incompressible Williamson nanofluid.

The flow is 2Dpast a stretching surface in a porous medium. Schematic diagram of a system under investigation is shown in the Figure 1. The plate is stretched along x – axes with velocity u=ax, (a>0). Amagnetic field is applied along the y –direction. The temperature at surface is  $T_w$ , fluid velocity is  $U_w$ ,  $C_w$  represents urface concentration. Moreover,  $T_m$  denotes the melting surface temperature and  $T_\infty$  denotes the free stream temperature of the nanofluid. The free stream temperature  $T_\infty$  is greater than the melting surface temperature  $T_m$ .

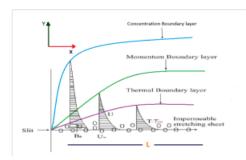


Figure 1: Geometry of the physical model.

Under the above constraints, the boundary layer equations are given as:

$$\frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \frac{\partial \mathbf{v}}{\partial \mathbf{x}} = 0 \tag{1}$$

$$u\frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \mathbf{v}\frac{\partial \mathbf{v}}{\partial \mathbf{v}} = v\frac{\partial^2 \mathbf{u}}{\partial \mathbf{v}^2} + \sqrt{2}v\frac{\partial \mathbf{u}}{\partial \mathbf{v}}\frac{\partial^2 \mathbf{u}}{\partial \mathbf{v}^2} - \sigma\frac{B_0^2}{\rho}\mathbf{u} - \frac{\mathbf{v}}{\mathbf{k}'}\mathbf{u}$$
(2)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho c)_f} \frac{\partial q_r}{\partial y} + \tau \left( D_B \left( \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} \right) + \frac{\partial T}{\partial y} \right) + \frac{1}{(\rho c)_f} \frac{\partial T}{\partial y} + \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y} - \frac{1}{(\rho c)_f} \frac{\partial q_r}{\partial y} + \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y} - \frac{1}{(\rho c)_f} \frac{\partial q_r}{\partial y} + \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y} - \frac{1}{(\rho c)_f} \frac{\partial q_r}{\partial y} + \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y} - \frac{1}{(\rho c)_f} \frac{\partial q_r}{\partial y} + \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y} - \frac{1}{(\rho c)_f} \frac{\partial q_r}{\partial y} + \frac{\partial T}{\partial y} = \alpha_m \frac{\partial T}{\partial y} - \frac{\partial T}{\partial y} + \frac{\partial T}{\partial y} = \alpha_m \frac{\partial T}{\partial y} + \frac{\partial T}{\partial y} - \frac{\partial T}{\partial y} + \frac{\partial T}{\partial$$

$$\frac{D_T}{T_{\infty}} \left( \frac{\partial \mathbf{T}}{\partial y} \right)^2 + \frac{\nu}{(\rho c)_f} \left( \frac{\partial u}{\partial y} \right)^2 \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B \left(\frac{\partial C}{\partial y}\right)^2 + \frac{D_T}{T_{co}} \left(\frac{\partial^2 T}{\partial y^2}\right) - k_0 \tag{4}$$

The associated boundary conditions for the modeled problem:

$$u = U_w(x) = ax, T = T_m, C = C_w \text{ at } y = 0,$$

$$u = 0, T \to T_\infty, C \to C_\infty \text{ as } y \to \infty,$$

$$k\left(\frac{\partial T}{\partial y}\right)_{y=0} = \rho[\beta + c_s(T_m - T_0)]v(x, 0).$$
(5)

where u and v denote the components of fluid velocity along x and y direction, respectively, T denotes  $\frac{Nu}{n}$  e<sub>-</sub> temperature of the nanofluid,  $\rho$  the nanofluid density,  $\alpha_{n}$  The thermal diffusivity of the nanofluid,  $\nu$  the kinematic viscosity,  $D_R$ thecoefficient of Brownian  $D_T$ thermophoresis thecoefficient of diffusion, k'the porous medium permeability,  $(\rho c)_f$  the heat capacity of the fluid and  $(\rho c)_n$  denotes the heat capacity of the nanoparticle.

# III. NON-DIMENSIONAL FORM OF THE GOVERNING EQUATIONS:

The dimensional PDEs are converted into the nondimensional form by similarity transformation. We introduce the following dimensionless similarity variable

$$\eta = y \sqrt{\frac{a}{v}}, \psi = \sqrt{av} x f(\eta), \theta(\eta) = \frac{T - T_m}{T_{\infty} - T_m}, \beta(\eta) = \frac{C - C_{\infty}}{C_w - C_{\infty}}$$
(6)

After using similarity transformation, the equations take the form

$$ff'' - (f')^2 + W_e f'' f''' - (Q + k_p) f' + f''' = 0$$
 (7)

$$\frac{\left(1+\frac{4}{3}R\right)\theta^{\prime\prime}}{Pr} + f\theta^{\prime} + Nb\theta^{\prime}\beta^{\prime} + Nt(\theta^{\prime})^{2} + Ec(f^{\prime\prime\prime})^{2} = 0$$
 (8)

$$\beta'' + Lef \beta' + \frac{Nt}{Nb} \theta'' - Le \gamma \beta = 0$$
 (9)

The relevant boundary conditions are:

$$f'(0) = 1, \Pr f(\theta) + M \theta'(0) = 0, \theta(0) = 0,$$

$$\beta(0) = 0, \operatorname{at} \eta = 0,$$

$$f'(\infty) \to 0, \theta(\infty) \to 1, \beta(\infty) \to 1 \operatorname{as} \eta \to \infty$$

$$(10)$$

The physical constraints appeared in Eq. (7) to Eq. (10), we represents

$$\begin{split} Pr &= \frac{v}{\alpha}, R = \frac{-4T_{\infty}^{3}\sigma^{*}}{3k^{*}k}, k_{p} = \frac{v}{k'a}, W_{e} = \gamma x \sqrt{\frac{2a^{3}}{v}}, \\ \gamma &= \frac{k_{0}U(C_{\infty} - C_{W})}{v}, Nt = \frac{\tau D_{T}(T_{W} - T_{m})}{vT_{\infty}}, Q = \frac{\sigma B_{0}^{2}}{\rho a}, Le = \frac{v}{D_{B}}, \\ Nb &= \frac{\tau D_{B}(C_{\infty} - C_{m})}{v}, Ec = \frac{u_{w}^{2}}{\rho_{f}(T_{W} - T_{m})}. \end{split}$$

where Pr is the representation of the Prandtl number, R represents the thermal radiation parameter,  $k_p$  the

permeability parameter, W the non-Newtonian Williamson parameter, Y the chemical reaction parameter, Y the thermophoresis parameter, Y the magnetic parameter, Y the Lewis number, Brownian motion parameter is denoted by Y and Y the Eckert number.

Nusselt number, Sherwood number and the skin friction coefficients are expressed as:

coefficients are expressed as: 
$$Nu_x = \frac{xq_w}{k((T_\infty - T_m))}$$
,  $Sh_x = \frac{xq_m}{D_B((C_\infty - C_w))}$  and  $C_f = \frac{\tau_w}{\rho U_w^2}$ , (11) where,  $\tau_w$  is the shear stress along the stretching surface,

where,  $\tau_w$  is the shear stress along the stretching surface,  $q_w$  is the heat fluxfrom the stretching surface and  $q_m$  is the wall mass flux, are given as:

$$\begin{split} q_w &= -k \left(\frac{\partial T}{\partial y}\right)_{y=0}, q_m = -D_B \left(\frac{\partial C}{\partial y}\right)_{y=0}, \tau_w = \mu \left(\frac{\partial u}{\partial y} + \right. \\ &\Gamma 2 \partial u \partial y 2 y = 0, \end{split} \tag{12}$$

Using the dimensionless variables, we get

$$-\theta'(0), \frac{Sh_x}{\sqrt{R_x}} = -\beta'(0),$$

$$C_f \sqrt{R_x} = -f''(0) + \frac{\lambda}{2} f''(0)^2$$
(13)

where  $Re_x$  denotes the Reynolds number and is expressed as  $Re_x = \frac{xU_W(x)}{v}$ .

#### IV. NUMERICAL SOLUTION

The system of higher order ODE is converted into first order ODEs. The prescribed boundary conditions are also converted into first order system of ordinary differential equations (ODEs).





The first order system of ordinary differential equations including the related boundary conditions is solved numerically using Fortran Language.

$$f''' = \frac{1}{1 + \lambda f''} (-ff'' + (f')^2 + (Q + k_p)f')$$

$$\theta'' = \frac{3Pr}{3 + 4R} (-f\theta' - Nb\theta'\beta' - Nt(\theta')^2 - Ec(f'')^2$$

$$\beta^{\prime\prime} = -Lef\beta^{\prime} - \frac{Nt}{Nb}\theta^{\prime\prime} + Le\gamma\beta$$

By using the following notations,

$$f = y_{1}, f' = y'_{1} = y_{2}, f'' = y'_{2} = y_{3},$$

$$\theta = y_{4}, \theta' = y'_{4} = y_{5}, \theta'' = y'_{5},$$

$$\beta = y_{6}, \beta' = y'_{6} = y_{7}, \beta'' = y'_{7}.$$
The system of first order ODEs are:

$$y'_1 = y_2,$$
  $y_1(0) = r$  (15)  
 $y'_2 = y_3,$   $y_2(0) = 1$  (16)

$$y_2' = y_3, y_2(0) = 1 (16)$$

$$y_3' = \frac{1}{1+\lambda y_3} (-y_1 y_3 + (y_2)^2 + (Q + k_p) y_2$$
$$y_3(0) = s \tag{17}$$

$$y_4' = y_5 y_4(0) = 0 (18)$$

$$y_5' = -\frac{{}_{3+4R}}{{}_{3+4R}}(y_1y_5 + Nby_5y_7 + Nty_5^2 + Ecy_3^2)$$
 
$$y_5(0) = -\frac{{}_{P}r}{{}_{M}}$$
 
$$y_6' = y_7 \qquad \qquad y_6(0) = 0$$

$$y_5(0) = -\frac{Pr}{M}$$
 (19)

$$y_6' = y_7 y_6(0) = 0 (20)$$

$$y_7' = -Ley_1y_7 - \frac{Nt}{Nb}y_5' + Le\gamma y_6, y_7(0) = t$$
 (21)  
To solve the above system of equations, the unbounded

domain  $[0,\eta_{\infty}]$  is restricted to bounded domain  $[0,\eta_{e}]$ where  $\eta_e = 5$ . This is since increasing the value of  $\eta_e$  beyond 5 gives negligible variation in the numerical results. In the modeled problem, $r^{(0)}$ ,  $s^{(0)}$ ,  $t^{(0)}$  are initial guesses which are required to solve the above first order system of ordinary differential equations with fourth order Adams-Bashforth Moulton Method. Newton iterative scheme is used to refine those initial guesses. The iterative process is repeated until the following conditions is met.

$$max(|y_2(\eta_\infty)|, |y_4(\eta_\infty) - 1|, |y_6(\eta_\infty) - 1|) < 10^{-3}$$

## V.RESULTS AND DISCUSSION

The main purpose of this part is to analyze velocity, temperature, and concentration profiles. Table 1is prepared to study the impact of various parameters on the  $-\theta'(0)$  and  $-\beta'(0)$ . From this table, we can see that by increasing the values of permeability parameter, non-Newtonian Williamson parameter, chemical reaction parameter, Eckert number,  $-\theta'(0)$  decreasing whereas the  $-\beta'(0)$  is increased. By increasing the dimensionless melting parameter  $-\theta'(0)$  increasing and Sherwood number is decreased.

The impact of dimensionless melting parameter on velocity profile  $f'(\eta)$  and dimensionless temperature profile  $\theta(\eta)$  is shown in Figure 2and 3respectively. The graphical demonstration shows that for the increasing values of dimensionless melting parameter, the velocity profile and thickness of the boundary layerincreases slightly and the temperature distribution decreases. It is found that an increase in the dimensionless melting parameter increases the melting intensity, which acts as boundary condition at the stretching surface and tends to make the boundary layer thicker.

Impact of Radiation parameter is shown in Figure 4. It is seen that for gradually enhancing values of R energy profile also decreased. Due to increase in thermalRadiation parameter more heat to fluid produces that enhance the energy filedwith chemical effect on the melting surface. Figure 5also represents that concentration profile is increased for increasing values of radiation parameter.

From the figure 6, it is observed that by increasing values of Lewis number temperature near the surface of plate decreases and away from the surface of plate increases. Figure 7 reflects the effect of Lewis number on concentration profile. Lewis number can be defined as the ratio of thermal diffusion to the molecular diffusion. It is convenient of help us find the relation between mass and heat transfer coefficient. By increasing Lewis number, the concentration profile becomes steeper.

Figures 8 and 9 demonstrate the influence of the magnetic parameter on the dimensionless profile of velocity distribution  $f'(\eta)$  and dimensionless profile of temperature distribution  $\theta(\eta)$ , respectively. From these figures, with increasing values of the magnetic parameter, profile of velocity decreases. It is also observed that temperature distribution  $\theta(\eta)$  shows increasing effects as the magnetic parameter increases. The reason beyond this electrically conducting fluid produces a resistive force known as Lorentz force, which opposes the flow and tends to make the fluid motion slowdown in the boundary layer and therefore reduces the profile of velocity whereas its temperature  $\theta(n)$ increases with the increase in magnetic parameter.





Table 1: $-\theta'(0)$ and $-\beta'(0)$ for different parameter
---

$k_p$	М		γ	Ec	- heta'( <b>0</b> )	$-oldsymbol{eta}'(0)$
r		$W_{\cdot}$				• • •
0.5	0.5	0.2	0.01	0.2	-2.240853000	0.524976800
1.0					-2.166060000	0.549167900
1.5					-2.098345000	0.574106300
2.0					-2.037475000	0.600233300
	1.0				-1.363586000	0.331753700
	1.5				-1.044776000	0.225277800
	2.0				-0.856633100	0.172704300
	2.5				-0.732235500	0.143191300
		0.0			-2.082783000	0.574948000
		0.05			-2.072699000	0.579855900
		0.1			-2.061879000	0.585522400
		0.2			-2.037475000	0.600233300
			0.0		-2.094427000	0.595451500
			0.1		-1.770716000	0.638036800
			0.2		-1.679036000	0.640612900
			0.3		-1.650183000	0.625154200
				0.0	-1.736491000	0.276181700
				0.3	-2.182688000	0.741150400
				0.7	-2.743267000	1.193489000
			<u> </u>	1.0	-3.164277000	1.442901000

The impact of Pr on the profile of temperature field in the presence of meltingparameter is displayed in Figure 10. From figure, we deduce that by increasingthe values of Prandtl number, temperature profile increases. This is because the larger values of Prandtl number possess smaller thermal diffusivity and smaller Prandtl number have stronger thermal diffusivity.

The impact of thermophoresis parameter on dimensionless profile of temperature distribution  $\theta(\eta)$  and dimensionless profile of concentration distribution  $\beta(\eta)$  are presented respectively in Figures 11and 12. It is clear, from these figuresprofile of temperature and their associative thickness of thermal boundary layer ofthe thermal field increase with the increasing values of thermophoresis parameter.It is also noticed that for varying values of Nt concentration profile  $\beta(\eta)$  and related thickness of boundary layer decreases.

Figures 13and 14 depict that by increasing Brownian motion parameter, temperature profile and thickness of boundary layer increases slightly whereas concentration profile decreases significantly.

Figures 15and 16indicate the influence of the permeability parameter on the dimensionless profile of velocity distribution  $f'(\eta)$  and dimensionless profile of thetemperature produces a resistive force, that has a tendency to slow down the fluid motion. It is observed that resistance decreases in the fluid motion by increasing values of the permeability parameter. Therefore, it is concluded that velocity profile  $f'(\eta)$  and temperature profile  $\theta(\eta)$  decreases by increasing values of permeability parameter.

From Figures 17and 18, we observe the effect of non-Newtonian Williamson parameter on the velocity and temperature profiles respectively. It is observed that the velocity profile and thickness of boundary layer decrease by increasing values of  $W_e$ . It is also observed that the profile of temperature and thickness of thermal boundary layer decreases.

The effects of chemical reaction coefficient for concentration profile are observed in the Figure 19, which show that by increasing  $\gamma$ , the concentration profile decreases, and it also decreases inthe thickness of concentration.

In Figure 20, it indicates that increasing the value of Eckert number Ec has the enhancing effect on temperature profile and increases the thermal boundary layer thickness in the flow field. The temperature increases due to increasing the Eckert number *Ec* that generates heat in fluid.

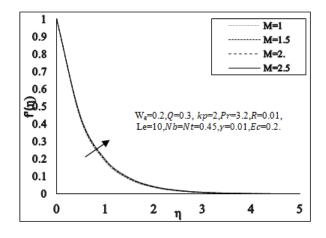


Figure 2: Impact of M on the Velocity Profile





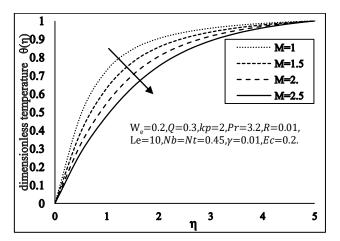


Figure 3: Impact of M on the Temperature Field

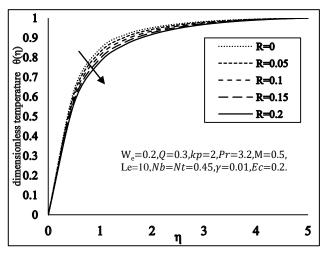


Figure 4: Impact of R on the Dimensionless Temperature

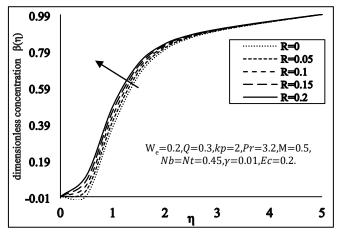


Figure 5: Impact of R on the dimensionless concentration

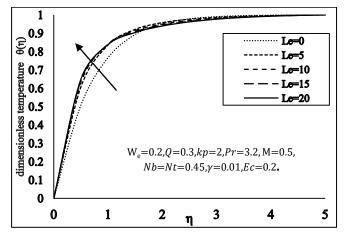


Figure 6: Impact of Leon the dimensionless temperature

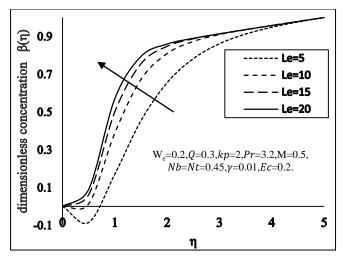


Figure 7: Impact of *Le* on the dimensionless concentration

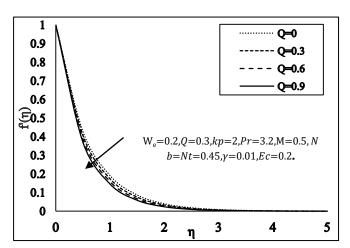
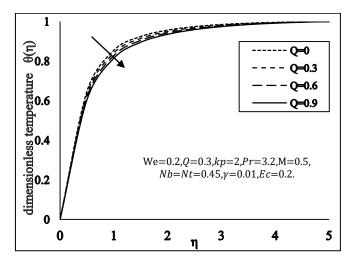


Figure 8: Impact of Q on the Velocity Field







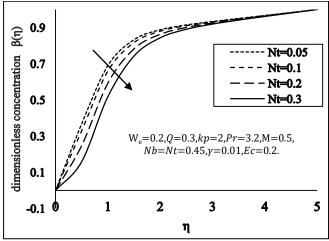


Figure 9: Impact of Q on the Temperature Profile 1  $\theta$ Pr=2.5 temperature 9.0 Pr=3.2 Pr=4.2 Pr=6.2 dimensionless to 0.4  $W_e=0.2, Q=0.3, kp=2, Pr=3.2, M=0.5,$  $Nb=Nt=0.45, \gamma=0.01, Ec=0.2.$ 0 1 2 3 4 5

Figure 12: Impact of *Nt* on the dimensionless concentration

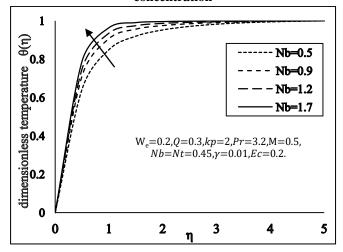


Figure 10: Impact of Pr on the dimensionless temperature

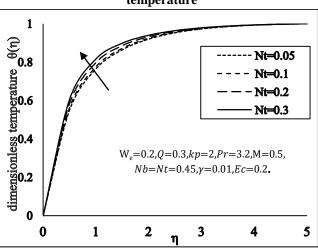


Figure 13: Impact of *Nb*on the dimensionless temperature

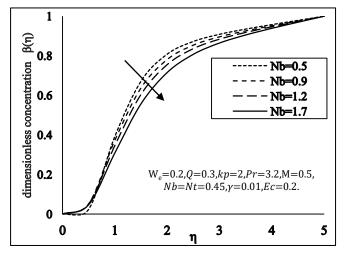


Figure 11: Impact of Nt on the dimensionless temperature

Figure 14: Impact of *Nb* on the dimensionless concentration





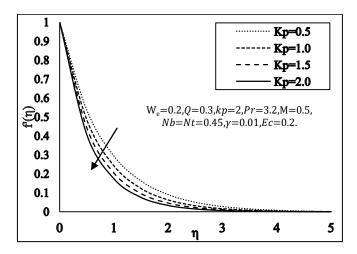


Figure 15: Impact of  $k_p$  on the dimensionless velocity

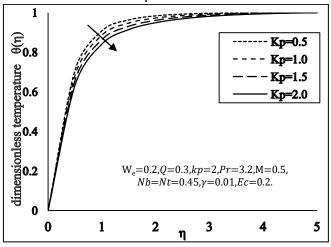


Figure 16: Impact of  $k_p$  on the dimensionless

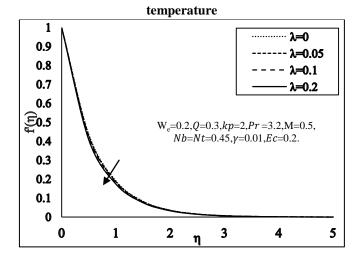


Figure 17: Impact of  $\lambda$ on the dimensionless velocity

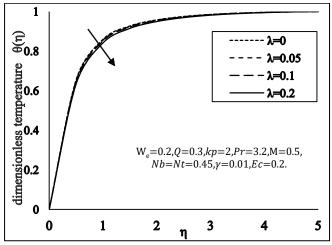


Figure 18: Impact of  $\lambda$ on the dimensionless temperature

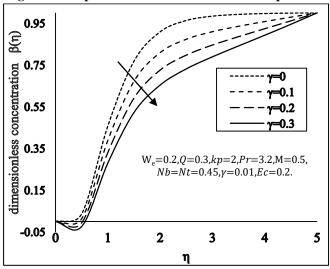


Figure 19: Impact of γon the dimensionless concentration

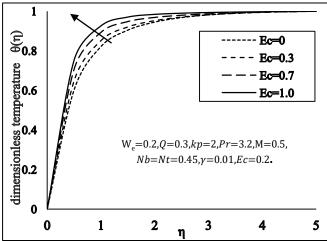


Figure 20: Impact of Econ dimensionless temperature

### VI. CONCLUSION

The concluding remarks and the important findings of the investigation from graphical

representation below:



are



- It is reported that for enhancing values of the radiation parameter and Magnetic parameter, temperature increases whereas the concentration decreases.
- Increases the velocity profile decreases gradually and increasing effects on the temperature.
- Decrement in temperature profile is observed for increasing values of the Prandtl number.
- For increasing the Brownian parameter, the temperature profile increases and the concentration profile.
- Velocity field and Temperature field both are decreases by enlarging permeability parameter  $k_n$ .
- An increase in non-Newtonian Williamson parameter sources decrease in the velocity profile and similar effects on temperature.
- By increasing the thermophoresis parameter Nt, momentum increases, and concentration decreases.

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