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Some Topological and Modal Operators over the Intuitionistic Fuzzy Sets of Cube Root Type

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Abstract**:** ***In this paper, we defined newly topological operators and modal-like operators and negation over Intuitionistic Fuzzy Sets of Cube Root Type (IFSCRT) are proposed and few theorems are proved. In addition, some of the basic properties of the new operators are discussed***

Keywords **:** ***Intuitionistic Fuzzy Set, Intuitionistic Fuzzy Sets of Cube Root Type, Level Operator, Modal Operator, Topological Operator.***

I. INTRODUCTION

In 1965, Fuzzy sets theory was proposed by L.A. Zadeh[1]. In 1986, the concept of intuitionistic fuzzy sets (IFSs)where the information of membership function and non member function are consider, as a generalization of fuzzy set were introduced by K.T Atanassov[2]. Modal operators, level operator and topological operators are different groups of operators over Intuitionistic Fuzzy Set(IFS) were introduce by K.T Atanassov [2].After the introduction of IFS in terms of different operators, the study is expected to show interest of IFS to applied for uncertain data processing such as pattern recognition, fault diagnosis, reliability optimization of complex systems, machine learning, image processing, decision making and etc.

In this paper, our aim is to define different modal type operators over IFSCRT and to derive their properties. The rest of this paper are organized as follows. In Section 2, we summarize some definitions of IFS and their extensional set IFSCRT are definition. Diagrammatically representations of IFSCRT were shown in this section. In Section 3,we state some results of existing operators over IFSCRT. In the Section 4,we defines operators over on IFSCRT and derive their properties. Finally, we present the conclusion in Section 5.

II. PRELIMINARIES

In this section we recall some operators and definition on IFS are reviewed in this section which will be useful in further study of the paper. Let X be non empty set.

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Definition 2.1 (K.T.Atanassov, 1986)

An IFS A in X is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A(x) : X \rightarrow [0, 1]$ and $\nu_A(x) : X \rightarrow [0, 1]$ denote the membership and non-membership function of A respectively, and

$0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$

Furthermore, we have $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ called the intuitionistic fuzzy set index or hesitation margin of x in A. $\pi_A(x)$ is the degree of indeterminacy of $x \in X$ to the IFS A and $\pi_A(x) \in [0, 1]$ i.e $\pi_A(x) : X \rightarrow [0, 1]$ and $0 \leq \pi_A(x) \leq 1$ for each $x \in X$. $\pi_A(x)$ expresses the lack of knowledge of whether x belongs to IFS A or not.

Definition 2.2 (U Rizwan and I Nabeel , 2015)

An Intuitionistic Fuzzy Set of Cube Root Type(IFSCRT) A in X is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A(x) : X \rightarrow [0, 1]$ and $\nu_A(x) : X \rightarrow [0, 1]$ denote the membership and non-membership function of A respectively, and

$0 \leq \frac{\sqrt[3]{\mu_A(x)} + \sqrt[3]{\nu_A(x)}}{\sqrt{3}} \leq 1$ for each $x \in X$

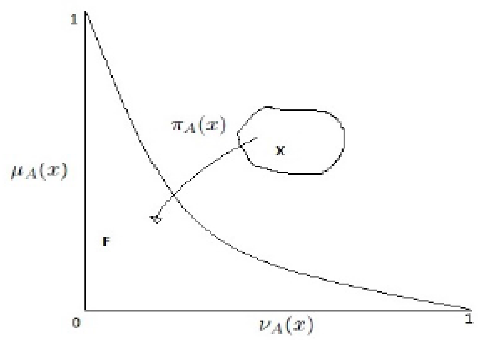

Fig. 1.Geometrical representation of IFSCRT

Definition 2.3 (U Rizwan and I Nabeel , 2015)

The degree of non determinacy (uncertainty) of an element $x \in X$ to the IFSCRT A is defined by

$$\pi_A(x) = \left(1 - \sqrt[3]{\mu_A(x)} - \sqrt[3]{\nu_A(x)}\right)^3$$

It can be easily shown that

$$\sqrt[3]{\mu_A(x)} + \sqrt[3]{\nu_A(x)} + \sqrt[3]{\pi_A(x)} = 1$$

Definition 2.4 (U Rizwan and I Nabeel , 2015)

Let X be a non empty set.Let A and B be two IFSCRTs such that

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$
$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$$

We define the following relations and operations

(i) $A \subset B$ if and only $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)\ \forall\ x \in X$

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(ii) $A \supset B$ if and if only $\mu_A(x) \geq \mu_B(x)$ and $\nu_A(x) \leq \nu_B(x)\ \forall\ x \in X$

(iii) $A = B$ if and if only $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)\ \forall\ x \in X$

(iv) $A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)),\ \max(\nu_A(x), \nu_B(x))\rangle : x \in X\}$

(v) $A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)),\ \min(\nu_A(x), \nu_B(x))\rangle : x \in X\}$

(vi) The complement of A is defined by

$$\bar{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$$

III. Remarks on the Intuitionistic Fuzzy Set of Cube Root Type

Let X denote a non empty finite set. Let $\alpha, \beta \in [0,1]$ and $A \in$ IFSCRT as

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

(i) $\square A = \left\{\langle x, \mu_A(x), \left(1 - \sqrt[3]{\mu_A(x)}\right)^3 \rangle : x \in X\right\}$ *(modal logic :necessity measure)*

(ii) $\Diamond A = \left\{\langle x, \left(1 - \sqrt[3]{\nu_A(x)}\right)^3, \nu_A(x) \rangle : x \in X\right\}$ *(modal logic :possibility measure)*

(iii) $C(A) = \{\langle x, K, L \rangle : x \in X\}$,

$K = \max_{y \in X} \mu_A(y)$, $L = \min_{y \in X} \nu_A(y)$ *(Topological: closure)*

(iv) $I(A) = \{\langle x, k, l \rangle : x \in X\}$,

$k = \min_{y \in X} \mu_A(y)$, $l = \max_{y \in X} \nu_A(y)$ *(Topological: intersection)*

(v) $D_\alpha(A) = \{\langle x, (\mu_A(x)^{1/3} + \alpha\pi_A(x)^{1/3})^3, (\nu_A(x)^{1/3} + (1-\alpha)\pi_A(x)^{1/3})^3\rangle : x \in X\}$

(vi) $F_{\alpha,\beta}(A) = \{\langle x, (\mu_A(x)^{1/3} + \alpha\pi_A(x)^{1/3})^3, (\nu_A(x)^{1/3} + \beta\pi_A(x)^{1/3})^3\rangle : x \in X\}$

(vii) $G_{\alpha,\beta}(A) = \left\{\langle x, \alpha^3\mu_A(x), \beta^3\nu_A(x)\rangle : x \in X\right\}$

Theorem 3.1 For every IFSCRT A and B, we have

(i) $\square\square A = \square A$

(ii) $\Diamond\Diamond A = \Diamond A$

(iii) $\square \Diamond A = \Diamond A$

(iv) $\Diamond \square A = \square A$

(v) $D_\alpha(\square A) = F_{\alpha,\beta}(\square A) = \square A$

(vi) $D_\alpha(\Diamond A) = F_{\alpha,\beta}(\Diamond A) = \Diamond A$

(vii) $D_\alpha(\bar{A}) = \overline{D_{1-\alpha}(A)}$

(viii) $G_{\alpha,\beta}(A \cap B) = G_{\alpha,\beta}(A) \cap G_{\alpha,\beta}(B)$

(ix) $G_{\alpha,\beta}(A \cup B) = G_{\alpha,\beta}(A) \cup G_{\alpha,\beta}(B)$

(x) $\pi_{D_\alpha(A)}(x) = 0$

(xi) $F_{\alpha,\beta}(D_\alpha(A)) = D_\alpha(A)$

Proof: Proof is obvious

IV. Main Result

Here we introduce new operators over the IFSCRT which extend two operators in the literature related to IFS. Let X is a non empty finite set

Definition 4.1

Let $\alpha, \beta \in [0,1]$ and $A \in$ IFSCRT, we define the operators as follows

(i) $d_\alpha(A) = \{\langle x, (\nu_A(x)^{1/3} + \alpha\pi_A(x)^{1/3})^3, (\mu_A(x)^{1/3} + (1-\alpha)\pi_A(x)^{1/3})^3\rangle : x \in X\}$

(ii) $f_{\alpha,\beta}(A) = \{\langle x, (\nu_A(x)^{1/3} + \alpha\pi_A(x)^{1/3})^3, (\mu_A(x)^{1/3} + \beta\pi_A(x)^{1/3})^3\rangle : x \in X\}$

(iii) $g_{\alpha,\beta}(A) = \left\{\langle x, \alpha^3\nu_A(x), \beta^3\mu_A(x)\rangle : x \in X\right\}$

(iv) $J_{\alpha,\beta}(A) = \{\langle x, (\mu_A(x)^{1/3} + \alpha\pi_A(x)^{1/3})^3, (\beta(\nu_A(x)^{1/3})^3\rangle : x \in X\}$

(v) $j_{\alpha,\beta}(A) = \{\langle x, (\nu_A(x)^{1/3} + \alpha\pi_A(x)^{1/3})^3, (\beta(\mu_A(x)^{1/3})^3\rangle : x \in X\}$

(vi) $H_{\alpha,\beta}(A) = \{\langle x, (\alpha\mu_A(x)^{1/3})^3, (\nu_A(x)^{\frac{1}{3}} + \beta\pi_A(x)^{1/3})^3\rangle : x \in X\}$

(vii) $h_{\alpha,\beta}(A) = \{\langle x, (\alpha\nu_A(x)^{1/3})^3, (\mu_A(x)^{\frac{1}{3}} + \beta\pi_A(x)^{1/3})^3\rangle : x \in X\}$

Theorem 4.2

For every $A \in$ IFSCRT and for every $\alpha, \beta \in [0,1]$, we have

(i) $d_\alpha(A) \in$ IFSCRT

(ii) $\alpha \leq \beta \Longrightarrow d_\alpha(A) \subset d_\beta(A)$

(iii) $d_0(A) = \overline{\Diamond A}$

(iv) $d_1(A) = \overline{\square A}$

(v) $d_\alpha(\square A) = \overline{\square A}$

(vi) $d_\alpha(\Diamond A) = \overline{\Diamond A}$

Proof

Proof of (i) and (ii) are obvious

(iii) Note that

$$d_0(A) = \{\langle x, (\nu_A(x)^{1/3} + 0\pi_A(x)^{1/3})^3, (\mu_A(x)^{1/3} + (1-0)\pi_A(x)^{1/3})^3 \rangle : x \in X\}$$
$$= \{\langle x, \nu_A(x), (\mu_A(x)^{1/3} + \pi_A(x)^{1/3})^3\rangle : x \in X\}$$

Since $\pi_A(x)^{1/3} = 1 - \nu_A(x)^{1/3} - \mu_A(x)^{1/3}$ we have

$$d_0(A) = \{\langle x, \nu_A(x), (1 - \nu_A(x)^{1/3})^3\rangle : x \in X\} = \overline{\Diamond A}$$

This proof is complete.

(iv) Follows by the definition that

$$d_1(A) = \{\langle x, (\nu_A(x)^{1/3} + 1\pi_A(x)^{1/3})^3, (\mu_A(x)^{1/3} + (1-1)\pi_A(x)^{1/3})^3 \rangle : x \in X\}$$
$$= \{\langle x, (\nu_A(x)^{1/3} + \pi_A(x)^{1/3})^3, \mu_A(x)\rangle : x \in X\}$$
$$= \{\langle x, (1 - \mu_A(x)^{1/3})^3, \mu_A(x)\rangle : x \in X\} = \overline{\square A}$$

(v) Note that

$$d_\alpha(\square A) = \{\langle x, (\nu_{\square A}(x)^{1/3} + \alpha\pi_{\square A}(x)^{1/3})^3, (\mu_{\square A}(x)^{1/3} + (1-\alpha)\pi_{\square A}(x)^{1/3})^3\rangle : x \in X\}$$

Since $\pi_{\square A}(x) = 0$, we have

$$d_\alpha(\square A) = \{\langle x, \nu_{\square A}(x), \mu_{\square A}(x)\rangle : x \in X\} = \overline{\square A}$$

The proof is complete. The proof of (vi) is similar to that of (v).

Theorem 4.3

For every $A \in$ IFSCRT and for every $\alpha, \beta \in [0,1]$ where $\alpha + \beta \leq 1$, we have

(i) $f_{\alpha,\beta}(A) \in$ IFSCRT

(ii) $0 \leq \gamma \leq \alpha \Rightarrow f_{\gamma,\beta}(A) \subset f_{\alpha,\beta}(A)$

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(iii) $0 \le \gamma \le \beta \Rightarrow f_{\alpha,\beta}(A) \subset f_{\alpha,\gamma}(A)$
(iv) $d_\alpha(A) = f_{\alpha,1-\alpha}(A)$
(v) $f_{0,1}(A) = \overline{\Diamond A}$
(vi) $f_{1,0}(A) = \overline{\Box A}$

(vii) $f_{\alpha,\beta}(A) = F_{\alpha,\beta}(\bar{A})$
(viii) $\overline{f_{\alpha,\beta}(\bar{A})} = f_{\beta,\alpha}(A)$
(ix) $f_{0,0}(A) = \bar{A}$
(x) $f_{\alpha,\beta}(\Box A) = \overline{\Box A}$
(xi) $f_{\alpha,\beta}(\Diamond A) = \overline{\Diamond A}$

Proof

(i) Follows by noting that

$$\mu_{f_{\alpha,\beta}(A)}(x)^{1/3} + \nu_{f_{\alpha,\beta}(A)}(x)^{1/3} = [(\nu_A(x)^{\frac{1}{3}}+\alpha\pi_A(x)^{\frac{1}{3}})^3]^{\frac{1}{3}} +[\mu_A(x)^{1/3}+\beta\pi_A(x)^{\frac{1}{3}})^3]^{\frac{1}{3}}$$
$$= \nu_A(x)^{\frac{1}{3}} + \mu_A(x)^{1/3} + \pi_A(x)^{\frac{1}{3}}(\alpha+\beta)$$
$$\le \nu_A(x)^{\frac{1}{3}} + \mu_A(x)^{1/3} + \pi_A(x)^{\frac{1}{3}} = 1.$$

Proofs of (ii) and (iii) are obvious

(vi) Follows by noting that

$$f_{\alpha,1-\alpha}(A) = \{\langle x,(\nu_A(x)^{1/3} + \alpha\pi_A(x)^{1/3})^3, (\mu_A(x)^{1/3} + (1-\alpha)\pi_A(x)^{1/3})^3\rangle : x \in X\} = d_\alpha(A)$$

(v) By (vi) we have $f_{0,1}(A) = d_0(A)$ and $f_{1,0}(A) = d_1(A)$

It follows by theorem 4.2-(iii) that $f_{0,1}(A) = \overline{\Diamond A}$.

(vi) It follows by the theorem 4.2-(iv) that $f_{0,1}(A) = \overline{\Box A}$.

(viii) since

$$f_{\beta,\alpha}(A) = \{\langle x,(\nu_A(x)^{1/3} + \beta\pi_A(x)^{1/3})^3, (\mu_A(x)^{1/3} + \alpha\pi_A(x)^{1/3})^3\rangle : x \in X\}$$

$$f_{\beta,\alpha}(\bar{A}) = \{\langle x,(\mu_A(x)^{1/3} + \alpha\pi_A(x)^{1/3})^3, (\nu_A(x)^{1/3} + \beta\pi_A(x)^{1/3})^3\rangle : x \in X\}$$

we have

$$\overline{f_{\beta,\alpha}(\bar{A})} = \{\langle x,(\nu_A(x)^{1/3} + \beta\pi_A(x)^{1/3})^3, (\mu_A(x)^{1/3} + \alpha\pi_A(x)^{1/3})^3\rangle : x \in X\}$$

$$\overline{f_{\beta,\alpha}(\bar{A})} = f_{\beta,\alpha}(A).$$

Since $\pi_{\Box A}(x) = 0$ and $\pi_{\Diamond A}(x) = 0$, the proof (vii),(x) and (xi) are obvious.

Theorem 4.4

For every A∈ IFSCRT and real number $\alpha, \beta, \gamma \in [0,1]$

(i) $g_{\alpha,\beta}(A)$ ∈IFSCRT
(ii) $\alpha \le \gamma \Rightarrow g_{\alpha,\beta}(A) \subset g_{\gamma,\beta}(A)$
(iii) $\beta \le \gamma \Rightarrow g_{\alpha,\beta}(A) \subset g_{\alpha,\gamma}(A)$
(iv $g_{\alpha,\beta}(C(A)) = I(g_{\alpha,\beta}(A))$
(v) $g_{\alpha,\beta}(I(A)) = C(g_{\alpha,\beta}(A))$
(vi) $\overline{g_{\alpha,\beta}(A)} = g_{\beta,\alpha}(A)$
(vii) $g_{1,1}(A) = \bar{A}$
(viii) $g_{\alpha,\beta}(A \cap B) = g_{\alpha,\beta}(A) \cap g_{\alpha,\beta}(B)$
(ix) $g_{\alpha,\beta}(A \cup B) = g_{\alpha,\beta}(A) \cup g_{\alpha,\beta}(B)$

Proof

(i) Since

$$g_{\alpha,\beta}(A) = \{\langle x, \alpha^3\nu_A(x), \beta^3\mu_A(x)\rangle : x \in X\}$$

$$\mu_{g_{\alpha,\beta}(A)}(x)^{1/3} + \nu_{g_{\alpha,\beta}(A)}(x)^{1/3} = (\alpha^3\nu_A(x))^{\frac{1}{3}} + (\beta^3\mu_A(x))^{\frac{1}{3}}$$
$$= \alpha\nu_A(x)^{1/3} + \beta\mu_A(x)^{1/3}$$
$$\le \nu_A(x)^{1/3} + \mu_A(x)^{1/3} \le 1$$

We have $g_{\alpha,\beta}(A)$ ∈IFSCRT

(ii) Note that

$$g_{\alpha,\beta}(A) = \{\langle x, \alpha^3\nu_A(x), \beta^3\mu_A(x)\rangle : x \in X\}$$

and

$$g_{\gamma,\beta}(A) = \{\langle x, \gamma^3\nu_A(x), \beta^3\mu_A(x)\rangle : x \in X\}$$

Since $\alpha \le \gamma$ we have $\alpha^3 \le \gamma^3, \alpha^3\nu_A(x) \le \gamma^3\nu_A(x)$ and so

$$g_{\alpha,\beta}(A) \subset g_{\gamma,\beta}(A)$$

The proof of (iii) is similar to that (ii)

(iv) Follow by definition that

$$C(A) = \{\langle x, \max_{y\in X}\mu_A(y), \min_{y\in X}\nu_A(y)\rangle : x \in X\},$$

and

$$g_{\alpha,\beta}(C(A)) = \{\langle x, \alpha^3\min_{y\in X}\nu_A(y), \beta^3\max_{y\in X}\mu_A(y)\rangle : x \in X\},$$
$$= I(g_{\alpha,\beta}(A))$$

(v) Follows by nothing that

$$I(A) = \{\langle x, \min_{y\in X}\mu_A(y), \max_{y\in X}\nu_A(y)\rangle : x \in X\},$$

$$g_{\alpha,\beta}(I(A)) = \{\langle x, \alpha^3\max_{y\in X}\nu_A(y), \beta^3\min_{y\in X}\mu_A(y)\rangle : x \in X\}$$
$$= \{\langle x, \max_{y\in X}\alpha^3\nu_A(y), \min_{y\in X}\beta^3\mu_A(y)\rangle : x \in X\}$$

$$= C(g_{\alpha,\beta}(A)), \alpha,\beta \in [0,1]$$

(vi)Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ be a IFSCRT. Then

$$\bar{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$$
$$g_{\beta,\alpha}(A) = \{\langle x, \beta^3\nu_A(x), \alpha^3\mu_A(x)\rangle : x \in X\}$$
$$g_{\alpha,\beta}(\bar{A}) = \{\langle x, \alpha^3\nu_A(x), \beta^3\mu_A(x)\rangle : x \in X\}$$

and

$$\overline{g_{\alpha,\beta}(\bar{A})} = \{\langle x, \beta^3\nu_A(x), \alpha^3\mu_A(x)\rangle : x \in X\}$$

So $\overline{g_{\alpha,\beta}(A)} = g_{\beta,\alpha}(A)$ .Proof of (vii),(viii) and (ix) are obvious

Theorem 4.5

For every A∈ IFSCRT and real number $\alpha, \beta, \gamma \in [0,1]$

(i) $J_{\alpha,\beta}(A)$ ∈IFSCRT
(ii) $\alpha \le \gamma \Rightarrow J_{\alpha,\beta}(A) \subset J_{\gamma,\beta}(A)$
(iii) $\beta \le \gamma \Rightarrow J_{\alpha,\beta}(A) \subset J_{\alpha,\gamma}(A)$
(iv) $\Diamond A = J_{1,1}(A)$
(v) $A = J_{0,1}(A)$

Proof

(i) Since

$$J_{\alpha,\beta}(A) = \{\langle x,(\mu_A(x)^{1/3} + \alpha\pi_A(x)^{1/3})^3, (\beta(\nu_A(x)^{1/3})^3\rangle : x \in X\}$$

and

$$\mu_{J_{\alpha,\beta}(A)}(x)^{1/3} + \nu_{J_{\alpha,\beta}(A)}(x)^{1/3}$$
$$= ((\mu_A(x)^{1/3} + \alpha\pi_A(x)^{1/3})^3)^{1/3} + (\beta\nu_A(x))^{1/3}$$
$$\le \nu_A(x)^{1/3} + \mu_A(x)^{1/3} + \pi_A(x)^{1/3} = 1$$

we have $J_{\alpha,\beta}(A)$ ∈ IFSCRT

(ii) Note that

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$$J_{\alpha,\beta}(A) = \{\langle x,(\mu_A(x)^{1/3} + \alpha\pi_A(x)^{1/3})^3, (\beta(\nu_A(x)^{1/3})^3\rangle : x \in X\}$$

And

$$J_{\gamma,\beta}(A) = \{\langle x,(\mu_A(x)^{1/3} + \gamma\pi_A(x)^{1/3})^3, (\beta(\nu_A(x)^{1/3})^3\rangle : x \in X\}$$

Since $\alpha \leq \gamma$ we have

$$(\mu_A(x)^{1/3} + \alpha\pi_A(x)^{1/3})^3 \leq (\mu_A(x)^{1/3} + \gamma\pi_A(x)^{1/3})^3$$

and so $J_{\alpha,\beta}(A) \subset J_{\gamma,\beta}(A)$

The proof of (iii) is similar to that (ii).Proofs of (iv) and (v) are obvious.

Theorem 4.6

For every A∈ IFSCRT and real number $\alpha, \beta, \gamma \in [0,1]$

(i) $j_{\alpha,\beta}(A)$ ∈IFSCRT

(ii) $\alpha \leq \gamma \Rightarrow j_{\alpha,\beta}(A) \subset j_{\gamma,\beta}(A)$

(iii) $\beta \leq \gamma \Rightarrow j_{\alpha,\beta}(A) \subset j_{\alpha,\gamma}(A)$

(iv) $j_{\alpha,\beta}(\bar{A}) = j_{\alpha,\beta}(A)$

(v) $\overline{\Box A} = j_{1,1}(A)$

(vi) $\bar{A} = j_{0,1}(A)$

Proof

(i) Since

$$j_{\alpha,\beta}(A) = \{\langle x,(\nu_A(x)^{1/3} + \alpha\pi_A(x)^{1/3})^3, (\beta(\mu_A(x)^{1/3})^3\rangle : x \in X\}$$

and

$$\mu_{j_{\alpha,\beta}(A)}(x)^{1/3} + \nu_{j_{\alpha,\beta}(A)}(x)^{1/3} = \left((\nu_A(x)^{1/3} + \alpha\pi_A(x)^{1/3})^3\right)^{1/3} + \left(\beta\mu_A(x)\right)^{1/3}$$
$$= \nu_A(x)^{1/3} + \alpha\pi_A(x)^{1/3} + \beta\mu_A(x)^{1/3}$$
$$\leq \nu_A(x)^{1/3} + \mu_A(x)^{1/3} + \pi_A(x)^{1/3} = 1$$

Finally, it can be concluded that $j_{\alpha,\beta}(A)$ ∈IFSCRT

Note that

$$j_{\alpha,\beta}(A) = \{\langle x,(\nu_A(x)^{1/3} + \alpha\pi_A(x)^{1/3})^3, (\beta(\mu_A(x)^{1/3})^3\rangle : x \in X\}$$

and

$$j_{\gamma,\beta}(A) = \{\langle x,(\nu_A(x)^{1/3} + \gamma\pi_A(x)^{1/3})^3, (\beta(\mu_A(x)^{1/3})^3\rangle : x \in X\}$$

Since $\alpha \leq \gamma$ we have

$$(\nu_A(x)^{1/3} + \alpha\pi_A(x)^{1/3})^3 \leq (\nu_A(x)^{1/3} + \gamma\pi_A(x)^{1/3})^3$$

and so $j_{\alpha,\beta}(A) \subset j_{\gamma,\beta}(A)$

The proof of (iii) is similar to that (ii).Proofs of (iv), (v) and (vi) are obvious.

Theorem 4.7

For every A∈ IFSCRT and real number $\alpha, \beta, \gamma \in [0,1]$

(i) $H_{\alpha,\beta}(A)$ ∈IFSCRT

(ii) $\alpha \leq \gamma \Rightarrow H_{\alpha,\beta}(A) \subset H_{\gamma,\beta}(A)$

(iii) $\beta \leq \gamma \Rightarrow H_{\alpha,\beta}(A) \subset H_{\alpha,\gamma}(A)$

(iv) $H_{1,0}(A) = A$

Proof (i) Since

$$H_{\alpha,\beta}(A) = \{\langle x,(\alpha\mu_A(x)^{1/3})^3, (\nu_A(x)^{\frac{1}{3}} + \beta\pi_A(x)^{1/3})^3\rangle : x \in X\}$$

and

$$\mu_{H_{\alpha,\beta}(A)}(x)^{1/3} + \nu_{H_{\alpha,\beta}(A)}(x)^{1/3} = \left((\alpha\mu_A(x)^{1/3})^3\right)^{1/3} + \left((\nu_A(x)^{1/3} + \beta\pi_A(x)^{1/3})^3\right)^{1/3}$$
$$= \nu_A(x)^{1/3} + \alpha\mu_A(x)^{1/3} + \beta\pi_A(x)^{1/3}$$
$$\leq \nu_A(x)^{1/3} + \mu_A(x)^{1/3} + \pi_A(x)^{1/3} = 1$$

we have $H_{\alpha,\beta}(A) \in$ IFSCRT

(ii) Note that

$$H_{\alpha,\beta}(A) = \{\langle x,(\alpha\mu_A(x)^{1/3})^3, (\nu_A(x)^{\frac{1}{3}} + \beta\pi_A(x)^{1/3})^3\rangle : x \in X\}$$

and

$$H_{\gamma,\beta}(A) = \{\langle x,(\gamma\mu_A(x)^{1/3})^3, (\nu_A(x)^{\frac{1}{3}} + \beta\pi_A(x)^{1/3})^3\rangle : x \in X\}$$

Since $\alpha \leq \gamma$ we have

$$\left(\alpha\mu_A(x)^{1/3}\right)^3 \leq \left(\gamma\mu_A(x)^{1/3}\right)^3$$

and so $H_{\alpha,\beta}(A) \subset H_{\gamma,\beta}(A)$

The proof of (iii) is similar to that (ii).Proofs of (iv) are obvious.

Theorem 4.8

For every A∈ IFSCRT and real number $\alpha, \beta, \gamma \in [0,1]$

(i) $h_{\alpha,\beta}(A)$ ∈IFSCRT

(ii) $\alpha \leq \gamma \Rightarrow h_{\alpha,\beta}(A) \subset h_{\gamma,\beta}(A)$

(iii) $\beta \leq \gamma \Rightarrow h_{\alpha,\beta}(A) \subset h_{\alpha,\gamma}(A)$

(iv) $h_{\alpha,\beta}(\bar{A}) = H_{\alpha,\beta}(A)$

(v) $h_{1,0}(A) = \bar{A}$

(vi) $h_{1,1}(A) = \overline{\Diamond A}$

Proof (i) Since

$$h_{\alpha,\beta}(A) = \{\langle x,(\alpha\nu_A(x)^{1/3})^3, (\mu_A(x)^{\frac{1}{3}} + \beta\pi_A(x)^{1/3})^3\rangle : x \in X\}$$

and

$$\mu_{h_{\alpha,\beta}(A)}(x)^{1/3} + \nu_{h_{\alpha,\beta}(A)}(x)^{1/3} = \left((\alpha\nu_A(x)^{1/3})^3\right)^{1/3} + \left((\mu_A(x)^{1/3} + \beta\pi_A(x)^{1/3})^3\right)^{1/3}$$
$$= \mu_A(x)^{1/3} + \alpha\nu_A(x)^{1/3} + \beta\pi_A(x)^{1/3}$$
$$\leq \nu_A(x)^{1/3} + \mu_A(x)^{1/3} + \pi_A(x)^{1/3} = 1$$

we have $h_{\alpha,\beta}(A) \in$ IFSCRT

(ii) Note that

$$h_{\alpha,\beta}(A) = \{\langle x,(\alpha\nu_A(x)^{1/3})^3, (\mu_A(x)^{\frac{1}{3}} + \beta\pi_A(x)^{1/3})^3\rangle : x \in X\}$$

and

$$h_{\gamma,\beta}(A) = \{\langle x,(\gamma\nu_A(x)^{1/3})^3, (\mu_A(x)^{\frac{1}{3}} + \beta\pi_A(x)^{1/3})^3\rangle : x \in X\}$$

Since $\alpha \leq \gamma$ we have

$$\left(\alpha\nu_A(x)^{1/3}\right)^3 \leq \left(\gamma\nu_A(x)^{1/3}\right)^3$$

and so $h_{\alpha,\beta}(A) \subset h_{\gamma,\beta}(A)$

The proof of (iii) is similar to that (ii).Proofs of (iv),(v) and (vi) are obvious.

V. CONCLUSION

We have defined level operators and modal-like operators over IFSCRT and proved their relationships. We have studied some desirable properties of the proposed operations such as closure, intersection, negation, necessity, possibility and topological over IFSCRT.

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