International Journal of Recent Technology and Engineering (IJRTE)
ISSN: 2277-3878, Volume-8 Issue-3, September 2019

Computation of State Vector of Networked Control System using Combination of Laplace Transform and Variational Iteration Method

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Abstract: Networked Control System (NCS) is a control system in which system components such as sensor, actuator and controller communicate over a network. Networked control system model is represented by $\dot{x}(t) = Ax(t) + Bu(t)$, where the state vector $x \in R$ and the control input $u \in R$. In this paper the state vector of the networked control system is computed by obtaining the solution of first order delay differential equation using the combination of Laplace and Variational Iteration method.

Keywords: Networked Control System, Delay Differential Equation, State Vector, Laplace Method and Variational Iteration Method. AMS Classification: Primary 93C15; Secondary 34K20

I. INTRODUCTION

Networked Control System (NCS) is a control system in which system components such as sensor, actuator and controller communicate over a network. Some of the advantages of NCS includes (a) efficiently sharing the data over the network, (b) increases the flexibility of the system thereby allowing the system to accept changes in the system's behavior and (c) complexity of the system can be reduced. However, technical problems such as limited bandwidth, delay in the network, poor scheduling affect the overall performance of the system leading to system's instability. In control theory, Delay differential equations are used many times. It is a sort of differential equation where the derivative of a function at a particular time depends not only at the present state but also at past state. The concept of delay enriches the dynamics and allows a correct narration in the real life situation. The analytical solution of the differential equation is identified in a most important way, but not limit to a predefined accuracy by successive iterations. Variational iteration method is an effective tool for solving a large class of nonlinear problems with approximations in an easier way.

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The following steps are involved in the Variational Iteration Method.

1. Acquiring the correction functional
2. Recognizing the Lagrange multiplier
3. Finding an initial approximation

The multi number of research outcomes has concern in time delay systems spread in the literature

The nonlinear wave equation, nonlinear fractional differential equation, nonlinear oscillation and nonlinear problem appearing in various engineering applications are discussed by He *et.al.*, in 2007 [1]. Tataria *et.al.*, suggested the variational iteration method for accelerating the convergence of the sequences in 2009 [2]. Variational Iteration Method and He's Polynomial method is used for solving various linear and nonlinear DDES of first, second and third order by Din *et.al.*, in 2010 [3]. Porshokouhi *et.al.*, used VIM method for solving DDE for 5th order in 2010 [4]. The Modified variational iteration method is used for solving Schrdinger and Laplace problems by Jassim in 2012 [5]. Elzaki *et.al.*, established a new sort of differential equations called nonlinear convolution ordinary differential equations and found the solution to them by combining new modified variational iteration method and Laplace transform in 2013 [6].

II. PROPOSED METHDOLOGY

A. System Description

The Networked Control System model can be represented by

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1)$$

Where $x(t) \in R$, $u(t) \in R$ are the plant's state and input respectively. A and B are real constant.

The control signal depends upon the plant's state at the instant $i_k h$ and hence the control law can be expressed as

$$u(t) = Kx(i_k h), t \in [i_k h + \tau_k, i_{k+1} h + \tau_k], k = 0,1,2,3,\ldots \quad (2)$$

where K denotes the state feedback gain vector, τ_k is the networked induced delay and h is the sampling period.

The number of consecutive packet dropouts for the time interval $(i_k h,\ i_{k+1} h)$ can be defined as follows:

$i_{k+1} - \tau_k = 1, 0$ packet is lost.

$i_{k+1} - i_k = 2, 1$ packet is lost.

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Manuscript published on 30 September 2019
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$i_{k+1} - i_k = p, p-1$ packets are lost.
Assume the constants η and τ, 0 ≤ τ ≤ η such that
$(i_{k+1} - i_k)h + \tau_{k+1} \leq \eta$
$\tau \leq \tau_k \qquad \forall k \in N$ (4)
Where τ and η denotes the lower and the upper bounds of the total delay involving both transmission delays and packet dropouts respectively.
Using (1) - (4), the system model can be expressed as
$\dot{x} = Ax(t) + A_d x(t-\tau)\ , t > 0$ (5)
$x(t) = \varphi(t), \qquad t \in (-\tau, 0)$

where A_d = BK and τ -delay time, φ(t) is the state's initial function, which is a first order differential equation with finite delay. Solution of the equation (5) represents the state vector of the given NCS model (1).

B. Computation of state vector of Networked Control System using Variational Iteration Method combined with Laplace Transform

Consider the first order delay differential equation
$\dot{x} = Ax(t) + A_d x(t-\tau),\ t > 0$
$x(t) = \varphi(t), \qquad t \in [-\tau, 0)$ (6)
Taking Laplace transform
$sX(s) - x(0) = L[Ax(t) + A_d x(t-\tau)]$
Iteration formula is given by
$X_{n+1}(s) = X_n(s) + \lambda(s)[sX_n(s) - x(0) - L[Ax(t) +$
$Adxt-\tau]$
Lagrangian multiplier is taken as $\lambda(s) = -\frac{1}{s}$
Taking inverse Laplace transform
$x_{n+1}(t) = x_n(t) - L^{-1}[\frac{1}{s}[sX_n(s) - x(0) - L[Ax_n(t) +$
$Adxnt-\tau]$ (7)
Assume $x(t) = 0,\ t < -\tau$, then $x(t-\tau) = 0,\ t \in [-\tau, 0]$. From the Eqn.(6), we get $\dot{x} = Ax(t)$ and the solution of this ODE is $x(t) = Ce^{At}$. Assume $x(0) = 1$, then C=1.
Eqn.(7) becomes
$x_{n+1}(t) = 1 + L^{-1}[\frac{1}{s}L[Ax_n(t) + A_d x_n(t-\tau)]]$
which gives the general solution of equation (6) with initial iteration $x_0(t) = 1$.
We can get the successive approximations begins with an initial approximation $x_0(t)$, and the exact solutions can be determined by using $x(t) = lim_{m\to\infty} x_m(t)$.

C. Numerical examples

In this section, few examples were illustrated using the above algorithm.

Example 1

If $A > 0, A_d > 0$, i.e. assume $A = 3, A_d = 5$ in (6)
$x(t) = 3x(t) + 5x(t-\tau), t > 0$ (8)
$x(t) = \varphi(t), t \in (-\tau, 0)$

Then the general solution of (8) is given by
$x_{n+1}(t) = 1 + L^{-1}[\frac{1}{s}L[3x_n(t) + 5x_n(t-\tau)]]$
$x_1(t) = 1 + 8t$
$= a_{10} + a_{11}t$
$x_2(t) = 1 + (8a_{10} - 5a_{11}\tau)t + 4a_{11}t^2$
$= a_{20} + a_{21}t + a_{22}t^2$
$x_3(t) = 1 + (8a_{20} - 5a_{21}\tau + 5a_{22}\tau^2)t +$
$(4a_{21} - 5a_{22}\tau)t^2 + (8/3)a_{22}t^3$
$= a_{30} + a_{31}t + a_{32}t^2 + a_{33}t^3$
$x_4(t) = a_{40} + a_{41}t + a_{42}t^2 + a_{43}t^3 + a_{44}t^4$
Here
$a_{40} = 1$
$a_{41} = 8a_{30} - 5a_{31}\tau + 5a_{32}\tau^2 - 5a_{33}\tau^3$
$a_{42} = 4a_{31} - 5a_{32}\tau + (15/2)a_{33}\tau^2$
$a_{43} = (8/3)a_{32} - 5a_{33}\tau$
$a_{44} = 2a_{33}$
$x_5(t) = a_{50} + a_{51}t + a_{52}t^2 + a_{53}t^3 + a_{54}t^4 + a_{55}t^5$
Here
$a_{50} = 1$
$a_{51} = 8a_{40} - 5a_{41}\tau + 5a_{42}\tau^2 - 5a_{43}\tau^3 + 5a_{44}\tau^4$
$a_{52} = 4a_{41} - 5a_{42}\tau + (\frac{15}{2})a_{43}\tau^2 - 10a_{44}\tau^3$
$a_{53} = (8/3)a_{42} - 5a_{43}\tau + 10a_{44}\tau^2$
$a_{54} = 2a_{43} - 5a_{44}\tau$
$a_{55} = (8/5)a_{44}$
And so on…

Example 2

If $A < 0, A_d < 0$ i.e., assume $A = -2, A_d = -1$ in (6)
$\dot{x}(t) = -2x(t) - x(t-\tau), t > 0$
$x(t) = \varphi(t), t \in [-\tau, 0)$ (9)
Then the general solution of (9) is given by
$x_{n+1}(t) = 1 + L^{-1}[\frac{1}{s}L[-2x_n(t) - x_n(t-\tau)]]$
$x_1(t) = 1 - 3t$
$= a_{10} - a_{11}t$
$x_2(t) = 1 - (3a_{10} - a_{11}\tau)t + (3/2)a_{11}t^2$
$= a_{20} + a_{21}t + a_{22}t^2$
$x_3(t) = 1 - (3a_{20} - a_{21}\tau - a_{22}\tau^2)t + ((3/2)a_{21} -$
$a22\tau t2 - a22t3$
$= a_{30} + a_{31}t + a_{32}t^2 + a_{33}t^3$
$x_4(t) = a_{40} + a_{41}t + a_{42}t^2 + a_{43}t^3 + a_{44}t^4$
Here
$a_{40} = 1$
$a_{41} = -3a_{30} + a_{31}\tau - a_{32}\tau^2 - a_{33}\tau^3$
$a_{42} = (-3/2)a_{31} + a_{32}\tau + (3/2)a_{33}\tau^2$
$a_{43} = -a_{32} - a_{33}\tau$
$a_{44} = (3/4)a_{33}$
$x_5(t) = a_{50} + a_{51}t + a_{52}t^2 + a_{53}t^3 + a_{54}t^4 + a_{55}t^5$
Here
$a_{50} = 1$
$a_{51} = -3a_{40} + a_{41}\tau - a_{42}\tau^2 + a_{43}\tau^3 - a_{44}\tau^4$
$a_{52} = (-3/2)a_{41} - 2a_{42}\tau - (3/2)a_{43}\tau^2 + 2a_{44}\tau^3$
$a_{53} = -a_{42} + a_{43}\tau - 2a_{44}\tau^2$
$a_{54} = (-3/4)a_{43} - \tau$
$a_{55} = (-3/5)a_{44}$
and so on…

Example 3

If $A > 0, A_d < 0$, i.e., assume $A = 1, A_d = -2$ in (6)
$\dot{x}(t) = x(t) - 2x(t-\tau), t > 0$ (10)
$x(t) = \varphi(t), t \in [-\tau, 0)$
Then the general solution of (10) is given by
$x_{n+1}(t) = 1 + L^{-1}[\frac{1}{s}L[x_n(t) - 2x_n(t-\tau)]]$
$x_1(t) = 1 - t$
$= a_{10} - a_{11}t$
$x_2(t) = 1 + (-a_{10} - 2a_{11}\tau)t + (1/2)a_{11}t^2$
$= a_{20} + a_{21}t + a_{22}t^2$
$x_3(t) = 1 + (-a_{20} +$
$2a_{11}\tau - 2a_{22}\tau^2)t +$
$((-1/2)a_{21} +$
$a_{22}\tau)t^2 + (-1/3)a_{22}t^3$

Retrieval Number C4598098319/2019©BEIESP
DOI: 10.35940/ijrte.C4598.098319
Journal Website: www.ijrte.org

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

$$= a_{30} + a_{31}t + a_{32}t^2 + a_{33}t^3$$
$$x_4(t) = a_{40} + a_{41}t + a_{42}t^2 + a_{43}t^3 + a_{44}t^4$$

Here

$$a_{40} = 1$$
$$a_{41} = -a_{30} + 2a_{31}\tau - 2a_{32}\tau$$
$$a_{42} = (-1/2)a_{31} + 2a_{32}\tau$$
$$a_{43} = (-1/3)a_{32}$$
$$a_{44} = (1/4)a_{33}$$
$$x_5(t) = a_{50} + a_{51}t + a_{52}t^2 + a_{53}t^3 + a_{54}t^4 + a_{55}t^5$$

Here

$$a_{50} = 1$$
$$a_{51} = -a_{40} + 2a_{41}\tau - 2a_{42}\tau^2 + 2a_{43}\tau^3 - 2a_{44}\tau^4$$
$$a_{52} = (-1/2)a_{41} + 2a_{42}\tau - 3a_{43}\tau^2 + 4a_{44}\tau^3$$
$$a_{53} = (-1/3)a_{42} + 2a_{43}\tau - 4a_{44}\tau^2$$
$$a_{54} = (-1/4)a_{43} + 2a_{44}\tau$$
$$a_{55} = (-1/5)a_{44}$$

and so on…

Example 4

If $A < 0, A_d > 0$, i.e., assume $A = -1, A_d = 2$ in (6)

$$\dot{x}(t) = -x(t) + 2x(t-\tau), t > 0 \qquad (11)$$
$$x(t) = \varphi(t), t \in [-\tau, 0)$$

Then the general solution of (11) is given by

$$x_{n+1}(t) = 1 + L^{-1}[\tfrac{1}{s}L[-x_n(t) + 2x_n(t-\tau)]]$$
$$x_1(t) = 1 + t$$
$$= a_{10} + a_{11}t$$
$$x_2(t) = 1 + (a_{10} - 2a_{11}\tau)t + ((1/2)a_{11})t^2$$
$$= a_{20} + a_{21}t + a_{22}t^2$$
$$x_3(t) = 1 + (a_{20} - 2a_{21}\tau + 2a_{22}\tau^2)t + ((1/2)a_{21} -$$
2a22τt2+(13a22)t3
$$= a_{30} + a_{31}t + a_{32}t^2 + a_{33}t^3$$
$$x_4(t) = a_{40} + a_{41}t + a_{42}t^2 + a_{43}t^3 + a_{44}t^4$$

Here

$$a_{40} = 1$$
$$a_{41} = 3a_{30} - 2a_{31}\tau + 2a_{31}\tau^2 - 2a_{33}\tau^3$$
$$a_{42} = (3/2)a_{31} - 2a_{32}\tau + 3a_{33}\tau^2$$
$$a_{43} = a_{32} - 2a_{33}\tau$$
$$a_{44} = (3/4)a_{33}$$
$$x_3(t) = a_{50} + a_{51}t + a_{52}t^2 + a_{53}t^3 + a_{54}t^4 + a_{55}t^5$$

Here

$$a_{50} = 1$$
$$a_{51} = 3a_{40} - 2a_{41}\tau - 2a_{42}\tau^2 - 2a_{43}\tau^3 + 2a_{44}\tau^4$$
$$a_{52} = (3/2)a_{41} - 2a_{42}\tau + 3a_{43}\tau^2 - 4a_{44}\tau^3$$
$$a_{53} = (2/3)a_{42} + (1/3)a_{42}\tau - 2a_{43}\tau^2 + 4a_{44}\tau^3$$
$$a_{54} = (1/2)a_{43} + (1/4)a_{43}\tau^2 - 2a_{44}\tau$$
$$a_{55} = (2/5)a_{44}$$

and so on…

III. RESULT ANALYSIS

The analysis carried out in this paper shows that the VIM can be effectively worked to get an approximate solutions of Delay Differential Equations. Here we have combined Laplace Transform and VIM method for determining the state vector of the networked control systems which is represented by Delay Differential Equations. In this method an iteration formula is built by a Lagrange multiplier. It can be determined by the variational theory. Using this multiplier and initial approximation x_0, we will be obtained the successive approximationx_{m+1}, $m \geq 0$, of the solution x. Then the exact solution is given by $x(t) = lim_{m\to\infty} x_m$. From our examples, we can see that successive approximate solutions of the exact solution x.

IV. CONCLUSION

In this paper, we have combined Laplace Transform and Variational Iteration Method for computing the state vector of the Networked Control System by obtaining the solution of first order delay differential equation. The obtained solution shows that a combination of a Laplace and the variational iteration method is a very convenient and effective method for finding the state vector of the networked control system model. Some examples highlight the efficiency of the method proposed in the paper.

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