

Generative Aspects of Oxide Pictures by Oxide Tile Rewriting Grammar



P. Helen Vijitha, Robinson Thamburaj

Abstract: In formal languages, picture language is generalization of string language theory to two dimensions. Pictures which may be regarded as two-dimensional objects occur in studies concerning recognition of patterns, images and various computational fields. Several studies have been done for generating and/or recognizing higher dimensional objects using formal models. Tile rewriting grammar (TRG) is yet another model introduced for generating picture languages. TRG combines isometric rewriting rules with the Giammaresi and Restivo's Tiling system. This rewriting grammar generates spirals, square and rectangular grids. The power of generating pictures by tile rewriting grammar is more than REC. Sweety et al have generated hexagonal pictures, introducing hexagonal Tile Rewriting Grammar. Kuberalet al have introduced Triangular Tile Rewriting Grammar to generate Triangular Pictures.

A special class of objects namely Oxide pictures have been of interest recently. Oxide network is a special case of Silicate network. The silicates are a complicated class of minerals made up of tetrahedral silicates. A basic silicate tetrahedron unit SiO_4 is formed with Oxygen ions in the corners and a Silicate ion in the center. In a two dimensional plane a ring of tetrahedrons that are shared by Oxygen nodes forms a silicate sheet. In this paper, Oxide Tile Rewriting Grammar (OXTRG) is proposed for generating Oxide pictures. The motivation for the study is derived from the Oxide network which is obtained by deleting all the silicon nodes of a silicate network. Closure properties of OXTRG are discussed. When compared with schemes such as Oxide Tiling System and Oxide Sgraffito Automaton, OXTRG is found to be more powerful.

Keywords: Oxide pictures, Oxide tiling systems, Oxide tile rewriting grammars, Oxide Sgraffito automata.

I. INTRODUCTION

In formal languages images can be generated using two dimensional grammars. Two dimensional grammars can be classified as array grammars and matrix grammars respectively.

An array of elements can be viewed as a One to One correspondence from the integer-coordinate set into the alphabets, where all but finitely many integer-coordinates are mapped into # ("blank"). An array grammar replaces sub arrays by sub arrays, whereas substrings are replaced by substrings in a string grammar. A problem arises, when the replacement still needs to be an array. If two sub arrays, call them A and B are not identical in rule $A \rightarrow B$, an undesirable non-local changes may occur in the host array.

This problem do not arise in string problem; even if the length of the substring that needs to be replaced is not of the same length. This difficulty is solved by requiring A and B in any array grammar production $A \rightarrow B$ to be geometrically identical. A grammar of this type is called "isometric".

Rosenfeld[11] introduced the matrix grammars, imposing the constraint that in a rewriting rule the right and left parts must be geometrically identical arrays (isometric arrays); this constraint trounces the problem of "shearing". Siromoney's array grammar [4][10][13] is sequential and also parallel. Using horizontal productions, a string of non terminals is derived sequentially and then applying vertical productions in parallel, the vertical derivations proceed. Matz introduced the context-free picture grammar [8]. The right hand part of the grammar is a two dimensional regular expression. Matz's grammar depends on column concatenation, row concatenation and their closures. As row concatenation is defined only on pictures of identical width, the problem of shearing is avoided. CrespiReghizzi and Pradella [14] introduced tile rewriting grammars. The context-free grammars for one dimensional language are extended to two dimensions. Tile rewriting grammar (TRG) is a newly introduced model that generate images combining Rosenfeld's rewriting rules with the Giammarresi and Restivo's tiling system [1,3]. The rules of TRG have a non terminal alphabet in the left and a set of tiles over terminals and non terminals in the right part. This rule replaces the rectangular sub picture of the current picture with a geometrically identical (isometric) rectangle that belongs to the local picture language. This generates spirals, square and rectangular grids. This is more powerful in generating picture than Recognizable languages. Sweety et al [15] have introduced and defined Hexagonal Tile Rewriting Grammar to generate hexagonal pictures. A HTRG (hexagonal tile rewriting grammar) is a four tuple $(N, \Sigma \cup \{b\}, S, R)$, where

N a set of nonterminals is, $\Sigma \cup \{b\}$ is the end alphabet that includes the boundary symbol, S the starting element and R is a set of rules that contains two types of rules. Type 1 matches a sub picture of bounded size that is geometrically same as the right part, type 2 is intended to match any sized sub picture that is tiled with all the alphabets of the given set. Kuberalet al [7] have introduced Triangular Tile Rewriting Grammars (TTRG). A Triangular Tile Rewriting Grammar is a four tuple (N, Σ, S, R) . R consists of rules of fixed size and Variable size rules. A fixed size rule matches a bounded sub picture that is similar with the right side of the rule. A variable size rule matches of any sized subpicture. In this paper, we have introduced Oxide tile rewriting grammar (OXTRG) for generating Oxide pictures. We have compared OXTRG with Oxide tiling system. We have also proved that this model generates Oxide pictures with greater power than Oxide tiling system.

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II. PRELIMINARIES

We recall some basic definitions of Oxide pictures, Oxide Tiling system and Oxide Sgraffito automata from [5][6]. The silicates are the most complicated class of minerals. SiO₄ tetrahedra (Fig. 1) with oxygen ions in the corner vertices and silicate ion in the center vertex form the basic unit of silicates. In a two dimensional plane a ring of tetrahedrons is linked by oxygen nodes to other rings that forms a silicate sheet. A silicate network is a fixed parallel interconnection of silicate sheets.

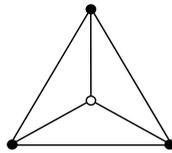


Fig. 1. SiO₄ tetrahedra

Oxide Network is a new network obtained by deleting all the silicon nodes from a silicate network. An *n*-dimensional oxide network is denoted by *OX* (*n*). A coordinate system for Oxide network assigns an *id* to each Oxygen node. $\alpha = 0, \beta = 0, \text{ and } \gamma = 0$ are the three coordinate axes, α -lines, β -lines and γ -lines are the lines parallel to the coordinate axes that are at mutual angle of 120 degrees between any two of them[9,16].

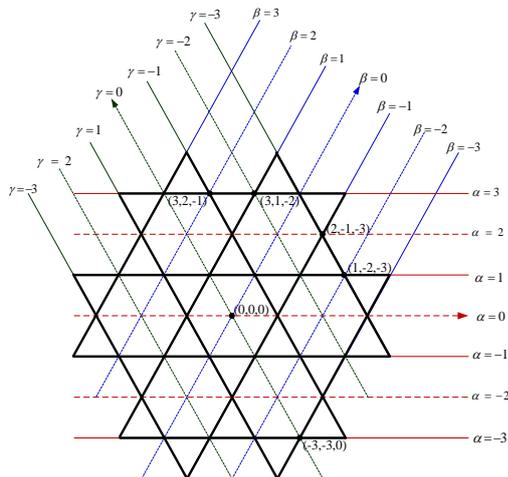


Fig. 2. Coordinate System in Oxide Networks

Definition 2.1: An Oxide picture *OX_p* is an oxide assemblage over Σ . Such set of Oxide pictures of symbols of Σ may be represented by Σ^{**OX_p} . An Oxide picture language over the finite element Σ is a subset of Σ^{**OX_p} .

Example 2.1.

An Oxide picture over the elements {a, b, c} is shown in Fig. 1

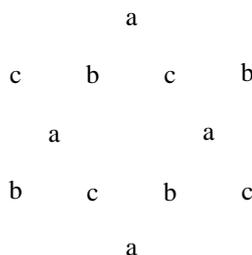


Fig. 1. An Oxide Picture

Definition 2.2: Let Σ and Γ be two finite elements and $OX_p \in \Gamma^{**OX_p}$ be an Oxide picture. The projection of OX_p by a mapping π is the Oxide picture $OX'_p \in \Sigma^{**OX_p}$ such that $OX'_p = \pi(OX_p(a, b, c))$

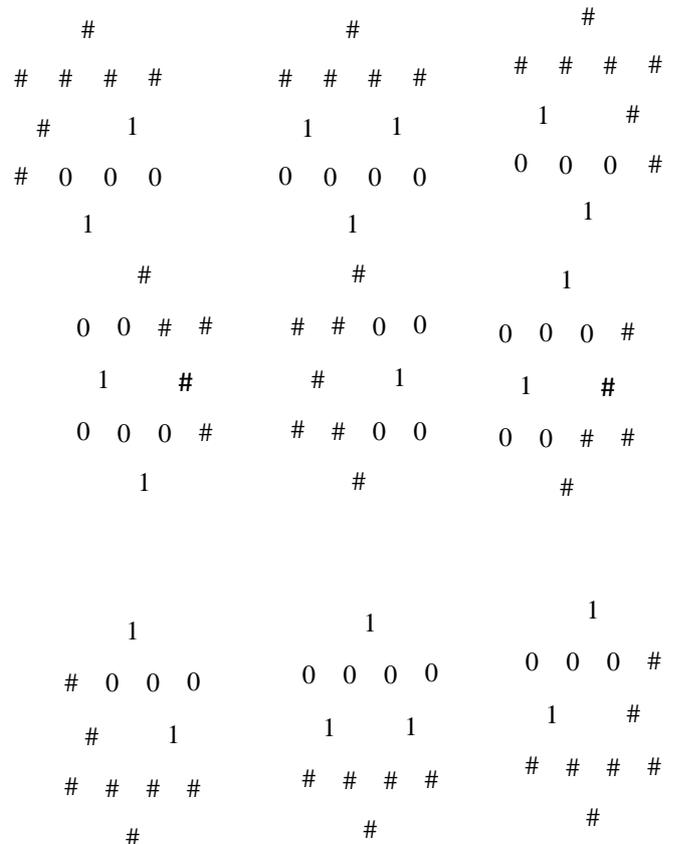
Definition 2.3: Consider an Oxide picture language $L \subseteq \Gamma^{**OX_p}$. The projection of L by a mapping π is $L' = \{OX'_p / OX'_p = \pi(OX_p), \forall OX_p \in L\} \subseteq \Sigma^{**OX_p}$

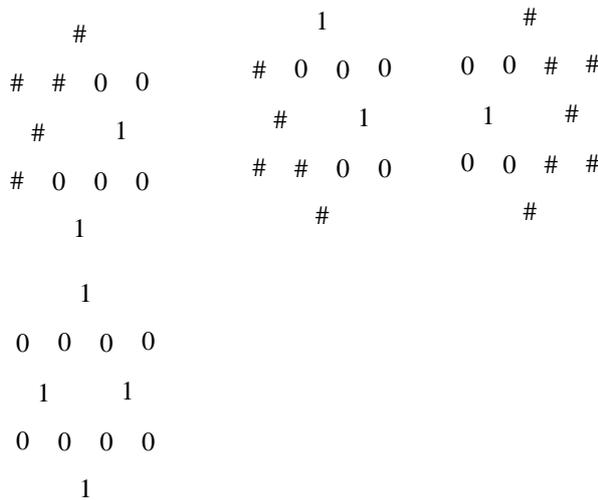
We get bordered version of Oxide picture say $O\hat{X}_p$ when $\# \notin \Sigma$ is added to the Oxide picture as boundary.

Definition 2.4: An Oxide picture language is local if a set of finite blocks of Star of David tiles is present in $\Gamma \cup \{\#\}$ such a way that $L = \{OX_p \in \Gamma^{**OX_p} / OX_p(1) \subseteq S(\theta)\}$.

Example 2.2 Let $\Gamma = \{0, 1\}$ and let $S(\theta)$ be a set of Star of David tiles over Γ .

$L(S(\theta))$ is the local Oxide picture language of oxide pictures that have '1' in the position of even ' α ' and '0' in the remaining position.





Definition 2.5: An Oxide picture language is recognizable by an Oxide Tiling system if there exists a local Oxide picture language $L(S(\theta))$ over an element Γ and π , a mapping defined from $\Gamma \rightarrow \Sigma$ such that the projection of L is $(L(\theta))$.

Definition 2.6: An oxide tiling system is a 4 tuple $(\Gamma, \Sigma, \pi, S(\theta))$ where Γ and Σ are finite elements. $\pi : \Gamma \rightarrow \Sigma$ is a protrusion. $S(\theta)$ is a set of Star of David tiles in $\Gamma \cup \{\#\}$

In the visual arts, the Italian word sgraffito meaning scratched is a technique used in painting glass and pottery, that puts down a basic surface, covers or color it with another color, and then scratch the top layer so that the resulting shape or pattern will be of the lighter color.

This classical art which had contributed much during the years of renaissance has been the motivation for introducing sgraffito automata to recognize pictures.

A computing device – sgraffito automaton[2] that recognize pictures, works with a finite control on a picture with elements of distinct weights and rewrite the symbol it visits with a lighter weight symbol.(similar to the technique of scratching the superficial layers in the visual arts).

Definition 2.7: A sgraffito automata on an Oxide picture is a seven tuple $SOX = (\Sigma, \Gamma, Q, q_0, Q_F, \delta, \mu)$, $(\Sigma, \Gamma, Q, q_0, Q_F, \delta, \mu)$ is a Turing machine that recognizes Oxide pictures and μ is a weight function.

A SOX moves its head in the directions determined by the Turing machine based on its position. In each step the scanned symbol is rewritten by a lighter symbol only. If the automaton enters into a final state, it is accepted.

III. OXIDE TILE REWRITING GRAMMAR

We define Oxide tile rewriting grammars (OXTRG) to generate Oxide pictures.

Definition 3.1: An Oxide Tile Rewriting grammar is a seven tuple $(N_T, \Sigma_T, S, S_L, S_R, S_{UR}, R_T)$ where

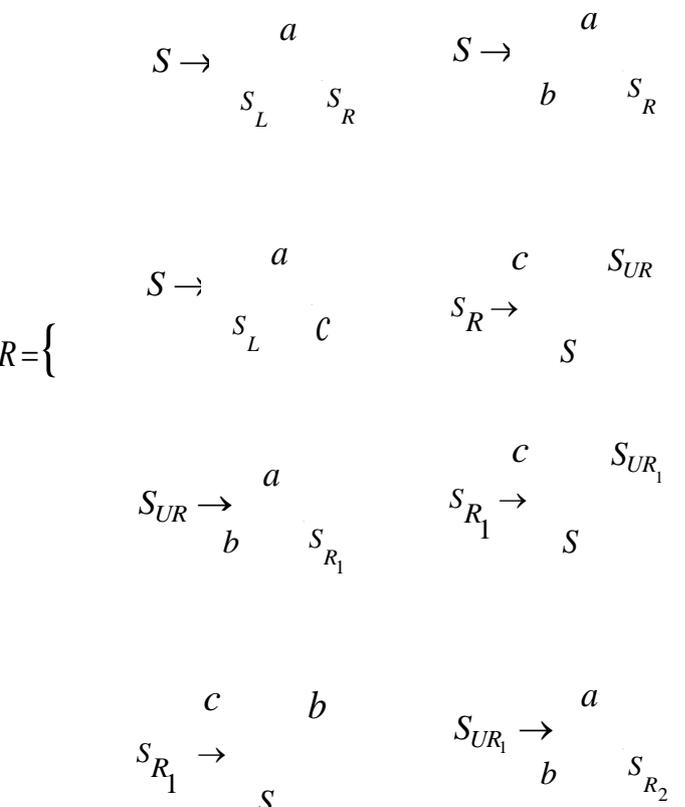
N_T - set of nonterminals ,
 Σ_T - Terminal elements,
 S - starting A array (triangular) of elements over Σ ,
 S_L, S_R
 S_L - a scheme that have V array(triangular) of elements over terminals and nonterminals to the left .
 S_R - a scheme that have V array(triangular) of elements over terminals and nonterminals to the right .
 R_T - a set of rules which assign to the left a nonterminal elements and triangular arrays (A array and V array) of elements on nonterminal and terminal symbols to the right.
 The nonterminal element in the left side of R_T denotes homogeneous triangular arrays (A array and V array).

Example 3.1
 Consider an Oxide Tile Rewriting Grammar $(N_T, \Sigma_T, S, S_L, S_R, S_{UR}, R_T)$ where

$$\Sigma_T = \{a, b, c\}$$

$$N_T = \{S, S_L, S_{UR}\}$$

S_R - V array of elements over terminal and nonterminal symbols.



$$S_{R_2} \rightarrow \begin{matrix} c & S_{UR} \\ a & \end{matrix} \quad S_{R_2} \rightarrow \begin{matrix} c & b \\ S & \end{matrix}$$

$$S_{R_3} \rightarrow \begin{matrix} b & c & b & S_{UR} \\ S & & S & \end{matrix}$$

$$S_L \rightarrow \begin{matrix} c & b \\ S & \end{matrix} \quad S_R \rightarrow \begin{matrix} c & b \\ S & \end{matrix}$$

$$S_R \rightarrow \begin{matrix} c & b \\ a & \end{matrix}$$

We get an Oxide picture language such that $L = \{\text{Set of Oxide picture of size 'n' that have 'a' in the position of even '}\alpha\text{' , 'b' and 'c' in the remaining position.}\}$

Example 3.2: $L = \{\text{Set of Oxide picture of size '2n' that have 'a' in the position of even '}\alpha\text{' , 'b' and 'c' in the remaining position.}\}$

Removing the rule $S_{R_1} \rightarrow \begin{matrix} c & b \\ S & \end{matrix}$

from Example 3.1, we generate Oxide pictures of size '2n'.

Example 3.3: $L = \{\text{Set of Oxide picture of size '2n+1' that have 'a' in the position of even '}\alpha\text{' , 'b' and 'c' in the remaining position.}\}$

Remove the rule $S_{R_2} \rightarrow \begin{matrix} c & b \\ S & \end{matrix}$

We generate Oxide pictures of size 3,5,7...

Example 3.4: $L = \{\text{Set of Oxide picture of size '3n' that have 'a' in the position of even '}\alpha\text{' , 'b' and 'c' in the remaining position.}\}$

Remove the rule $S_{R_2} \rightarrow \begin{matrix} c & b \\ S & \end{matrix}$

rule from Example 3.1 and insert the rules

$$S_{R_2} \rightarrow \begin{matrix} c & S_{UR} \\ S & \end{matrix} \quad S_{UR_2} \rightarrow \begin{matrix} a & \\ b & S_{R_3} \end{matrix}$$

We generate Oxide pictures of size 3,6,9...

Theorem 3.1 The family $L(\text{OXT RG})$ is closed under union, rotation about 180° and alphabetical projection.

Proof:

Consider grammars $G_{T_1} = (\Sigma_{T_1}, N_{T_1}, S_{L_A}, S_{R_A}, S_{UR_A}, R_{T_1})$ and $G_{T_2} = (\Sigma_{T_2}, N_{T_2}, S_{L_B}, S_{R_B}, S_{UR_B}, R_{T_2})$. For simplicity we suppose that $N_{T_1} \cap N_{T_2} = \emptyset$, the starting symbol

$$S \notin N_{T_1} \cup N_{T_2}$$

G_{T_1}, G_{T_2} produces Oxide pictures with size atleast 2. Then it

is easy to show that the Oxide tile rewriting grammar $G_T = (\Sigma_T, N_T \cup N_{T_1} \cup N_{T_2}, \{S\}, S_{L_A}, S_{R_A}, S_{UR_A}, S_{L_B}, S_{R_B}, S_{UR_B}, R_{T_1} \cup R_{T_2})$ where Union \cup :

$$R_T = \left\{ \begin{matrix} S \rightarrow \begin{matrix} a & \\ S_{L_A} & S_{R_A} \end{matrix} \quad S \rightarrow \begin{matrix} b & \\ S_{L_B} & S_{R_B} \end{matrix} \end{matrix} \right.$$

is in such a way that $L(G_T) = L(G_{T_1}) \cup L(G_{T_2})$.

Rotation R about 180° :

Consider the grammar $(N_T, \Sigma_T, S, S_L, S_R, S_{UR}, R_T)$. It is easy to verify that $L(G_T) = L(G_T)^R$.

Projection π :

Consider a grammar $G_{T_1} = (\Sigma_{T_1}, N_{T_1}, S_L, S_R, S_{UR}, R_{T_1})$, and a projection $\pi : \Sigma_1 \rightarrow \Sigma_2$. We construct $G_{T_2} = (\Sigma_{T_2}, N_{T_2}, S_L, S_R, S_{UR}, R_{T_2})$, such that

$L(G_{T_2}) = \pi L(G_{T_1})$: Let Σ_{T_2} be a new set of nonterminals in correspondence with elements of Σ_{T_1} . Then



$N_{T_2} = N_{T_1} \cup \Sigma_{T_2} \cup R_{T_2} = \phi(R_{T_1}) \cup R_T$ and $\phi: \Sigma_{T_1} \rightarrow \Sigma_{T_2}$,
 $\pi(a_1) = a_2$. It can be extended to rules of OXTRG.

IV. COMPARISON OF OXIDE TILE REWRITING GRAMMAR WITH OTHER MODELS

we compare Oxide tiling systems, Oxide wang system and Oxide Sgraffito automata with Oxide Tile Rewriting Grammar .

The comparison of Oxide Tile Rewriting Grammar with Oxide tiling systems and Oxide wang system has few difficulties: OXTS are defined by local Oxide languages with # (boundary elements). This is not present in OXTRG. For this first we show that a class of local Oxide languages is included strictly in L(OXTRG).

Lemma 4.1

$$L(\text{LOCu,eq}(\text{OX}(n))) \subseteq L(\text{OXTRG})$$

Proof:

Consider $L(\theta)$, a local Oxide picture language (without boundaries) in Σ that is defined by a set of allowed star of David tiles $(\{\theta_1, \theta_2, \theta_3 \dots \theta_n\})$. We generate star of David tile $\{\theta_i\}$ using rewriting rules of an equivalent Oxide rewriting grammar $(\Sigma, N, S, S_L, S_R, S_{UR}, R)$.

We reformulate the Definition 2.4 to simplify the comparison of OXTRG with OXTS, showing their equivalence. Then we prove $\text{OXTS} \subseteq \text{OXTRG}$.

Definition 4.1. The Oxide tiling systems OXTS eqO and OXTS u,eqO are the same as a OXTS, with the following respective changes:

- The local Oxide language as defined in (2.4) is replaced with $\text{LOCeqO}(S\{\theta_1, \theta_2, \theta_3 \dots \theta_n\})$, $S(\{\theta_i\})$'s are finite set of star of David tiles in Γ .
- The local Oxide language as defined in (2.4) is replaced with $\text{LOCu,eqO}(S\{\theta_1, \theta_2, \theta_3 \dots \theta_n\})$. Here $S(\{\theta_i\})$'s are finite set of star of David tiles in $\Gamma \cup \#$. There is no boundary symbol # in OXTSu,eqO.

Lemma 4.2 :

$$L(\text{OXTSu,eqO}) \equiv L(\text{OXTSeqO})$$

First we show that $L(\text{OXTSeqO}) \subseteq L(\text{OXTSu,eqO})$.

Consider $\text{OXT} = (\Sigma, \Gamma, \pi, S(\{\theta_1, \theta_2, \theta_3 \dots \theta_n\}))$.

Let this be an Oxide tiling system OXTSeqO.

For every Star of David tile set $S(\{\theta_i\})$.

let $\theta_i = \theta_i' \cup \theta_i''$ with $\theta_i' = \{ \text{tiles containing the boundary symbol } \# \text{ separated from the other tiles } \theta_i'' \}$. Introduce $\Gamma' \notin \Gamma \cup \Sigma$, new alphabets to encode boundary and a mapping $bl: \Gamma \rightarrow \Gamma'$.

δ_i : for every Star of David tile h in θ_i'' , if we have a tile in θ_i' that overlaps with Star of David tile h , then rename this as a new tile h' and include this in the set δ_i .

Consider an OXTSu,eqO

Let $\text{OXT}' = (\Sigma, \Gamma \cup \Gamma', \pi', \delta_i')$ be an OXTSu,eqO.

Define $\pi'(bl(a_1)) = \pi'(a_1) = \pi(a_1), a_1 \in \Gamma$

$gh(a_1) = a_1, a_1 \notin \Gamma'$ and $gh(a_1) = h^{-1}a_1, a_1 \in \Gamma'$ and

$$\delta_i' = \{S(\theta) / S(\theta) \subseteq S(\theta_i') \cup \delta_i \wedge gh(S(\theta)) = S(\theta_i') \wedge S(\theta) \cap \delta_i \neq \emptyset, 1 \leq i \leq n\}$$

We can easily prove that $L(\text{OXT}) = L(\text{OXT}')$.

$L(\text{OXTSu,eqO}) \subseteq L(\text{OXTSeqO})$:

Let $\text{OXT} = (\Sigma, \Gamma, \pi, S(\{\theta_1, \theta_2, \theta_3 \dots \theta_n\}))$ be an OXTSu,eqO.

An OXTSeqO can be constructed introducing δ_i , the tile sets with boundary, for every Star of David tile in $\{\theta_i\}$

Consider an OXTSeqO $\text{OXT}' = (\Sigma, \Gamma, \pi', \delta_i')$ where

$$\delta_i' = \{S(\theta) \cup S(\theta_i) / S(\theta) \subseteq \delta_i \wedge S(\theta) \neq \emptyset, 1 \leq i \leq n\}$$

Then $L(\text{OXT}) = L(\text{OXT}')$.

Theorem 4.1:

$$L(\text{OXTS}) \subseteq L(\text{OXTRG})$$

PROOF. It follows from lemma 5.1,5.2, $L(\text{OXTSu,eqO})$ and $L(\text{OXLOCu,eqO})$ is closed with respect to projection.

Theorem 4.2:

$$L(\text{OXTS}) \neq L(\text{OXTRG})$$

Consider example 3.3, $L = \{ \text{Set of Oxide picture any size } '2n+1' \text{ that have } 'a' \text{ in the position of even } 'a', 'b' \text{ and } 'c' \text{ in the remaining position.} \}$

There is an Oxide rewriting grammar that generate such Oxide pictures. But Oxide tiling system does not recognize such Oxide pictures.

Theorem4.3:

$$L(SOXREC) \equiv L(OXTRG)$$

Proof:

Let $OX_p \in \Gamma^{**OX_p}$ be an input Oxide picture $SOX = (Q, \Sigma, \Gamma, q_0, Q_F, \delta, \mu)$

Let L be an Oxide picture language in SOXREC. We can easily construct a grammar $(\Sigma, N, S, S_L, S_R, S_{UR}, R)$ that generates this oxide picture.

Let L be an Oxide picture generated by Oxide rewriting grammar. Then there is Sgraffito automaton SOX that goes through the input Oxide picture from the left most position and writes the state of each position in the corresponding tape field. When OXT reaches a state in Q_F the input Oxide picture is accepted.

V. CONCLUSION

In this paper we have defined Oxide rewriting grammar to generate Oxide pictures. We started with generation of Oxide pictures, then compared Oxide rewriting grammar with the Oxide tiling system and the recognizing device Sgraffito automata to recognize such pictures and found that OXTRG has greater capacity to generate Oxide pictures than Oxide tiling system. Many further properties can be obtained relating to the classes. Practical applicability to such as pattern recognition needs to be investigated.

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