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Algebraic Structures of m-polar Fuzzy Matrices

Ramakrishna Mankena, T.V. Pradeep Kumar, Ch. Ramprasad, J. Vijaya Kumar

***Abstract:** Problems from real life situations related to multiple agents $(n \geq 5)$ and Big data are efficiently solved by Computational Mathematics using N-Dimensional Polar information. This information cannot be well-represented by means of fuzzy matrices or bipolar fuzzy matrices. Therefore, m-polar fuzzy matrix theory is applied to graphs to describe the relationships among several individuals. In this paper, some operations are defined to formulate these matrices. we proved the properties of m-polar fuzzy matrices by exploiting the binary operations ring sum ($\oplus$) and ring subtraction ($\ominus$). In addition to this we also extended various operations such as reflexive, irreflexive, maximum and minimum for the idea of m-polar fuzzy matrices.*

***Keywords:** m-polar fuzzy matrix, m-polar fuzzy operators, reflexive, irreflexive*

I. INTRODUCTION

Converting 3D real image to 2D realistic screening motions was made possible by Computer based modeling. Literature related to Mathematical modeling problems using classical matrices is widely available however material on computation fuzzy matrices is limited. Recently fuzzy matrices are used in several applications that deal with the processing of vague non-deterministic and uncertain relational data. This triggered a demand for fuzzy based m-polar fuzzy matrix.

For example, Hashimoto [2], Kim et al. [6] and Kolodziejczyk [7] have studied the canonical form of a transitive, generalized fuzzy matrix and strongly transitive fuzzy matrix respectively. Properties like iterations, min-max composition, and convergence power of a fuzzy matrix and power sequence of different types of fuzzy matrices were studied by Hemasinha et al. [3], Ragab et al. [9], Thomason [11] and Zhou-Tian et al. [17] respectively. Xin [12, 13] understood controllable fuzzy matrices. Kim et al. [5] and Ragab et al. [10], studied the properties of determinant and adjoint of a square fuzzy matrix respectively. Pal [8] defined intuitionistic fuzzy determinant. Khan et al. [4] prefaced intuitionistic fuzzy matrices.

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* Correspondence Author
Ramakrishna Mankena*, Research scholar of Acharya Nagarjuna University and Department of Mathematics, Malla Reddy College of Engineering, Hyderabad, India
T.V. Pradeep Kumar, Department of Mathematics, University College of Engineering, Acharya Nagarjuna University, Nagarjuna Nagar, India
Ch. Ramprasad, Department of Mathematics, Vasireddy Venkatadri Institute of Technology, Namburu, India
J. Vijaya Kumar, Department of Mathematics, Vasireddy Venkatadri Institute of Technology, Namburu, India

Contour set theory was generalized to sets of objects with vague boundaries by Zadeh [14] which spelt the origin of fuzzy set theory. Further, Zhang [15, 16] presented the idea of bipolar fuzzy sets. The idea which lies behind such illustration is associated with bipolar information (i.e., affirming information and negating information) about the given set. In addition to the growth of social networking internet and other technologies have given rise to uncertainty alongside certainty of information. It is in these cases where we have multiple attributes that are uncertain, we need fuzzy sets that can go beyond the domains of bipolar fuzzy sets and the concept of m-polar fuzzy sets has been introduced. M-polar fuzzy sets are theorized by Chen et al. [1] to include relationships among several individual elements of a set. In this set, each element has a value between 0 and 1 indicating the degree of absence or presence of a particular predicate. They extended this concept to show that cryptographic mathematical notions can be concisely obtained for m-polar fuzzy sets. This theory has applications in solving real world problems even when the number of predicates (parameters or measurements) is more than two.

M-polar fuzzy sets can be utilized to investigate agreeable amusements, weighted recreations, and multi-esteemed relations. In basic leadership issues, m-polar fuzzy sets are useful for the multi-criteria choice of articles dependent on the multi-polar data. For instance, m-polar fuzzy sets can be connected when an organization chooses to make a product; a nation chooses its pioneers, a gathering of companions who needs to watch a movie. In remote correspondence, it may be utilized well to talk about the contentions and perplexities of correspondence signals. Accordingly, m-polar fuzzy sets have applications in numerical speculations as well as in genuine issues. Bipolar fuzzy idea cross-section can be utilized to contemplate the two-sided unverifiable conduct of items yet on the off chance that the information has multi-polar data to be managed; the bipolar fuzzy idea grid can't give suitable outcomes. Hence, we require the hypothesis of m-polar fuzzy idea cross-section to deal with information and data having numerous susceptibilities.

In Neurobiology, multi-polar neurons in cerebrum assemble a lot of data from different neurons. In Information Technology, multi-polar technology can be abused to work extensive scale structures. Thinking about realistic structures, m-polar fuzzy sets can be utilized to depict the relationship among a few people. Specifically, m-polar fuzzy sets are to be helpful in an adjustment of exact issues on the off chance that it is important to make judgments with a gathering of understandings.

For example, the exact estimation of media transmission security of individuals is a point which lies in $[0,1]^m$ $(m \approx 7\times 10^9)$ since different individuals are observed on different occasions. A few applications incorporate n-valued logic, assessment, and ordering of options.

In the present work, we introduce the concept of m-polar fuzzy matrix and some binary operations on them including ring sum and ring product. Further, some properties of m-polar fuzzy matrices with respect to the new operations as well as pre-defined operations are presented. These outcomes fortify decisiveness in problematic situations.

II. PRELIMINARIES

Some basic operators on m-polar fuzzy matrices are explained and their notations are introduced below.

Definition 1: Let W be an m-polar fuzzy set on X and $l = \langle l_1, l_2, \cdots, l_m\rangle$, $n = \langle n_1, n_2, \cdots, n_m\rangle$ be two elements of W where $l_1, l_2, \cdots, l_m$ and $n_1, n_2, \cdots, n_m \in [0, 1]$.

Then for any $\alpha \in [0, 1]$, we define

i) Maximum of $\{l, n\} = l \vee n = \langle l_1, l_2, \cdots, l_m\rangle \vee \langle n_1, n_2, \cdots, n_m\rangle = \langle l_1 \vee n_1, l_2 \vee n_2, \cdots, l_m \vee n_m\rangle$

ii) Minimum of $\{l, n\} = l \wedge n = \langle l_1, l_2, \cdots, l_m\rangle \wedge \langle n_1, n_2, \cdots, n_m\rangle = \langle l_1 \wedge n_1, l_2 \wedge n_2, \cdots, l_m \wedge n_m\rangle$

iii) Ring subtraction of $\{l, n\} = l \ominus n = \langle l_1, l_2, \cdots, l_m\rangle \ominus \langle n_1, n_2, \cdots, n_m\rangle$

$$= \begin{cases} \langle l_1, l_2, \cdots, l_m\rangle, & \text{if } \langle l_1, l_2, \cdots, l_m\rangle > \langle n_1, n_2, \cdots, n_m\rangle \\ 0, & \text{if } \langle l_1, l_2, \cdots, l_m\rangle \le \langle n_1, n_2, \cdots, n_m\rangle \end{cases}$$

vi) Upper α - cut of $l = l^{(\alpha)}$

$$= \begin{cases} \langle 1, 1, \cdots, 1\rangle, & \text{if } \langle l_1, l_2, \cdots, l_m\rangle \ge \langle \alpha, \alpha, \cdots, \alpha\rangle \\ \langle 0, 0, \cdots, 0\rangle, & \text{if } \langle l_1, l_2, \cdots, l_m\rangle < \langle \alpha, \alpha, \cdots, \alpha\rangle \end{cases}$$

v) Lower α - cut of $l = l_{(\alpha)}$

$$= \begin{cases} \langle l_1, l_2, \cdots, l_m\rangle, & \text{if } \langle l_1, l_2, \cdots, l_m\rangle \ge \langle \alpha, \alpha, \cdots, \alpha\rangle \\ \langle 0, 0, \cdots, 0\rangle, & \text{if } \langle l_1, l_2, \cdots, l_m\rangle < \langle \alpha, \alpha, \cdots, \alpha\rangle \end{cases}$$

Now, we introduce two more new operators on W, $\oplus$ and $\odot$ as follows:

vi) Ring sum of $\{l, n\} = l \oplus n = \langle l_1, l_2, \cdots, l_m\rangle \oplus \langle n_1, n_2, \cdots, n_m\rangle = \langle l_1 + n_1 - l_1.n_1, l_2 + n_2 - l_2.n_2, \cdots, l_m + n_m - l_m.n_m\rangle$

vii) Ring product of $\{l, n\} = l \odot n = \langle l_1, l_2, \cdots, l_m\rangle \odot \langle n_1, n_2, \cdots, n_m\rangle = \langle l_1.n_1, l_2.n_2, \cdots, l_m.n_m\rangle$,

where the operators '$.$', '$-$' and '$+$' are ordinary multiplication, subtraction and addition on real numbers respectively.

Immediately, we can observe that

i) $\langle 1, 1, \cdots, 1\rangle \oplus \langle l_1, l_2, \cdots, l_m\rangle = \langle 1, 1, \cdots, 1\rangle$,

ii) $\langle 1, 1, \cdots, 1\rangle \odot \langle l_1, l_2, \cdots, l_m\rangle = \langle l_1, l_2, \cdots, l_m\rangle$,

iii) $\langle 0, 0, \cdots, 0\rangle \oplus \langle l_1, l_2, \cdots, l_m\rangle = \langle l_1, l_2, \cdots, l_m\rangle$,

iv) $\langle 0, 0, \cdots, 0\rangle \odot \langle l_1, l_2, \cdots, l_m\rangle = \langle 0, 0, \cdots, 0\rangle$.

Definition 2: [M-polar fuzzy matrix] An m-polar fuzzy matrix $X = \left[\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}}\rangle\right]$ is a matrix on fuzzy algebra. The zero matrix O_r of order $r \times r$ is the matrix where all the elements are $O_m = \langle 0, 0, \cdots, 0\rangle$ and the identity matrix I_r of order $r \times r$ is the matrix where all the diagonal elements are $I_m = \langle 1, 1, \cdots, 1\rangle$ and other entries are $O_m = \langle 0, 0, \cdots, 0\rangle$.

The set of all rectangular m-polar fuzzy matrices of order $r \times k$ is denoted by M_{rk} and that of square matrices of order $r \times r$ is denoted by M_r.

At present, we are confining to some innovative operations on m-polar fuzzy matrices.

Let $W = \left[\langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}}\rangle\right]$ and $F = \left[\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}}\rangle\right]$ be two m-polar fuzzy matrices of order $r \times s$. Then

i) Ring sum of $\{W, F\} = W \oplus F = \left[\langle w_{1_{lk}} + f_{1_{lk}} - w_{1_{lk}}.\, f_{1_{lk}}, w_{2_{lk}} + f_{2_{lk}} - w_{2_{lk}}.\, f_{2_{lk}}, \cdots, w_{m_{lk}} + f_{m_{lk}} - w_{m_{lk}}.\, f_{m_{lk}}\rangle\right]$

ii) Ring product of $\{W, F\} = W \odot F = \left[\langle w_{1_{lk}}.\, f_{1_{lk}}, w_{2_{lk}}.\, f_{2_{lk}}, \cdots, w_{m_{lk}}.\, f_{m_{lk}}\rangle\right]$

iii) Maximum of $\{W, F\} = W \vee F = \left[\langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}}\rangle \vee \langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}}\rangle\right]$

iv) Minimum of $\{W, F\} = W \wedge F = \left[\langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}}\rangle \wedge \langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}}\rangle\right]$

v) Ring subtraction of $\{W, F\} = W \ominus F =$

$\left[\left\langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}} \right\rangle \ominus \left\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \right\rangle\right]$

vi) The transpose m-polar fuzzy matrix of $W = W^T = \left[\left\langle w_{1_{kl}}, w_{2_{kl}}, \cdots, w_{m_{kl}} \right\rangle\right]$

vii) $W \le F$ if and only if $w_{1_{lk}} \le f_{1_{lk}}, w_{2_{lk}} \le f_{2_{lk}}, \cdots, w_{m_{lk}} \le f_{m_{lk}}$ for all l, k.

viii) For any two m-polar fuzzy matrices W and F, $W \wedge F = \min\{W, F\}$.

Notations 3: If W is any m-polar fuzzy matrix, then we denote $W \oplus W$ as $[2]W$ and in general we have $[h+1]W = [h]W \oplus W$. Similarly, $W \odot W = W^{[2]}$ and $W^{[h+1]} = W^{[h]} \odot W$ for all h.

Further, we define some types of m-polar fuzzy matrices.

Definition 4: Let $Q = \left[\left\langle q_{1_{lk}}, q_{2_{lk}}, \cdots, q_{m_{lk}} \right\rangle\right]$ be a matrix of order n. Then we say

a. Q is reflexive if $\left\langle q_{1_{ll}}, q_{2_{ll}}, \cdots, q_{m_{ll}} \right\rangle = \langle 1, 1, \cdots, 1\rangle$ for all $l = 1, 2, \cdots, n$.

b. Q is irreflexive if $\left\langle q_{1_{ll}}, q_{2_{ll}}, \cdots, q_{m_{ll}} \right\rangle = \langle 0, 0, \cdots, 0\rangle$ for all $l = 1, 2, \cdots, n$.

c. Q is nearly irreflexive if $\left\langle q_{1_{ll}}, q_{2_{ll}}, \cdots, q_{m_{ll}} \right\rangle \le \left\langle q_{1_{lk}}, q_{2_{lk}}, \cdots, q_{m_{lk}} \right\rangle$ for all $l, k = 1, 2, \cdots, n$.

d. Q is symmetric if $Q^T = Q$.

e. Q is constant if $\left\langle q_{1_{lk}}, q_{2_{lk}}, \cdots, q_{m_{lk}} \right\rangle = \left\langle q_{1_{ik}}, q_{2_{ik}}, \cdots, q_{m_{ik}} \right\rangle$ for all $l, k, i = 1, 2, \cdots, n$.

f. Q is identity if $\left\langle q_{1_{ll}}, q_{2_{ll}}, \cdots, q_{m_{ll}} \right\rangle = \langle 1, 1, \cdots, 1\rangle$ and $\left\langle q_{1_{lk}}, q_{2_{lk}}, \cdots, q_{m_{lk}} \right\rangle = \langle 0, 0, \cdots, 0\rangle$ $(l \ne k)$ for all l, k.

g. Q is weakly reflexive if $\left\langle q_{1_{ll}}, q_{2_{ll}}, \cdots, q_{m_{ll}} \right\rangle \ge \left\langle q_{1_{lk}}, q_{2_{lk}}, \cdots, q_{m_{lk}} \right\rangle$ for all l, k.

h. Q is diagonal if $\left\langle q_{1_{ll}}, q_{2_{ll}}, \cdots, q_{m_{ll}} \right\rangle \ge \langle 0, 0, \cdots, 0\rangle$ and $\left\langle q_{1_{lk}}, q_{2_{lk}}, \cdots, q_{m_{lk}} \right\rangle = \langle 0, 0, \cdots, 0\rangle$ $(l \ne k)$ for all l, k.

Notations 5: If all the elements of a matrix are $\langle 0, 0, \cdots, 0\rangle$ then we denote it by O and all the elements of a matrix are $\langle 1, 1, \cdots, 1\rangle$ then we denote it by U. Generally, the identity matrix of order $m \times m$ is denoted by I_m.

III. THEORETICAL RESULTS ON M-POLAR FUZZY MATRICES

From now onwards, we use the notation $X = [x_{lk}], Y = [y_{lk}], Z = [z_{lk}]$ and $T = [t_{lk}]$ to denote m-polar fuzzy matrices.

In the following properties, we have given detailed properties of the m-polar fuzzy matrices.

Property 6: *Let X be an m-polar fuzzy matrix of order $r \times r$. Then*

a. $I_r \oplus \left(X \oplus X^T\right)$ *is reflexive and symmetric,*

b. $X \ominus I_r$ *is irreflexive,*

c. $\left(X \oplus X^T\right)$ *is nearly irreflexive and symmetric,*

d. $I_r \oplus \left(X \oplus X^T\right) = I_r \vee \left(X \oplus X^T\right)$.

Proof: *a.* $X \oplus X^T = \left[\left\langle x_{1_{lk}} + x_{1_{kl}} - x_{1_{lk}}.x_{1_{kl}}, x_{2_{lk}} + x_{2_{kl}} - x_{2_{lk}}.x_{2_{kl}}, \cdots, x_{m_{lk}} + x_{m_{kl}} - x_{m_{lk}}.x_{m_{kl}} \right\rangle\right]$ and $I_r \oplus \left(X \oplus X^T\right) = \left[\left\langle n_{1_{lk}}, n_{2_{lk}}, \cdots, n_{m_{lk}} \right\rangle\right]$, where $\left\langle n_{1_{ll}}, n_{2_{ll}}, \cdots, n_{m_{ll}} \right\rangle = \langle 1, 1, \cdots, 1\rangle$ and $\left\langle n_{1_{lk}}, n_{2_{lk}}, \cdots, n_{m_{lk}} \right\rangle = \left\langle x_{1_{lk}} + x_{1_{kl}} - x_{1_{lk}}.x_{1_{kl}}, x_{2_{lk}} + x_{2_{kl}} - x_{2_{lk}}.x_{2_{kl}}, \cdots, x_{m_{lk}} + x_{m_{kl}} - x_{m_{lk}}.x_{m_{kl}} \right\rangle$, for $l \ne k$.

Now, $\left\langle n_{1_{kl}}, n_{2_{kl}}, \cdots, n_{m_{kl}} \right\rangle = \left\langle x_{1_{kl}} + x_{1_{lk}} - x_{1_{kl}}.x_{1_{lk}}, x_{2_{kl}} + x_{2_{lk}} - x_{2_{kl}}.x_{2_{lk}}, \cdots, x_{m_{kl}} + x_{m_{lk}} - x_{m_{kl}}.x_{m_{lk}} \right\rangle = \left\langle n_{1_{lk}}, n_{2_{lk}}, \cdots, n_{m_{lk}} \right\rangle$. That is, every diagonal element of $I_r \oplus \left(X \oplus X^T\right)$ is $\langle 1, 1, \cdots, 1\rangle$ and all non-diagonal elements are $\left\langle x_{1_{lk}} + x_{1_{kl}} - x_{1_{lk}}.x_{1_{kl}}, x_{2_{lk}} + x_{2_{kl}} - x_{2_{lk}}.x_{2_{kl}}, \cdots, x_{m_{lk}} + x_{m_{kl}} - x_{m_{lk}}.x_{m_{kl}} \right\rangle$. Therefore, $I_r \oplus \left(X \oplus X^T\right)$ is reflexive and also symmetric.

b. The diagonal elements of $X \ominus I_r$ are $\langle 0, 0, \cdots, 0\rangle$ because $\left\langle x_{1_{ll}}, x_{2_{ll}}, \cdots, x_{m_{ll}} \right\rangle \le \langle 1, 1, \cdots, 1\rangle$. Hence, $X \ominus I_r$ is irreflexive.

c. Let $S = \left(X \oplus X^T\right)$, i.e., $\left\langle n_{1_{lk}}, n_{2_{lk}}, \cdots, n_{m_{lk}} \right\rangle = \left\langle x_{1_{lk}} + x_{1_{kl}} - x_{1_{lk}}.x_{1_{kl}}, x_{2_{lk}} + x_{2_{kl}} - x_{2_{lk}}.x_{2_{kl}}, \cdots, x_{m_{lk}} + x_{m_{kl}} - x_{m_{lk}}.x_{m_{kl}} \right\rangle = \left\langle n_{1_{kl}}, n_{2_{kl}}, \cdots, n_{m_{kl}} \right\rangle$.

Therefore, S is symmetric. Again, $\left\langle n_{1_{ll}}, n_{2_{ll}}, \cdots, n_{m_{ll}} \right\rangle = \left\langle 2x_{1_{ll}} - x_{1_{ll}}^2, 2x_{2_{ll}} - x_{2_{ll}}^2, \cdots, 2x_{m_{ll}} - x_{m_{ll}}^2 \right\rangle$. Since X is nearly irreflexive, $\left\langle x_{1_{ll}}, x_{2_{ll}}, \cdots, x_{m_{ll}} \right\rangle \le \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle$.

Therefore, $\langle 1-x_{1_{ll}}, 1-x_{2_{ll}}, \cdots, 1-x_{m_{ll}}\rangle \geq \langle 1-x_{1_{lk}}, 1-x_{2_{lk}}, \cdots, 1-x_{m_{lk}}\rangle$.

Now, $\langle n_{1_{lk}}, n_{2_{lk}}, \cdots, n_{m_{lk}}\rangle - \langle n_{1_{ll}}, n_{2_{ll}}, \cdots, n_{m_{ll}}\rangle =$

$\{\langle 1-(1-x_{1_{lk}}).(1-x_{1_{kl}}), 1-(1-x_{2_{lk}}).(1-x_{2_{kl}}), \cdots, 1-(1-x_{m_{lk}}).(1-x_{m_{kl}})\rangle\} - \{\langle 1-(1-x_{1_{ll}}).(1-x_{1_{ll}}), 1-(1-x_{2_{ll}}).(1-x_{2_{ll}}), \cdots, 1-(1-x_{m_{ll}}).(1-x_{m_{ll}})\rangle\}$

$= \langle (1-x_{1_{ll}}).(1-x_{1_{ll}}) - (1-x_{1_{lk}}).(1-x_{1_{kl}}), (1-x_{2_{ll}}).(1-x_{2_{ll}}) - (1-x_{2_{lk}}).(1-x_{2_{kl}}), \cdots, (1-x_{m_{ll}}).(1-x_{m_{ll}}) - (1-x_{m_{lk}}).(1-x_{m_{kl}})\rangle$

$\geq \langle 0, 0, \cdots, 0\rangle$.

Therefore, $(X \oplus X^T)$ is nearly irreflexive and symmetric.

d. Since $\langle 1\vee(x_{1_{lk}}+x_{1_{kl}}-x_{1_{lk}}.x_{1_{kl}}), 1\vee(x_{2_{lk}}+x_{2_{kl}}-x_{2_{lk}}.x_{2_{kl}}), \cdots, 1\vee(x_{m_{lk}}+x_{m_{kl}}-x_{m_{lk}}.x_{m_{kl}})\rangle = \langle 1, 1, \cdots, 1\rangle$ and

$\langle 0\vee(x_{1_{lk}}+x_{1_{kl}}-x_{1_{lk}}.x_{1_{kl}}), 0\vee(x_{2_{lk}}+x_{2_{kl}}-x_{2_{lk}}.x_{2_{kl}}), \cdots, 0\vee(x_{m_{lk}}+x_{m_{kl}}-x_{m_{lk}}.x_{m_{kl}})\rangle = \langle x_{1_{lk}}+x_{1_{kl}}-x_{1_{lk}}.x_{1_{kl}}, x_{2_{lk}}+x_{2_{kl}}-x_{2_{lk}}.x_{2_{kl}}, \cdots, x_{m_{lk}}+x_{m_{kl}}-x_{m_{lk}}.x_{m_{kl}}\rangle$,

$I_r \vee (X \oplus X^T) = [\langle n_{1_{lk}}, n_{2_{lk}}, \cdots, n_{m_{lk}}\rangle]$ where $\langle n_{1_{ll}}, n_{2_{ll}}, \cdots, n_{m_{ll}}\rangle = \langle 1, 1, \cdots, 1\rangle$ and

$\langle n_{1_{lk}}, n_{2_{lk}}, \cdots, n_{m_{lk}}\rangle = \langle x_{1_{lk}}+x_{1_{kl}}-x_{1_{lk}}.x_{1_{kl}}, x_{2_{lk}}+x_{2_{kl}}-x_{2_{lk}}.x_{2_{kl}}, \cdots, x_{m_{lk}}+x_{m_{kl}}-x_{m_{lk}}.x_{m_{kl}}\rangle$, for $l \neq k$. Again $I_r \oplus (X \oplus X^T) = [\langle n_{1_{lk}}, n_{2_{lk}}, \cdots, n_{m_{lk}}\rangle]$,

where $\langle n_{1_{ll}}, n_{2_{ll}}, \cdots, n_{m_{ll}}\rangle = \langle 1, 1, \cdots, 1\rangle$ and

$\langle n_{1_{lk}}, n_{2_{lk}}, \cdots, n_{m_{lk}}\rangle = \langle x_{1_{lk}}+x_{1_{kl}}-x_{1_{lk}}.x_{1_{kl}}, x_{2_{lk}}+x_{2_{kl}}-x_{2_{lk}}.x_{2_{kl}}, \cdots, x_{m_{lk}}+x_{m_{kl}}-x_{m_{lk}}.x_{m_{kl}}\rangle$, $l \neq k$.

Hence the result hold.

Next, we get an inequality involving $\oplus$, $\ominus$ and $\vee$. □

Property 7: *Let X and Y be any two m-polar fuzzy matrices. Then $X \oplus Y \geq X \vee Y \geq X \ominus Y$.*

Proof: Let $\langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}}\rangle$, $\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}}\rangle$ and $\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}}\rangle$ be the lk th element of the matrices $X \oplus Y$, $X \vee Y$ and $X \ominus Y$ respectively.

Now $\langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}}\rangle = \langle x_{1_{lk}}+y_{1_{lk}}-x_{1_{lk}}.y_{1_{lk}}, x_{2_{lk}}+y_{2_{lk}}-x_{2_{lk}}.y_{2_{lk}}, \cdots, x_{m_{lk}}+y_{m_{lk}}-x_{m_{lk}}.y_{m_{lk}}\rangle$

$$= \begin{cases} \langle x_{1_{lk}}+y_{1_{lk}}.(1-x_{1_{lk}}), x_{2_{lk}}+y_{2_{lk}}.(1-x_{2_{lk}}), \cdots, x_{m_{lk}}+y_{m_{lk}}.(1-x_{m_{lk}})\rangle \geq \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}}\rangle \\ \langle y_{1_{lk}}+x_{1_{lk}}.(1-y_{1_{lk}}), y_{2_{lk}}+x_{2_{lk}}.(1-y_{2_{lk}}), \cdots, y_{m_{lk}}+x_{m_{lk}}.(1-y_{m_{lk}})\rangle \geq \langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}}\rangle \end{cases}$$

$\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}}\rangle = \{\langle x_{1_{lk}}\vee y_{1_{lk}}, x_{2_{lk}}\vee y_{2_{lk}}, \cdots, x_{m_{lk}}\vee y_{m_{lk}}\rangle\}$

$\leq \langle x_{1_{lk}}+y_{1_{lk}}-x_{1_{lk}}.y_{1_{lk}}, x_{2_{lk}}+y_{2_{lk}}-x_{2_{lk}}.y_{2_{lk}}, \cdots, x_{m_{lk}}+y_{m_{lk}}-x_{m_{lk}}.y_{m_{lk}}\rangle = \langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}}\rangle$.

Thus, $\langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}}\rangle \geq \langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}}\rangle$ for all l, k. Hence, $X \oplus Y \geq X \vee Y$.

Again, $\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}}\rangle = \langle x_{1_{lk}}\ominus y_{1_{lk}}, x_{2_{lk}}\ominus y_{2_{lk}}, \cdots, x_{m_{lk}}\ominus y_{m_{lk}}\rangle$

$$= \begin{cases} \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}}\rangle, & \text{if } \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}}\rangle > \langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}}\rangle \\ \langle 0, 0, \cdots, 0\rangle, & \text{if } \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}}\rangle \leq \langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}}\rangle. \end{cases}$$

That is, $\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}}\rangle \leq \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}}\rangle \leq \langle x_{1_{lk}}\vee y_{1_{lk}}, x_{2_{lk}}\vee y_{2_{lk}}, \cdots, x_{m_{lk}}\vee y_{m_{lk}}\rangle$ for all l, k.

Thus, $X \ominus Y \leq X \vee Y$.

Finally, $X \oplus Y \geq X \vee Y \geq X \ominus Y$. □

Property 8: *Let X and Y be any two m-polar fuzzy matrices. Then*

a. $(X \vee Y)\vee(X \ominus Y) = (X \vee Y)$,

b. $(X \vee Y)\ominus(X \ominus Y) \leq Y$,

c. $X \oplus Y \geq (X \vee Y)\vee(X \ominus Y)$,

d. $X \oplus Y \geq (X \vee Y)\ominus(X \ominus Y)$.

Proof: *a.* Let $\langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}}\rangle$, $\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}}\rangle$ and $\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}}\rangle$ be the lk th elements of $X \vee Y$, $X \ominus Y$ and $(X \vee Y)\vee(X \ominus Y)$ respectively.

Then $\left\langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}} \right\rangle =$
$\left\langle x_{1_{lk}} \vee y_{1_{lk}}, x_{2_{lk}} \vee y_{2_{lk}}, \cdots, x_{m_{lk}} \vee y_{m_{lk}} \right\rangle$,

$$\left\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \right\rangle = \begin{cases} \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle, & \text{if } \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle > \left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle \\ \left\langle 0, 0, \cdots, 0 \right\rangle, & \text{if } \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle \le \left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle. \end{cases}$$

$$\left\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}} \right\rangle =$$

$$= \begin{cases} \max\left\{ \left\langle x_{1_{lk}} \vee y_{1_{lk}}, x_{2_{lk}} \vee y_{2_{lk}}, \cdots, x_{m_{lk}} \vee y_{m_{lk}} \right\rangle, \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle \right\}, \text{if } \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle > \left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle \\ \max\left\{ \left\langle x_{1_{lk}} \vee y_{1_{lk}}, x_{2_{lk}} \vee y_{2_{lk}}, \cdots, x_{m_{lk}} \vee y_{m_{lk}} \right\rangle, \left\langle 0, 0, \cdots, 0 \right\rangle \right\}, \text{if } \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle \le \left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle \end{cases}$$

$$= \begin{cases} \left\{ \left\langle x_{1_{lk}} \vee x_{1_{lk}}, x_{2_{lk}} \vee x_{2_{lk}}, \cdots, x_{m_{lk}} \vee x_{m_{lk}} \right\rangle \right\}, \\ \text{if } \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle > \left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle \\ \left\{ \left\langle y_{1_{lk}} \vee 0, y_{2_{lk}} \vee 0, \cdots, y_{m_{lk}} \vee 0 \right\rangle \right\}, \\ \text{if } \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle \le \left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle \end{cases}$$

$$= \begin{cases} \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle, \\ \text{if } \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle > \left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle \\ \left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle, \\ \text{if } \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle \le \left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle. \end{cases}$$

That is, lk th element $\left\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}} \right\rangle$ of $(X \vee Y) \vee (X \ominus Y)$ is either $\left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle$ or $\left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle$ according as $\left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle >$ or $\le \left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle$.

Also, the lk th element of $\left\langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}} \right\rangle$ is either $\left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle$ or $\left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle$ according as $\left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle >$ or $\le$ $\left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle$.

Therefore, $\left\langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}} \right\rangle = \left\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}} \right\rangle$ for all l, k.

Hence, $(X \vee Y) \vee (X \ominus Y) = (X \vee Y)$.

b. Let $\left\langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \right\rangle$ be the lk th element of $(X \vee Y) \ominus (X \ominus Y)$. Then lk th element $\left\langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}} \right\rangle$ of $X \vee Y$ is $\left\langle w_{1_{lk}}, w_{2_{lk}}, \cdots, w_{m_{lk}} \right\rangle =$ $\left\langle x_{1_{lk}} \vee y_{1_{lk}}, x_{2_{lk}} \vee y_{2_{lk}}, \cdots, x_{m_{lk}} \vee y_{m_{lk}} \right\rangle$

$$= \begin{cases} \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle, \\ \text{if } \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle > \left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle \\ \left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle, \\ \text{if } \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle \le \left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle, \end{cases}$$

and the lk th element of $X \ominus Y$ is

$$\left\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \right\rangle = \begin{cases} \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle, \\ \text{if } \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle > \left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle \\ \left\langle 0, 0, \cdots, 0 \right\rangle, \\ \text{if } \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle \le \left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle. \end{cases}$$

Therefore,

$$\left\langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \right\rangle = \begin{cases} \left\langle 0, 0, \cdots, 0 \right\rangle, \\ \text{if } \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle > \left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle \\ \left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle, \\ \text{if } \left\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \right\rangle \le \left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle. \end{cases}$$

That is, the elements of $(X \vee Y) \ominus (X \ominus Y)$ are either $\left\langle 0, 0, \cdots, 0 \right\rangle$ or $\left\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \right\rangle$. Hence, $(X \vee Y) \ominus (X \ominus Y) \le Y$.

c. It is obvious that $(X \vee Y) \vee (X \ominus Y) = X \vee Y \le X \oplus Y$.

d. It is obvious that $Y \le X \vee Y$. Hence $(X \vee Y) \ominus (X \ominus Y) \le Y \le X \vee Y \le X \oplus Y$. □

Property 9: *Let X, Y and Z be any three m-polar fuzzy matrices. Then*

a. $X \oplus (Y \vee Z) = (X \oplus Y) \vee (X \oplus Z)$,

b. $X \oplus (Y \ominus Z) \ge (X \oplus Y) \ominus (X \oplus Z)$,

c. $X \ominus (Y \oplus Z) \le (X \ominus Y) \oplus (X \ominus Z)$,

d. $X \ominus (Y \vee Z) \le (X \ominus Y) \vee (X \ominus Z)$'

e. $X \vee (Y \oplus Z) \leq (X \vee Y) \oplus (X \vee Z)$,

f. $X \vee (Y \ominus Z) \geq (X \vee Y) \ominus (X \vee Z)$.

Proof: *a.* Let $\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \rangle$, $\langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \rangle$, $\langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \rangle$, $\langle i_{1_{lk}}, i_{2_{lk}}, \cdots, i_{m_{lk}} \rangle$ and $\langle j_{1_{lk}}, j_{2_{lk}}, \cdots, j_{m_{lk}} \rangle$ be the lk th elements of $Y \vee Z,\ X \oplus Y,\ X \oplus Z,\ X \oplus (Y \vee Z)$ and $(X \oplus Y) \vee (X \oplus Z)$ respectively. Then

$$\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \rangle = \langle y_{1_{lk}} \vee z_{1_{lk}}, y_{2_{lk}} \vee z_{2_{lk}}, \cdots, y_{m_{lk}} \vee z_{m_{lk}} \rangle,$$

$$\langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \rangle = \langle x_{1_{lk}} + y_{1_{lk}} - x_{1_{lk}}.\, y_{1_{lk}}, x_{2_{lk}} + y_{2_{lk}} - x_{2_{lk}}.\, y_{2_{lk}}, \cdots, x_{m_{lk}} + y_{m_{lk}} - x_{m_{lk}}.\, y_{m_{lk}} \rangle,$$

$$\langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \rangle = \langle x_{1_{lk}} + z_{1_{lk}} - x_{1_{lk}}.\, z_{1_{lk}}, x_{2_{lk}} + z_{2_{lk}} - x_{2_{lk}}.\, z_{2_{lk}}, \cdots, x_{m_{lk}} + z_{m_{lk}} - x_{m_{lk}}.\, z_{m_{lk}} \rangle,$$

$$\langle i_{1_{lk}}, i_{2_{lk}}, \cdots, i_{m_{lk}} \rangle = \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle \oplus \langle y_{1_{lk}} \vee z_{1_{lk}}, y_{2_{lk}} \vee z_{2_{lk}}, \cdots, y_{m_{lk}} \vee z_{m_{lk}} \rangle =$$

$$\begin{cases} \langle x_{1_{lk}} + y_{1_{lk}} - x_{1_{lk}}.\, y_{1_{lk}}, x_{2_{lk}} + y_{2_{lk}} - x_{2_{lk}}.\, y_{2_{lk}}, \cdots, x_{m_{lk}} + y_{m_{lk}} - x_{m_{lk}}.\, y_{m_{lk}} \rangle, \text{ if } \langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle > \langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle \\ \langle x_{1_{lk}} + z_{1_{lk}} - x_{1_{lk}}.\, z_{1_{lk}}, x_{2_{lk}} + z_{2_{lk}} - x_{2_{lk}}.\, z_{2_{lk}}, \cdots, x_{m_{lk}} + z_{m_{lk}} - x_{m_{lk}}.\, z_{m_{lk}} \rangle, \text{ if } \langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle \leq \langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle. \end{cases}$$

$$\langle j_{1_{lk}}, j_{2_{lk}}, \cdots, j_{m_{lk}} \rangle = \langle g_{1_{lk}} \vee h_{1_{lk}}, g_{2_{lk}} \vee h_{2_{lk}}, \cdots, g_{m_{lk}} \vee h_{m_{lk}} \rangle$$

$$= \begin{cases} \langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \rangle, \text{ if } \langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \rangle > \langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \rangle \\ \langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \rangle, \text{ if } \langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \rangle \leq \langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \rangle. \end{cases}$$

If $\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle > \langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle$, then

$$\langle y_{1_{lk}}.(1-x_{1_{lk}}), y_{2_{lk}}.(1-x_{2_{lk}}), \cdots, y_{m_{lk}}.(1-x_{m_{lk}}) \rangle > \langle z_{1_{lk}}.(1-x_{1_{lk}}), z_{2_{lk}}.(1-x_{2_{lk}}), \cdots, z_{m_{lk}}.(1-x_{m_{lk}}) \rangle.$$

i.e., $\langle x_{1_{lk}} + y_{1_{lk}} - x_{1_{lk}}.\, y_{1_{lk}}, x_{2_{lk}} + y_{2_{lk}} - x_{2_{lk}}.\, y_{2_{lk}}, \cdots, x_{m_{lk}} + y_{m_{lk}} - x_{m_{lk}}.\, y_{m_{lk}} \rangle > \langle x_{1_{lk}} + z_{1_{lk}} - x_{1_{lk}}.\, z_{1_{lk}}, x_{2_{lk}} + z_{2_{lk}} - x_{2_{lk}}.\, z_{2_{lk}}, \cdots, x_{m_{lk}} + z_{m_{lk}} - x_{m_{lk}}.\, z_{m_{lk}} \rangle$

or, $\langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \rangle > \langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \rangle$.

Again, if $\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle \leq \langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle$, then $\langle x_{1_{lk}} + y_{1_{lk}} - x_{1_{lk}}.\, y_{1_{lk}}, x_{2_{lk}} + y_{2_{lk}} - x_{2_{lk}}.\, y_{2_{lk}}, \cdots, x_{m_{lk}} + y_{m_{lk}} - x_{m_{lk}}.\, y_{m_{lk}} \rangle \leq \langle x_{1_{lk}} + z_{1_{lk}} - x_{1_{lk}}.\, z_{1_{lk}}, x_{2_{lk}} + z_{2_{lk}} - x_{2_{lk}}.\, z_{2_{lk}}, \cdots, x_{m_{lk}} + z_{m_{lk}} - x_{m_{lk}}.\, z_{m_{lk}} \rangle$

or, $\langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \rangle \leq \langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \rangle$.

That is, $\langle j_{1_{lk}}, j_{2_{lk}}, \cdots, j_{m_{lk}} \rangle =$

$$\begin{cases} \langle x_{1_{lk}} + y_{1_{lk}} - x_{1_{lk}}.\, y_{1_{lk}}, x_{2_{lk}} + y_{2_{lk}} - x_{2_{lk}}.\, y_{2_{lk}}, \cdots, x_{m_{lk}} + y_{m_{lk}} - x_{m_{lk}}.\, y_{m_{lk}} \rangle, \\ \text{if } \langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle > \langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle \\ \langle x_{1_{lk}} + z_{1_{lk}} - x_{1_{lk}}.\, z_{1_{lk}}, x_{2_{lk}} + z_{2_{lk}} - x_{2_{lk}}.\, z_{2_{lk}}, \cdots, x_{m_{lk}} + z_{m_{lk}} - x_{m_{lk}}.\, z_{m_{lk}} \rangle, \\ \text{if } \langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle \leq \langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle. \end{cases}$$

Therefore, $\langle i_{1_{lk}}, i_{2_{lk}}, \cdots, i_{m_{lk}} \rangle = \langle j_{1_{lk}}, j_{2_{lk}}, \cdots, j_{m_{lk}} \rangle$ for all l, k.

Hence, $X \oplus (Y \vee Z) = (X \oplus Y) \vee (X \oplus Z)$.

b. Let $\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \rangle$, $\langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \rangle$, $\langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \rangle$, $\langle s_{1_{lk}}, s_{2_{lk}}, \cdots, s_{m_{lk}} \rangle$ and $\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}} \rangle$ be the lk th elements of $Y \ominus Z,\ X \oplus Y,\ X \oplus Z,\ X \oplus (Y \ominus Z)$ and $(X \oplus Y) \ominus (X \oplus Z)$ respectively. Then

$$\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \rangle = \begin{cases} \langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle, \\ \text{if } \langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle > \langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle \\ \langle 0, 0, \cdots, 0 \rangle, \\ \text{if } \langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle \leq \langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle. \end{cases}$$

$\langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \rangle = \langle x_{1_{lk}} + y_{1_{lk}} - x_{1_{lk}}.\, y_{1_{lk}},$

$x_{2_{lk}} + y_{2_{lk}} - x_{2_{lk}} . y_{2_{lk}}, \cdots, x_{m_{lk}} + y_{m_{lk}} - x_{m_{lk}} . y_{m_{lk}} \rangle$,

$\langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \rangle = \langle x_{1_{lk}} + z_{1_{lk}} - x_{1_{lk}} . z_{1_{lk}}, x_{2_{lk}} + z_{2_{lk}} - x_{2_{lk}} . z_{2_{lk}}, \cdots, x_{m_{lk}} + z_{m_{lk}} - x_{m_{lk}} . z_{m_{lk}} \rangle$,

$\langle s_{1_{lk}}, s_{2_{lk}}, \cdots, s_{m_{lk}} \rangle =$
$\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle \oplus \langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \rangle$

$$= \begin{cases} \langle x_{1_{lk}} + y_{1_{lk}} - x_{1_{lk}} . y_{1_{lk}}, x_{2_{lk}} + y_{2_{lk}} - x_{2_{lk}} . y_{2_{lk}}, \cdots, \\ x_{m_{lk}} + y_{m_{lk}} - x_{m_{lk}} . y_{m_{lk}} \rangle, \text{ if } \langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle > \\ \langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle \\ \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle, \text{ if } \langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle \leq \\ \quad \langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle. \end{cases}$$

$$\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}} \rangle = g_{lk} \ominus h_{lk} =$$
$$\begin{cases} \langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \rangle, \\ \text{if } \langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \rangle > \langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \rangle \\ \langle 0, 0, \cdots, 0 \rangle, \\ \text{if } \langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \rangle \leq \langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \rangle. \end{cases}$$

If $\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle > \langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle$, then

$\langle y_{1_{lk}}.(1 - x_{1_{lk}}), y_{2_{lk}}.(1 - x_{2_{lk}}), \cdots, y_{m_{lk}}.(1 - x_{m_{lk}}) \rangle$
$> \langle z_{1_{lk}}.(1 - x_{1_{lk}}), z_{2_{lk}}.(1 - x_{2_{lk}}), \cdots, z_{m_{lk}}.(1 - x_{m_{lk}}) \rangle$

as $\langle 0, 0, \cdots, 0 \rangle \leq \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle \leq \langle 1, 1, \cdots, 1 \rangle$.

i.e., $\langle x_{1_{lk}} + y_{1_{lk}} - x_{1_{lk}} . y_{1_{lk}}, x_{2_{lk}} + y_{2_{lk}} - x_{2_{lk}} . y_{2_{lk}}, \cdots, x_{m_{lk}} + y_{m_{lk}} - x_{m_{lk}} . y_{m_{lk}} \rangle \geq \langle x_{1_{lk}} + z_{1_{lk}} - x_{1_{lk}} . z_{1_{lk}}, x_{2_{lk}} + z_{2_{lk}} - x_{2_{lk}} . z_{2_{lk}}, \cdots, x_{m_{lk}} + z_{m_{lk}} - x_{m_{lk}} . z_{m_{lk}} \rangle$,

i.e., $\langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \rangle > \langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \rangle$.

But, when $\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle \leq \langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle$,

we have $\langle x_{1_{lk}} + y_{1_{lk}} - x_{1_{lk}} . y_{1_{lk}}, x_{2_{lk}} + y_{2_{lk}} - x_{2_{lk}} . y_{2_{lk}}, \cdots, x_{m_{lk}} + y_{m_{lk}} - x_{m_{lk}} . y_{m_{lk}} \rangle \leq \langle x_{1_{lk}} + z_{1_{lk}} - x_{1_{lk}} . z_{1_{lk}}, x_{2_{lk}} + z_{2_{lk}} - x_{2_{lk}} . z_{2_{lk}}, \cdots, x_{m_{lk}} + z_{m_{lk}} - x_{m_{lk}} . z_{m_{lk}} \rangle$.

i.e., $\langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \rangle \leq \langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \rangle$.

Thus, $\langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \rangle > \langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \rangle$ or $\langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \rangle \leq \langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \rangle$ according as $\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle > \langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle$ or $\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle \leq \langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle$.

Therefore, $\langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}} \rangle =$

$$\begin{cases} \langle x_{1_{lk}} + y_{1_{lk}} - x_{1_{lk}} . y_{1_{lk}}, x_{2_{lk}} + y_{2_{lk}} - x_{2_{lk}} . y_{2_{lk}}, \cdots, \\ x_{m_{lk}} + y_{m_{lk}} - x_{m_{lk}} . y_{m_{lk}} \rangle \text{if } \langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle > \\ \langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle \\ \langle 0, 0, \cdots, 0 \rangle, \text{ if } \langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle \leq \\ \langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle. \end{cases}$$

Hence, $\langle s_{1_{lk}}, s_{2_{lk}}, \cdots, s_{m_{lk}} \rangle \geq \langle t_{1_{lk}}, t_{2_{lk}}, \cdots, t_{m_{lk}} \rangle$, whatever may be the values of $\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle$, $\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle$ and $\langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle$, for all l, k.

Thus $X \oplus (Y \ominus Z) \geq (X \oplus Y) \ominus (X \oplus Z)$.

c. Let $\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \rangle$, $\langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \rangle$, $\langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \rangle$, $\langle i_{1_{lk}}, i_{2_{lk}}, \cdots, i_{m_{lk}} \rangle$ and $\langle j_{1_{lk}}, j_{2_{lk}}, \cdots, j_{m_{lk}} \rangle$ be the lk th elements of $Y \oplus Z$, $X \ominus Y$, $X \ominus Z$, $X \ominus (Y \oplus Z)$ and $(X \ominus Y) \oplus (X \ominus Z)$ respectively. Then

$\langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \rangle = \langle y_{1_{lk}} + z_{1_{lk}} - y_{1_{lk}} . z_{1_{lk}}, y_{2_{lk}} + z_{2_{lk}} - y_{2_{lk}} . z_{2_{lk}}, \cdots, y_{m_{lk}} + z_{m_{lk}} - y_{m_{lk}} . z_{m_{lk}} \rangle$,

$$\langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \rangle =$$
$$\begin{cases} \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle, \text{ if } \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle > \\ \quad \langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle \\ \langle 0, 0, \cdots, 0 \rangle, \qquad \text{if } \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle \leq \\ \langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle. \end{cases}$$

$$\langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \rangle$$
$$= \begin{cases} \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle, \text{ if } \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle > \\ \quad \langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle \\ \langle 0, 0, \cdots, 0 \rangle, \qquad \text{if } \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle \leq \\ \langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle. \end{cases}$$

$\langle i_{1_{lk}}, i_{2_{lk}}, \cdots, i_{m_{lk}} \rangle =$
$\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle \oplus \langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \rangle$

$$= \begin{cases} \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle, & \text{if } \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle > \langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \rangle \\ \langle 0, 0, \cdots, 0 \rangle, & \text{if } \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle \leq \langle f_{1_{lk}}, f_{2_{lk}}, \cdots, f_{m_{lk}} \rangle \end{cases}$$

$\langle j_{1_{lk}}, j_{2_{lk}}, \cdots, j_{m_{lk}} \rangle =$

$\langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \rangle \oplus \langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \rangle$.

Case 1. If

$\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle > \langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle$ and

$\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle > \langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle$, then

$\langle g_{1_{lk}}, g_{2_{lk}}, \cdots, g_{m_{lk}} \rangle = \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle$ and

$\langle h_{1_{lk}}, h_{2_{lk}}, \cdots, h_{m_{lk}} \rangle = \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle$.

Therefore, $\langle i_{1_{lk}}, i_{2_{lk}}, \cdots, i_{m_{lk}} \rangle =$

$$\begin{cases} \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle, & \text{if } \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle > \langle y_{1_{lk}} + z_{1_{lk}} - y_{1_{lk}} . z_{1_{lk}}, y_{2_{lk}} + z_{2_{lk}} - y_{2_{lk}} . z_{2_{lk}}, \cdots, y_{m_{lk}} + z_{m_{lk}} - y_{m_{lk}} . z_{m_{lk}} \rangle \\ \langle 0, 0, \cdots, 0 \rangle, & \text{if } \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle \leq \langle y_{1_{lk}} + z_{1_{lk}} - y_{1_{lk}} . z_{1_{lk}}, y_{2_{lk}} + z_{2_{lk}} - y_{2_{lk}} . z_{2_{lk}}, \cdots, y_{m_{lk}} + z_{m_{lk}} - y_{m_{lk}} . z_{m_{lk}} \rangle \end{cases}$$

and $\langle j_{1_{lk}}, j_{2_{lk}}, \cdots, j_{m_{lk}} \rangle =$

$2\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle - \langle x_{1_{lk}}^2, x_{2_{lk}}^2, \cdots, x_{m_{lk}}^2 \rangle$.

Thus, $\langle i_{1_{lk}}, i_{2_{lk}}, \cdots, i_{m_{lk}} \rangle \leq \langle j_{1_{lk}}, j_{2_{lk}}, \cdots, j_{m_{lk}} \rangle$.

Case 2. If

$\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle < \langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle$ and

$\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle < \langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle$, then

$\langle i_{1_{lk}}, i_{2_{lk}}, \cdots, i_{m_{lk}} \rangle = \langle 0, 0, \cdots, 0 \rangle$ and

$\langle j_{1_{lk}}, j_{2_{lk}}, \cdots, j_{m_{lk}} \rangle = \langle 0, 0, \cdots, 0 \rangle$. i.e.,

$\langle i_{1_{lk}}, i_{2_{lk}}, \cdots, i_{m_{lk}} \rangle = \langle j_{1_{lk}}, j_{2_{lk}}, \cdots, j_{m_{lk}} \rangle$.

Case 3. If $\langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle <$

$\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle < \langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle$,

then $\langle i_{1_{lk}}, i_{2_{lk}}, \cdots, i_{m_{lk}} \rangle = \langle 0, 0, \cdots, 0 \rangle$ and

$\langle j_{1_{lk}}, j_{2_{lk}}, \cdots, j_{m_{lk}} \rangle = \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle$.

i.e., $\langle i_{1_{lk}}, i_{2_{lk}}, \cdots, i_{m_{lk}} \rangle \leq \langle j_{1_{lk}}, j_{2_{lk}}, \cdots, j_{m_{lk}} \rangle$.

Case 4. If

$\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle < \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle$

$< \langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle$,

then $\langle i_{1_{lk}}, i_{2_{lk}}, \cdots, i_{m_{lk}} \rangle = \langle 0, 0, \cdots, 0 \rangle$ and

$\langle j_{1_{lk}}, j_{2_{lk}}, \cdots, j_{m_{lk}} \rangle = \langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle$.

i.e., $\langle i_{1_{lk}}, i_{2_{lk}}, \cdots, i_{m_{lk}} \rangle \leq \langle j_{1_{lk}}, j_{2_{lk}}, \cdots, j_{m_{lk}} \rangle$.

Therefore for all these cases

$\langle i_{1_{lk}}, i_{2_{lk}}, \cdots, i_{m_{lk}} \rangle \leq \langle j_{1_{lk}}, j_{2_{lk}}, \cdots, j_{m_{lk}} \rangle$, for whatever may be the values of $\langle x_{1_{lk}}, x_{2_{lk}}, \cdots, x_{m_{lk}} \rangle$,

$\langle y_{1_{lk}}, y_{2_{lk}}, \cdots, y_{m_{lk}} \rangle$ and $\langle z_{1_{lk}}, z_{2_{lk}}, \cdots, z_{m_{lk}} \rangle$.

i.e., $\langle i_{1_{lk}}, i_{2_{lk}}, \cdots, i_{m_{lk}} \rangle \leq \langle j_{1_{lk}}, j_{2_{lk}}, \cdots, j_{m_{lk}} \rangle$

for all l, k. Hence, $X \ominus (Y \oplus Z) \leq (X \ominus Y) \oplus (X \ominus Z)$.

Similarly, we can prove the other statements *d, e, f.* □

IV. CONCLUSIONS

It is a fact that m-polar fuzzy sets are one of the most ubiquitous models of both natural and human-made structures. m-polar fuzzy sets are beneficial to model the Biological, Physical, Computer Science and Social systems. Here, we prefaced m-polar fuzzy matrices and two operations between two m-polar fuzzy matrices. Several properties of m-polar fuzzy matrices are also obtained.

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