Performance Evaluation of Genetic Algorithm & Fuzzy Logic for Portfolio Optimization

Darsha Panwar, Manoj Jha, Namita Srivastava

Abstract- Teaching-learning based optimization (TLBO), biogeography-based optimization (BBO) and fuzzy multi-objective linear programming (FMOLP) are compared in this paper for portfolio optimization. A hybrid approach has been adopted for this comparative study which is a combination of a few methods, such as investor topology, cluster analysis, analytical hierarchy process (AHP) and optimization techniques. Return, risk, liquidity, coefficient of variation (CV) and AHP weighted scores are used as the objective function for optimization.

Keywords- Analytical hierarchy process; Biogeography-based algorithm; Cluster analysis; Fuzzy multi-objective linear programming; Portfolio optimization; Teaching-learning based algorithm.

I. INTRODUCTION

The key objective of portfolio selection is to achieve a perfect fraction of the assets, to confirm that the investor receives the highest returns at least risk. Professor Markowitz identified the problem of portfolio selection and introduced the Markowitz model The Markowitz model is a quadratic single objective programming model for choosing a diversified portfolio. This model was initially introduced by Professor Harry Markowitz [1]. This is based on the concept that holding two or more assets are less risky than holding one asset, and this has become a foundation of modern portfolio theory. This model is conceptually sound in analyzing the return and risk of a portfolio. Several linear programming models have been proposed with altered explanation of risk function and the purpose of improving and simplifying the Markowitz model. Konno and Yamazaki [2], proposed a mean absolute deviation model (MAD) and risk function was measured by the mean. Sprenza [3], addressed a linear programming model, in which the risk function measured by the semi-absolute deviation model. Branke *et. al.*, [4] has projected an envelope-based multi-objective evolutionary algorithm for portfolio selection, which is a mixture of a multi-objective algorithm with an embedded algorithm for parametric quadratic programming. In this proposed method, the evolutionary algorithm creates a bunch of convex subsets of the search space.

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An efficient frontier is generated by these subsets, called envelopes. Miyahara and Tsujii [5] studied the portfolio optimization problem in the case of the Levy process model and applied the “risk sensitive value measure method” for portfolio optimization, and for financial risk assets assumed the risk-sensitive value measure.

Two possibilistic mean–semi variance models, with real constraints, were given by Liu and Zhang [6] and the fuzzy multi-objective programming approach was used to solve this model, in which, return, risk, liquidity, and liquidity risk were assumed to be fuzzy variables and calculated by using possibilistic mean and possibilistic semi-variance. Bruni *et. al.* [7] advocated a linear bi-objective optimization to enhanced indexation (EI), which maximized return and minimized risk in the learning period. Hadi *et. al.*, [8] applied a pareto based enhanced genetic algorithm with four further constraints for portfolio optimization and used data selected from the Egyptian Exchange. They show that the proposed model is best among all the conventional optimization models. Mashayekhi and Omrani [9] applied the second version of a non-dominated sorting genetic algorithm (NSGA-II) for portfolio optimization, using data taken from Tehran Stock Exchange. Qu *et. al.* [10] presented an efficient model for solving portfolio problems, which was large-scale optimization problems, by introducing two asset preselection processes. He applied MOEA/D, MODE-SS, MODE-NDS, MOCLPSO, and NSGAII multi-objective evolutionary algorithms to check the efficiency of the proposed model.

Chen [11] presented an uncertain mean-variance-skewness portfolio selection model with the criteria transaction costs, bounds on holdings, cardinality of the portfolio, and minimum transaction lot constraints, and developed a hybrid approach firefly algorithm-genetic algorithm (FA-GA) for solving the proposed model. Saglam and Benson [12] presented the multi-period portfolio optimization problem in a mean-variance framework including diversification-by-sector constraints, buy-in-thresholds, transaction costs, and conditional value-at-risk. Zaheer [13] used the Shanghai Stock Exchange data to develop a hybrid PSO technique for portfolio optimization. He considered two different models with short sale and without short sale.

We observe from the literature review that k-means and fuzzy c-means algorithm are used for clustering. K-means calculates only the Euclidean distance and the initial selection of the centroid is random; at the same time c-means also measure the distance from the centroid which is a common drawback in both.

Hierarchy of AHP is focused only on the criteria of return, risk, liquidity, alpha-coefficient, beta-coefficient and value at risk. There are some more important and new factors which are not considered for valuation of stocks.

Generally, the methodologies involved in portfolio selection are the investor behavioural survey, cluster analysis, the analytical hierarchy process, the optimization technique, and ARIMA model; still, there are many opportunities in this research for further improvement in portfolio selection.

This paper compares teaching-learning based optimization (TLBO), biogeography-based optimization (BBO), and fuzzy multi-objective linear programming (FMOLP) for portfolio optimization. The comparative study adopts a hybrid approach which is a combination of a few methods, such as investor topology, cluster analysis, analytical hierarchy process (AHP) and optimization techniques. Return, risk, liquidity, coefficient of variation (CV) and AHP weighted scores are used as the objective function for optimization.

In this research, clustering is done using expectation maximization (EM) technique which is the extension of the k-means algorithm. EM calculates weighted distance while k-means calculates only Euclidean distance. The EM algorithm is an iterative statistical method.

Stock valuation is done by risk, return, liquidity, stock volatility (i.e. beta coefficient), comparison of return with respect to risk (i.e. alpha coefficient), CV and price-earnings ratio.

TLBO and BBO are from the genetic algorithm. Both are population-based techniques and require fewer parameters for solving the problem. Execution of the TLBO algorithm is not complex and has great feasibility for solving multi-objective problems. FMOLP is one of the most popular techniques for optimization.

Five objectives have been taken for optimization which are return, risk, liquidity, coefficient of variation (CV), price to earnings ratio (P/E ratio) and AHP weighted score. The data are taken from the Bombay Stock Exchange, Mumbai, India (https://www.bseindia.com) such as daily closing price, number of shares, turnover rate and price to earnings ratio, from February-'15 to January-'16.

This paper is organized in four Segments as mentioned: Segment 2 includes an account of the research methodology, the optimization techniques and their working process. Segment 3 presents the numerical illustrations, while Segment 4 contains comparison of the all three optimization techniques and Segment 5 presents the concluding remarks of research.

II. PROBLEM FORMULATION

All the notations and MOLP problem for portfolio selection along with the five objectives such as return, risk, liquidity, CV and AHP weight are as follows:

All objective functions and constraints are formulated as follows:

- Return

The return of the portfolio is written as:

$$f_1(x) = \sum_{i=1}^{n} r_i x_i$$

Where $r_i = \frac{1}{12}\sum_{i=1}^{12} r_{it}$.

- Risk

The semi-absolute deviation of return of the portfolio below the expected return over the past period t,

t = 1, 2, …T, can be written as:

$$k_t(x) = \left| min\left\{0, \sum_{i=1}^{n}(r_{it} - r_i)\right\} x_i \right| = \frac{|\sum_{i=1}^{n}(r_{it} - r_i)x_i| + \sum_{i=1}^{n}(r_i - r_{it})x_i}{2}$$

Consequently, the expected semi-absolute deviation of return of the portfolio x=$(x_1, x_2, x_3, \dots \dots x_n)$ below the expected return becomes

$$f_2(x) = k(x) = \frac{1}{T}\sum_{t=1}^{T} k_t(x) = \sum_{t=1}^{T}\frac{|\sum_{i=1}^{n}(r_{it}-r_i)x_i| + \sum_{i=1}^{n}(r_i - r_{it})x_i}{2T},$$

Where k (x) represents portfolio risk.

Above equation can be written as:

$$f_2(v) = k(v) = \frac{1}{T}\sum_{t=1}^{T} v_t ,$$

where $v_t + \sum_{t=1}^{T}(r_{it} - r_i)x_i \geq 0$.

- Liquidity

Liquidity is measured as the possibility of transformation of an investment into cash without affecting the asset's price. Possibility theory introduced by Zadeh and developed by Dubois and Prade for calculating turnover rate.

Fuzzy number F is known as trapezoidal with tolerance interval [a, b, α, β], if its membership function defined as:

$$\mu_F = \begin{cases} 1 - \dfrac{a-t}{\alpha} & if\ (a-\alpha) \leq t \leq a \\ 1 & if\ a \leq t \leq b \\ 1 - \dfrac{t-b}{\beta} & if\ b \leq t \leq (b+\beta) \\ 0 & otherwise. \end{cases}$$

Let the turnover rate of the i^{th} stock is denoted by a trapezoidal fuzzy number $L_i = (L_{ai}, L_{bi}, \alpha_i, \beta_i)$. Then, the portfolio turnover rate is given as$\sum_{i=1}^{n} L_i x_i$.The turnover rate of the portfolio by the fuzzy extension principle is given as;

$$f_4(x) = \sum_{i=1}^{n} L_i x_i = \sum_{i=1}^{n}\left(\frac{L_{ai}+L_{bi}}{2} + \frac{\beta_i - \alpha_i}{6}\right)x_i .$$

- Coefficient of variation (CV)

The CV of the portfolio is written as:

$$f_4(x) = \sum_{i=1}^{n} C_i x_i ,$$

Where $c_i = \frac{SD_i}{return_i}$ of the i^{th} stock.

- AHP weight

The AHP weight of the portfolio is written as:

$$f_7(x) = \sum_{i=1}^{n} w_{AHPi} x_i$$

Where w_{AHPi} is weight of i^{th} stock.

Constraints:

Investment economical restriction on the stocks:

- Sum of proportion of stocks should be 1

$$\sum_{i=1}^{n} x_i = 1$$

- Number of stocks held in a portfolio:

$$\sum_{i=1}^{n} b_i = a$$

- The maximum percentage of the investment which can be invested in a stock:

$x_i \leq u_i b_i$, $i = 1, 2, \dots, n$,

- The minimum percentage of the investment which can be invested in a stock:

$x_i \geq l_i b_i$, $i = 1, 2, \dots, n$,

The upper and lower bounds have been taken to avoid too many large and small investments.

The decision problem:

$\max f_1(x) = \sum_{i=1}^{n} r_i\, x_i$ (1)

$\min f_2(v) = \sum_{i=1}^{n} k_i\, x_i$ (2)

$\max f_3(x) = \sum_{i=1}^{n} L_i\, x_i$ (3)

$\min f_4(x) = \sum_{i=1}^{n} C_i\, x_i$ (4)

$\max f_5(x) = \sum_{i=1}^{n} w_{AHPi}\, x_i$ (5)

subject to $v_t + \sum_{t=1}^{T} (r_{it} - r_i) x_i \geq 0$ (6)

$\sum_{i=1}^{n} x_i = 1,$ (7)

$\sum_{i=1}^{n} b_i = a,$ (8)

$x_i \leq u_i b_i\,, i = 1, 2, \dots, n,$ (9)

$x_i \geq l_i b_i\,, i = 1, 2, \dots, n,$ (10)

$x_i \geq 0\,, i = 1, 2, \dots, n,$ (11)

$b_i \epsilon \{0, 1\}\,, i = 1, 2, \dots, n.$ (12)

Notations:

r_i : return of the i^{th} stock,

x_i : the proportion of the i^{th} stock which is invested,

b_i : a binary variable for the i^{th} stock and defined is as follows:

i.e. $b_i = \begin{cases} 1, & if\ i^{th}\ stock\ contained\ in\ portfolio \\ 0, & if\ not\ containing\ in\ portfolio \end{cases}$.

k_i : risk of the i^{th} stock.

C_i : coefficient of variation of the i^{th} stock,

w_{AHPi} :the AHP weight of the i^{th} stock,

L_i : liquidity of the i^{th} stock,

u_i : the maximum fraction of the i^{th} stock,

l_i : the minimum fraction of the i^{th} stock,

n : total number of stocks in each cluster,

a : number of stock in a selected portfolio.

III. METHODOLOGY

Portfolio selection problem is a multi-objective linear programming problem. Two hybrid approaches are proposed for portfolio selection using investor behavior, cluster analysis, AHP technique, and optimization technique. The data preparation for an experimental and numerical study has already been prepared and discussed in Chapter 3. The genetic algorithm and fuzzy decision theory are applied for portfolio selection. LINGO, MATLAB and RAPID-MINOR are used for solving the multi-objective problem, and cluster analysis, respectively.

A. Investor behavior pattern

The role of the behavior of a stockholder [14] is significant and they have a particular approach in picking the stocks. Here, we discuss different outlooks, styles, dependents, sources, and purpose for investment. According to the investors' topology, they do not favor making riskier investments; rather they desire a stable return. Investors have different objects for investing in the stock market. According to the survey, investors are inclined toward money, safety (liquidity), and low risk. Based on the above interest, the investors can be divided into three categories as follows:

a. Desire maximum returns.
b. Wish for low-risk stocks, even if the returns are less.
c. Interested in harmless investment (liquidity lovers).

Therefore, stocks are classified into three clusters namely high return, low risk, and liquid stocks, which satisfy the above-mentioned preference of the investors.

B. Cluster

"Cluster analysis is a technique of arrangement similar objects in the same cluster, which are different from other cluster's objects".

A different investor has different approach towards selecting the stocks. Generally, they are focused only on return, risk, and liquidity so that stocks are divided into three clusters namely high return stocks, less risky stocks and liquid stocks according to the investors' choice.

The projected research comprises investor topology. The purpose of investing in the stock market for each investor is different, as someone is more interested in higher return, someone wants to take the least risk and someone believes in a safe investment. Based on the preferences, the investors are divided into three different categories.

Therefore, on the basis of these three preferences discussed above, the stocks are divided into three clusters. To formulate clusters, EM clustering [15] is applied. This is a distribution model and maintains multivariate normal distribution.

C. AHP

Thomas L. Saaty introduced AHP, which is a multi-criteria decision making (MCDM) technique, in the 1970s [16] . AHP is a structured technique for organizing and analyzing complex decisions. It has a specific application in group decision-making. Another important advantage of AHP is that it gives inconsistency in judgment. The appeal of the AHP technique is that the execution is simple and intuitive. There are three leading phases of AHP for ranking the object:

1) Hierarchy structure design
2) Weight analysis
3) Consistency proof

AHP is used for evaluation of assets as per investor's preference. Ranking of assets can be done with the help of AHP.

Each entry of the judgmental matrix A is formed by following rules:

$a_{ij} > 0$

$a_{ij} = 1/a_{ji}$

$a_{ij} = 1$, when i=j, for all i,j

To compare two things, Satty [20] defined a 1-9 ranking scale.

For the matrix A of order 'n' the normalized Eigenvector is called Priority vector.

$Aw = \lambda_{max} w$,

Where 'w' is known as weight of the objects. The highest Eigen value of the matrix A is λ_{max}. The consistency index (CI) for each n^{th} order matrix is calculated as:

$CI = (\lambda_{max} - n)/ (n - 1)$

The consistency ratio (CR) is calculated as:

$CR = CI/RI$

Where RI the random index be determined by on the order of the matrix.

The matrix is consistent if $CR \leq 0.10$. There will be inconsistency if $CR > 0.10$ and judgmental matrix need to be revised.

D. Portfolio selection technique

This article involves all three optimization techniques for portfolio selection TLBO [17], FMOLP [18], and BBO [19].

TLBO

The TLBO algorithm was presented by Rao *et. al.*, in 2011 [20] and is encouraged by classroom strategy. The most intelligent (highest mean value) learner is treated as a teacher and the rest of the population is acknowledged as learners. These learners are trained by the teacher for betterment of the result.

FMOLP

FMOLP [21] is one of the most popular techniques for optimization. In this technique, multi-objectives are converted into a single objective problem.

BBO

The BBO technique is introduced by Dan Simon [22], who proposed a population based algorithm known as BBO algorithm. It is an evolutionary algorithm based on the concept of migration and mutation. The migration operator contains immigration and emigration probability.

IV. Numerical illustration

The data set of 144 assets registered in the BSE, Mumbai, India, (from February-'15 to January-'16) for numerical illustration are as follows:

Cluster analysis – Selected stocks are categorized into three clusters, with regard to quality, high liquidity stocks, low risk stocks, and high return stocks. Expectation maximization clustering technique is applied to this division. To perform cluster analysis**,** the EM clustering tool of the Rapid Miner version 5.2 software is used. The number of cluster should be set at k=3, as the stocks are categorized into three different groups. The initial distributions of the centroids are performed by k-means clustering. The mean result of the EM algorithm is shown in Table 1.

Table 1 Mean results for cluster analysis

Parameters	Cluster 1(16 stocks)	Cluster 2(97 stocks)	Cluster 3(31 stocks)
mean return	0.03987	0.02618	0.06782
mean risk	0.05175	0.04024	0.07227
Mean liquidity	0.00338	0.00035	0.00065
Category	Liquid stocks	Less risky stocks	High return stocks

According to the survey discussed in the 3.1 segment:

- **Cluster 1**: Investors who are looking for a secure investment should choose stocks that have highest liquidity, as in cluster 1.
- **Cluster 2**: Investors who are looking for a less risky investment should choose stocks that have least risk, as in cluster 2.
- **Cluster 3**: Investors who are looking for a high profit investment should choose stocks that have highest return, as in cluster 3.

For portfolio selection, select the 15 top most stocks from each cluster, which are highest in return, highest in liquidity and lowest in risk according to their cluster quality. Table 2 shows the cluster wise symbolic representation of the stocks.

Table 2 cluster wise selected stocks and symbols

Symbol	Cluster 1	Cluster 2	Cluster 3
S1	WHBRADY	SWARAJ ENGINE	CHENNAI PET.
S2	ASIANHOTNR	BAJFINANCE	THIRUMALAI
S3	FORCE MOTR	FINOLEX IND.	VADILAL IND
S4	TATA ELXSI	BHARAT PET.	GODFREY PH
S5	LYKA LABS	LAKSHMI MILL	H.P.COTTON
S6	AVON LIFESCIENCES	JSWSL	KINETIC ENG.
S7	SURYA ROSHNI	PFIZER LTD.	TOKYO PLAST
S8	NELCO LTD.	SABTN	KG DENIM
S9	NOCIL LTD	KAJARIA CER.	ZENITH FIBER
S10	CEAT LIMITED	ASIAN PAINTS	JENSON NICOLSON
S11	ESCORTS LTD.	LIC HOUS.FIN	NIIT LTD.
S12	REL INFRA	TATA COMM	CENTURY EXT
S13	CENTURY TEXT	ESSEL PROP	JASCH INDUST
S14	VEER ENERGY	CROMPT.GREAV	MEDI-CAPS
S15	SWAN ENG	GODREJ IND	PAS.ACRYLON

Numerical calculation of AHP weights

Stocks are ranked under the basic factor, and valuation factor. The basic factor contains basic parameters, risk, return, and liquidity, and valuation factor includes p/e ratio, CV, alpha, and beta-coefficient.

1) Coefficient of variation is the value of instability as compared to the return rate.
2) Alpha-coefficient compares return with respect to risk.
3) Beta-coefficient indicates volatility with the market of stock or portfolio. If the value of Beta is 1, then it means that stock is changing according to the market. Beta greater than 1 shows higher volatility and less than 1 shows less volatility than the market.
4) Price-earnings ratio (p/e ratio) facilitates the ratio of current share price and earning per share. This is a frequently used valuation factor. Its higher value represents higher earnings growth in future compared to lower value for a company. A company that is losing the money does not have p/e ratio.

A three level hierarchy structure is shown in Figure 1.

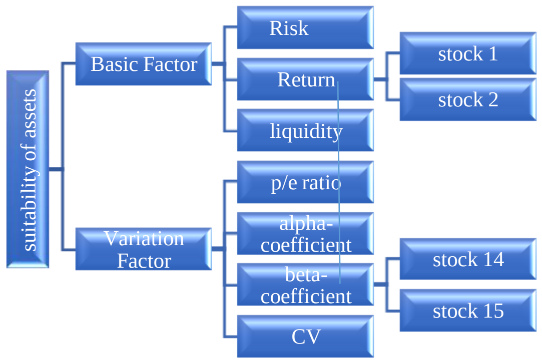

Figure 1 Hierarchy structure of AHP for comparative study

In this segment, the rank of stocks is calculated under the criteria and sub-criteria in AHP. The weights are given in Table 3 and input data are given in Table 4 to Table 6.

Table 3 Weights for each level

Criteria	Weight	sub-criteria	Weight
Basic factor	0.6000	Risk	0.2143
		Return	0.1714
		Liquidity	0.2143
Variation factor	0.4000	p/e ratio	0.1273
		Alpha-coefficient	0.0909
		Beta-coefficient	0.1091
		Coefficient of variation	0.0727

Table 4 Data for cluster 1

Stocks	Return	Risk	Liquidity	CV	Weight
S1	0.0628	0.0402	0.0173	1.5707	0.1266
S2	0.0087	0.0248	0.0091	7.4189	0.1071
S3	0.0907	0.0873	0.0065	2.4825	0.0988
S4	0.0805	0.0647	0.0049	1.8872	0.0843
S5	0.0947	0.0580	0.0036	1.5566	0.0973
S6	0.0549	0.0839	0.0027	3.7274	0.0548
S7	0.0264	0.0355	0.0024	3.5400	0.0549
S8	0.0296	0.0638	0.0021	5.2433	0.0687
S9	0.0369	0.0471	0.0019	3.0668	0.0533
S10	0.0314	0.0544	0.0016	4.8066	0.0387
S11	0.0179	0.0400	0.0016	5.7903	0.0476
S12	0.0151	0.0419	0.0015	6.2605	0.0394
S13	0.0113	0.0448	0.0015	10.6068	0.0351
S14	0.0226	0.0514	0.0014	6.3364	0.0437
S15	0.0186	0.0215	0.0013	2.9564	0.0496

Table 5 Data for cluster 2

Stocks	Return	Risk	Liquidity	CV	Weight
S1	0.0111	0.0172	0.0001	3.8099	0.0514
S2	0.0342	0.0174	0.0001	1.3198	0.0822
S3	0.0048	0.0188	0.0001	10.3879	0.0403
S4	0.0192	0.0190	0.0001	2.5065	0.0638
S5	0.0017	0.0197	0.0001	31.8555	0.0428
S6	0.0058	0.0203	0.0004	9.4639	0.0533
S7	0.0128	0.0218	0.0001	4.1962	0.0440
S8	0.0349	0.0220	0.0005	1.7178	0.1107
S9	0.0207	0.0229	0.0001	2.9862	0.0612
S10	0.0073	0.0232	0.0001	7.8238	0.0503
S11	0.0052	0.0235	0.0004	12.5956	0.0674
S12	0.0046	0.0236	0.0002	12.3072	0.1377
S13	0.0244	0.0237	0.0001	2.3202	0.0693
S14	0.0056	0.0237	0.0005	10.0820	0.0729
S15	0.0125	0.0238	0.0001	4.5062	0.0526

Table 6 Data for cluster 3

Stocks	Return	Risk	Liquidity	CV	Weight
S1	0.1004	0.0712	0.0010	1.7909	0.0608
S2	0.0992	0.0758	0.0017	2.0553	0.0804
S3	0.0975	0.0871	0.0021	1.2688	0.0975
S4	0.0971	0.0658	0.0002	1.7420	0.0725
S5	0.0953	0.0725	0.0003	2.0627	0.0540
S6	0.0924	0.0668	0.0006	1.7958	0.0535
S7	0.0915	0.0721	0.0008	1.8909	0.0592
S8	0.0892	0.0650	0.0017	1.7777	0.0830
S9	0.0883	0.0478	0.0007	1.2688	0.0684
S10	0.0860	0.0669	0.0011	2.2301	0.0599
S11	0.0807	0.0797	0.0014	2.4463	0.0835
S12	0.0779	0.1038	0.0003	3.1470	0.0455
S13	0.0113	0.0448	0.0015	10.6068	0.0351
S14	0.0226	0.0514	0.0014	6.3364	0.0437
S15	0.0186	0.0215	0.0013	2.9564	0.0496

FMOLP calculation

Assuming that after solving equation (1) with the constraints (6) to (12), the solution is X_1, then the values of all remaining objectives are to be solved at X_1. Repeat this process for the remaining objective functions from (2) to (5) and in this manner, with respect of each objective, seven solutions will be found.

Cluster wise upper and lower bound are given in Table 7.

Table 7 Upper bound and lower bound

	cluster 1		cluster 2		cluster 3	
Objective	Ub	Lb	Ub	Lb	Ub	Lb
Return	0.0913	0.0160	0.0337	0.0166	0.0997	0.0832
Risk	0.0694	0.0239	0.0230	0.0174	0.0829	0.0537
Liquidity	0.0134	0.0046	0.0005	0.0001	0.0019	0.0008
CV	4.6845	1.6217	7.7930	1.5565	1.8838	1.4877
AHP weight	0.1170	0.0731	0.1233	0.0628	0.0910	0.0691

Transform the multi-objective problem into a single objective using "weighted adaptive approach" and the weights are calculated by AHP. The converted single objective function is as follows:

$$\text{Max} \quad 0.1714a_1 + 0.2143a_2 + 0.2143a_3 + 0.0727a_4 + 0.0712a_5 \tag{13}$$

Subject to

$$f_k(x) - (upper\ bound - lower\ bound)a_k \geq lower\ bond, k = 1, 3, 5, \tag{14}$$

$$f_h(x) + (upper\ bound - lower\ bound)a_h \leq upper\ bond,\ \ h = 2, 4 \tag{15}$$

$$0 \leq \ a_j \leq 1, j = 1, 2, \ldots, 5 \tag{16}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{17}$$

$$\sum_{i=1}^{n} b_i = a \tag{18}$$

$$x_i - u_i b_i \geq 0\,, i = 1, 2, \ldots n, \tag{19}$$

$$x_i - l_i b_i \leq 0\,, i = 1, 2, \ldots n, \tag{20}$$

$$x_i \geq 0\,, i = 1, 2, \ldots n. \tag{21}$$

The first iteration will be found after solving the above-mentioned problem. If needed, the old lower bound will be altered with the first iteration. Repetition of this process depends on the investor's satisfaction.

The iterations for each cluster is given by Table 8.

Table 8 Iterations for each cluster

Iterations for cluster 1			Iterations for cluster 2			Iterations for cluster 3		
1	**2**	**3**	**1**	**2**	**3**	**1**	**2**	**3**
0.0446	0.0446	0.0446	0.0331	0.0335	0.0335	0.0943	0.0964	0.0964

0.0366	0.0366	0.0366	0.0201	0.0201	0.0201	0.0773	0.0773	0.0773
0.0134	0.0134	0.0134	0.0003	0.0003	0.0003	0.0019	0.0017	0.0017
3.7726	3.7721	3.7721	1.8069	1.6437	1.6437	1.4901	1.4877	1.4877
0.1170	0.1170	0.1170	0.0963	0.0965	0.0965	0.0902	0.0876	0.0876

A. Assets allocation

Table 9 represent the selected stocks for all three clusters.

Table 9 Cluster wise selected stocks

	TLBO			FMOLP			BBO		
Stock	cluster 1	cluster 2	cluster 3	cluster 1	cluster 2	cluster 3	cluster 1	cluster 2	cluster 3
S1	0.0855	0	0.1900	0.5555	0.0225	0.0225	0.4813	0.2289	0.0673
S2	0.0227	0.4203	0.1065	0.3717	0.3832	0.2488	0.0033	0.5215	0
S3	0.3112	0	0.3801	0.0227	0	0.5501	0.1138	0.0271	0.1573
S4	0.1071	0.1825	0.3059	0.0225	0.0225	0	0.3177	0.0497	0.2161
S5	0.4735	0	0.0175	0.0276	0	0	0.0839	0	0
S6	0	0	0	0	0	0	0	0.1728	0
S7	0	0	0	0	0	0	0	0	0
S8	0	0	0	0	0.5493	0.0225	0	0	0
S9	0	0	0	0	0	0.1561	0	0	0.3315
S10	0	0	0	0	0	0	0	0	0
S11	0	0	0	0	0	0	0	0	0
S12	0	0.1753	0	0	0	0	0	0	0
S13	0	0.1860	0	0	0.0225	0	0	0	0.2278
S14	0	0	0	0	0	0	0	0	0
S15	0	0.0359	0	0	0	0	0	0	0

V. COMPARISON

The risk/return ratio shows the risk-return trade-off and it is a key factor that helps in investment, and for those who have a little bit of knowledge about stock selection. Investors who are not interested in taking higher risks, would like to go with lower risk/return ratio. In the same way, investors whose risk tolerance level is high would like to invest with higher risk/return ratio.

Table 10 Risk/return ratio for all three clusters with respect to each technique.

Risk/return ratio			
Technique	Cluster 1	Cluster 2	Cluster 3
TLBO	0.7523	0.8538	0.7757
FMOLP	0.8206	0.6000	0.8019
BBO	0.7395	0.8003	0.7648

From the above results, BBO provides improved results for the investors who are risk averse, and the FMOLP technique is best for those investors who are concerned with higher returns.

VI. CONCLUSION

This research developed a hybrid approach to examine the portfolio selection problem. The important components of the hybrid approach involve investor topology, cluster analysis, AHP, and optimization techniques. Cluster analysis is done using the EM algorithm, which gives a better fit to the data in the clusters. The TLBO algorithm is effective, simple and strong and has a great possibility for solving multi-objective problems. In FMOLP, multi-objectives are converted into a single objective problem. The “weighted adaptive approach” is applied for this conversion and the weights are assessed by the AHP or investor.

The BBO algorithm needs fewer parameters for solving the problem. The key factor of all three models is, if the investor’s desire is not fulfilled by the resultant portfolio then the new or improved portfolio can be achieved by changing the fitness value, number of iterations and weights of objective functions or recalculate the AHP model, based on the preferences of the decision-maker.

This research also presents a comparative study for the problem of portfolio selection. As lower risk/return ratio is the sign of better risk-return trade-off for investment, the BBO technique gives a better result as compared to TLBO and FMOLP. BBO provides improved results for the investors who are risk averse, and the FMOLP technique is best for those investors who are concerned with higher returns.

One more most advantageous technique is if the investors are dissatisfied with any of the portfolios, new portfolios can be created by changing the number of habitat.

It is predictable that the proposed models shall be contributing some constructive study for portfolio selection. The research covers portfolio optimization which can be of interest for business and finance.

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