Solution of a Boundary Value Problem Involving I-Function and Struve's Function

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Abstract*: *Abstract: In the present paper, the authors established an integral involving I-function of two variables, Struve's function with extended general class of polynomials. Also solved a boundary value problem in the steady state temperature distribution of a rectangular plate using I-function, Struve's function and Extended general class of polynomials.

Keywords* : *I-function of two variables, Struve's function, Extended General class of polynomials and Boundary value problem.

I. INTRODUCTION

In [6], Shantha Kumari et.al., introduced and studied I-function of two variables which is due to the generalization of H-function. Srivastava et.al [9] obtained some results involving I-function of two variables due to [7]. Later, Singh et.al [8] observed some integrals involving $\overline{H}$ -function. Yashwant Singh et.al ([10], [11], [12]) established and obtained some recurrence relations, expansion formulae and solved boundary value problem for I-function and $\overline{H}$ -function. Neelampandey et.al [3] using I-function solved a boundary value problem for a rectangular plate.

In our previous paper [4], we established integral transforms like Mellin and Laplace using a product of I-function and some polynomials. Now, in the present study of this paper, we are applying the following:

Two variable I-function defined by Shanta Kumari et.al [6]

$$(1.1)\quad I[z_1, z_2] = I^{o,n_1;m_2,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2;p_3,q_3}\left[\begin{matrix} z_1 \\ z_2 \end{matrix}\left|\begin{matrix} (a_j;\alpha_j,A_j;\varepsilon_j)_{1,p_1} & :(c_j,C_j;U_j)_{1,p_2} & ;(e_j,E_j;P_j)_{1,p_3} \\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1} & :(d_j,D_j;V_j)_{1,q_2} & ;(f_j,F_j;Q_j)_{1,q_3} \end{matrix}\right.\right]$$

$$= \frac{1}{(2\pi i)^2}\int_{L_S}\int_{L_\tau}\phi(s,\tau)\theta_1(s)\theta_1(\tau) z_1^{\ s} z_2^{\ \tau}\, ds\, d\tau$$

Where

$$\phi(s,\tau) = \frac{\prod_{j=1}^{n_1}\Gamma^{\varepsilon_j}(1-a_j+\alpha_j s+A_j\tau)}{\prod_{j=n_1+1}^{p_1}\Gamma^{\varepsilon_j}(a_j-\alpha_j s-A_j\tau)\prod_{j=1}^{q_1}\Gamma^{\eta_j}(1-b_j+\beta_j s+B_j\tau)}$$

$$\theta_1(s) = \frac{\prod_{j=1}^{n_2}\Gamma^{U_j}(1-c_j+C_j s)\prod_{j=1}^{m_2}\Gamma^{V_j}(d_j-D_j s)}{\prod_{j=n_2+1}^{p_2}\Gamma^{U_j}(c_j-C_j s)\prod_{j=m_2+1}^{q_2}\Gamma^{V_j}(1-d_j+D_j s)}$$

$$\theta_2(\tau) = \frac{\prod_{j=1}^{n_3}\Gamma^{P_j}(1-e_j+E_j\tau)\prod_{j=1}^{m_3}\Gamma^{Q_j}(f_j-F_j\tau)}{\prod_{j=n_3+1}^{p_3}\Gamma^{P_j}(e_j-E_j\tau)\prod_{j=m_3+1}^{q_3}\Gamma^{Q_j}(1-f_j+F_j\tau)}$$

Where $n_j, p_j, q_j\ (j=1,2,3), m_j\ (j=2,3)$ are non-negative integers such that $0 \le n_j \le p_j, q_j > 0,\quad 0 \le m_j \le q_j$ $(j=2,3)$ (not all zero simultaneously). $\alpha_j, A_j\ (j=1,...,p_1)$, $\beta_j, B_j\ (j=1,...,q_1)$, $C_j\ (j=1,...,p_2), D_j\ (j=1,...,q_2)$ $E_j\ (j=1,...,p_3)$, $F_j\ (j=1,...,q_3)$ are positive quantities. $a_j\ (j=1,.....,p_1), b_j\ (j=1,.....,q_1), c_j\ (j=1,.....,p_2)$, $d_j\ (j=1,.....,q_2), e_j\ (j=1,.....,p_3), f_j\ (j=1,.....,q_3)$

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are complex numbers.

The exponents $\varepsilon_j, \eta_j, U_j, V_j, P_j, Q_j$ may take non-integer values. $L_S and L_\tau$ are suitable contours of Mellin-Barnes type. Moreover, the contour $L_S \in (\sigma_1 - i\infty, \sigma_1 + i\infty)$ s-plane, where σ_1 is real., so that all the poles of $\Gamma^{V_j}(d_j - D_j s)\ (j=1,....,m_2)$ lies to right of L_s and all poles $\Gamma^{U_j}(1-c_j - C_j s)\ (j=1,....,n_2)$ and $\Gamma^{\varepsilon_j}(1-a_j+\alpha_j s + A_j\tau)\ \ (j=1,....,n_1)$ lies to left of L_s. In complex τ -plane, L_τ satisfies above conditions (see [5]).

The Struve's function is defined as [5]

$$(1.2)\quad H^{\lambda,k}_{v,y,u}[x]$$

$$= \sum_{m=0}^{\infty} \frac{(-1)^m (z/2)^{v+2m+1}}{\Gamma(km+y)\Gamma(v+\lambda m+u)}$$

$$\mathrm{Re}(k) > 0, \mathrm{Re}(\lambda) > 0, \mathrm{Re}(y) > 0, \mathrm{Re}(v+u) > 0$$

In [2],

$$(1.3)\quad S^m_{n,t}[x] = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n+t,k}\, x^k$$

$n = 0, 1, 2,\ldots\ t = 0, 1, 2, \ldots..$

where 'm' is an arbitrary positive integer and the coefficients $A_{n+t,k}\ (n,k \ge 0)$ are arbitrary constants.

The orthogonal property for cosine functions [1]

$$(1.4)\quad \int_{-c}^{c} \cos\left(\frac{n\pi x}{c}\right)\cos\left(\frac{m\pi x}{c}\right)dx = \begin{cases} 0\ for\, n \ne m \\ c\ for\, n = m \end{cases}$$

The integral due to Kumar [1] is

$$(1.5)\quad \int_0^{a/2} \left(\cos\frac{\pi x}{a}\right)^n \cos\left(\frac{2\pi m x}{a}\right)dx$$

$$= \frac{a\Gamma(n+1)}{2^{n+1}\Gamma\left(\frac{n}{2}+m+1\right)\Gamma\left(\frac{n}{2}-m+1\right)}$$

II. MAIN INTEGRAL

Let us prove the following integral

$$(2.1)\quad \int_0^{a/2} \left(\cos\frac{\pi x}{a}\right)^n \cos\left(\frac{2\pi m x}{a}\right)$$

$$S^r_{l,t}\left[h\left(\cos\frac{\pi x}{a}\right)^{2p}\right] H^{\lambda,w}_{v,y,u}\left[b\left(\cos\frac{\pi x}{a}\right)^{2\rho}\right]$$

$$I^{0,n_1;m_2,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2;p_3,q_3}\left[\begin{matrix} z_1\left(\cos\frac{\pi x}{a}\right)^{2\sigma} \\ z_2\left(\cos\frac{\pi x}{a}\right)^{2\eta}\end{matrix}\right]dx$$

$$= \frac{a}{2^{n+1}} \sum_{k=0}^{[l/r]} \frac{(-l)_{r,k}}{k!} A_{l+t,k} \left(\frac{h}{4^p}\right)^k$$

$$\sum_{s=0}^{\infty} \frac{(-1)^s \left(\frac{b}{2^{2\rho+1}}\right)^{v+2s+1}}{\Gamma(\omega s+y)\Gamma(v+\lambda s+u)}$$

$$I^{0,n_1+1;m_2,n_2;m_3,n_3}_{p_1,q_1+2;p_2,q_2;p_3,q_3}\left[\begin{matrix} \frac{z_1}{4^\sigma} \\ \frac{z_2}{4^\eta}\end{matrix}\left|\begin{matrix}(-n-2pk-2g(\rho,s);2\sigma,2\eta;1); \\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1};\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}: & A,B \\ \left(\frac{-n}{2}-pk-g(\rho,s)-m;\sigma,\eta;1\right);\left(\frac{-n}{2}-pk-g(\rho,s)+m;\sigma,\eta;1\right): & C,D\end{matrix}\right]$$

where

$$(2.2)\quad g(\rho,r) = \rho(v+2r+1)$$

$$A = (c_j, C_j, U_j)_{1,p_2},\ B = (e_j, E_j, P_j)_{1,p_3}$$

$$C = (d_j, D_j, V_j)_{1,q_2},\ D = (f_j, F_j, Q_j)_{1,q_3}$$

Proof:

Substituting (1.1), (1.2) and (1.3) in L.H.S, and changing the order of integration, we get

$$= \sum_{k=0}^{[l/r]} \frac{(-l)_{r,k}}{k!} A_{l+t,k} h^k \sum_{s=0}^{\infty} \frac{(-1)^r \left(\frac{b}{2}\right)^{v+2s+1}}{\Gamma(\omega s+y)\Gamma(v+\lambda s+4)}$$

$$\frac{1}{(2\pi i)^2}\int_{L_1}\int_{L_2}\phi(t_1,t_2)\theta_1(t_1)\theta_2(t_2)z_1^{t_1}z_2^{t_2}$$

$$\int_0^{a/2}\left(\cos\frac{\pi x}{a}\right)^{n+2pk+2\rho(v+2s+1)+2\sigma t_1+2\eta t_2}\left(\cos\frac{2\pi mx}{a}\right)dx\,dt_1\,dt_2$$

Then using (1.1) and (1.5), we obtain (2.1).

Due to the convergence of integrals, change of order of integration is valid.

III. BOUNDARY VALUE PROBLEM

In this section, let us consider a boundary value problem in a rectangular plate. To evaluate steady-state temperature u(x, y) with insulated vertical edges x = 0 and x = a/2 .

$$(3.1)\quad \frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=a,\quad 0<x<a, 0<y<\frac{b}{2}$$

$$\left(\frac{\partial u}{\partial x}\right)_{x=0}=\left(\frac{\partial u}{\partial x}\right)_{x=\frac{a}{2}}=0$$

$$u(x,0)=0, 0<x<\frac{a}{2}$$

$$u\left(x,\frac{b}{2}\right)=f(x)$$

$$=\left[\cos\left(\frac{\pi x}{a}\right)\right]^n S_{l,t}^{r}\left[h\left(\cos\frac{\pi x}{a}\right)^{2p}\right]$$

$$H_{v,y,u}^{\lambda,k}\left[b\left(\cos\frac{\pi x}{a}\right)^{2\rho}\right] I_{p_1,q_1;p_2,q_2;p_3,q_3}^{0,n_1;m_2,n_2;m_3,n_3}\left[\begin{matrix} z_1\left(\cos\frac{\pi x}{a}\right)^{2\sigma} \\ z_2\left(\cos\frac{\pi x}{a}\right)^{2\eta}\end{matrix}\right]$$

The general solution of the above problem is given by Zill [13] as follows :

$$(3.2)\quad u(x,y)=A_0 y+\sum_{p=1}^{\infty}A_p\sinh\frac{2p\pi y}{a}\cos\frac{2p\pi x}{a},$$

$$0<x<\frac{a}{2}, 0<y<\frac{b}{2}$$

For y=b/2, we have

$$(3.3)\quad u\left(x,\frac{b}{2}\right)=f(x)$$

$$=A_0\frac{b}{2}+\sum_{p=1}^{\infty}A_p\sinh\frac{p\pi b}{a}\cos\frac{2p\pi x}{a},$$

$$0<x<\frac{a}{2}$$

To find A_0 , we integrate (3.3) both sides with respect to *x* from *0* to *a/2*, then

$$\int_0^{a/2}\left[\cos\left(\frac{\pi x}{a}\right)\right]^n S_{l,t}^{r}\left[h\left(\cos\frac{\pi x}{a}\right)^{2p}\right] H_{v,y,u}^{\lambda,k}\left[b\left(\cos\frac{\pi x}{a}\right)^{2\rho}\right]$$

$$I_{p_1,q_1;p_2,q_2;p_3,q_3}^{0,n_1;m_2,n_2;m_3,n_3}\left[\begin{matrix} z_1\left(\cos\frac{\pi x}{a}\right)^{2\sigma} \\ z_2\left(\cos\frac{\pi x}{a}\right)^{2\eta}\end{matrix}\right]dx$$

$$=\int_0^{a/2}\left(A_0\frac{b}{2}+\sum_{p=1}^{\infty}A_p\sinh\frac{p\pi b}{a}\cos\frac{2p\pi x}{a}\right)dx$$

Evaluating the integral on both sides and using (2.1), we get

$$(3.3)\quad A_0=\frac{1}{b2^{n-1}}\sum_{k=0}^{[l/r]}\frac{(-l)_{r,k}}{k!}A_{l+t,k}$$

$$\sum_{s=0}^{\infty}\frac{(-1)^r\left(\frac{b}{2^{2\rho+1}}\right)^{v+2s+1}}{\Gamma(\omega s+y)\Gamma(v+\lambda s+u)}$$

$$I_{p_1,q_1+2;p_2,q_2;p_3,q_3}^{0,n_1+1;m_2,n_2;m_3,n_3}\left[\begin{matrix}\frac{z_1}{4^{\sigma}}\\ \frac{z_2}{4^{\eta}}\end{matrix}\left|\begin{matrix}(-n-2pk-2g(\rho,s);2\sigma,2\eta;1);\\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1};\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}: \qquad A,B\\ \left(\frac{-n}{2}-pk-g(\rho,s);\sigma,\eta;1\right);\left(\frac{-n}{2}-pk-g(\rho,s);\sigma,\eta;1\right):C,D\end{matrix}\right]$$

where *A*, *B*, *C*, and *D* are as in (2.2).

Now to find A_p , multiply (3.3) both sides with $\cos\left(\frac{2m\pi x}{a}\right)$ and integrate with respect to *x* from *0* to *a/2*, we have

$$\int_0^{a/2}\left(\cos\frac{\pi x}{a}\right)^n \cos\left(\frac{2\pi mx}{a}\right)$$

$$S_{l,t}^{r}\left[h\left(\cos\frac{\pi x}{a}\right)^{2p}\right]H_{v,y,u}^{\lambda,k}\left[b\left(\cos\frac{\pi x}{a}\right)^{2\rho}\right]$$

$$I_{p_1,q_1;p_2,q_2;p_3,q_3}^{0,n_1;m_2,n_2;m_3,n_3}\left[\begin{matrix} z_1\left(\cos\frac{\pi x}{a}\right)^{2\sigma} \\ z_2\left(\cos\frac{\pi x}{a}\right)^{2\eta}\end{matrix}\right]dx$$

$$=\sum_{p=1}^{\infty}A_p \sinh\frac{p\pi b}{a}\int_0^{a/2}\cos\frac{2m\pi x}{a}\cos\frac{2p\pi x}{a}dx$$

The integral on R.H.S vanishes for $p \neq m$, according to (1.4). Now using (2.1) we get

$$(3.4)\quad A_m=\frac{1}{2^{n-1}\sinh\left(\frac{\pi mb}{a}\right)}\sum_{k=0}^{[l/r]}\frac{(-l)_{r,k}}{k!}A_{l+t,k}$$

$$\sum_{s=0}^{\infty}\frac{(-1)^r\left(\frac{b}{2^{2\rho+1}}\right)^{v+2s+1}}{\Gamma(\omega s+y)\Gamma(v+\lambda s+u)}$$

$$I_{p_1,q_1+2;p_2,q_2;p_3,q_3}^{0,n_1+1;m_2,n_2;m_3,n_3}\left[\begin{matrix}\frac{z_1}{4^{\sigma}}\\ \frac{z_2}{4^{\eta}}\end{matrix}\middle|\begin{matrix}(-n-2pk-2g(\rho,s);2\sigma,2\eta;1);\\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1};\end{matrix}\right.$$

$$(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}:$$
$$\left(\frac{-n}{2}-pk-g(\rho,s)-m;\sigma,\eta;1\right);$$

$$\left.\begin{matrix} A,B\\ \left(\frac{-n}{2}-pk-g(\rho,s)+m;\sigma,\eta;1\right):C,D\end{matrix}\right]$$

where *A*, *B*, *C*, and *D* are as in (2.2).

Hence the complete solution of Boundary Value Problem is

$$(3.5)\quad u(x,y)$$

$$=\sum_{k=0}^{[l/r]}\frac{(-l)_{r,k}}{k!}A_{l+t,k}\sum_{s=0}^{\infty}\frac{(-1)^r\left(\frac{b}{2^{2\rho+1}}\right)^{v+2s+1}}{\Gamma(\omega s+y)\Gamma(v+\lambda s+u)}$$

$$\left[\frac{y}{b2^{n-1}}I(\theta_1)+\sum_{m=0}^{\infty}\frac{\sinh\left(\frac{2\pi my}{a}\right)\cos\left(\frac{2\pi mx}{a}\right)}{2^{n-1}\sinh\left(\frac{\pi mb}{a}\right)}I(\theta_2)\right]$$

where

$$(3.6)\quad I(\theta_1)$$

$$=I_{p_1,q_1+2;p_2,q_2;p_3,q_3}^{0,n_1+1;m_2,n_2;m_3,n_3}\left[\begin{matrix}\frac{z_1}{4^{\sigma}}\\ \frac{z_2}{4^{\eta}}\end{matrix}\middle|\begin{matrix}(-n-2pk-2g(\rho,s);2\sigma,2\eta;1);\\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1};\end{matrix}\right.$$

$$(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}$$
$$\left(\frac{-n}{2}-pk-g(\rho,s);\sigma,\eta;1\right);$$

$$\left.\begin{matrix}:A,B\\ \left(\frac{-n}{2}-pk-g(\rho,s);\sigma,\eta;1\right):C,D\end{matrix}\right]$$

and

$$I(\theta_2)=$$

$$I_{p_1,q_1+2;p_2,q_2;p_3,q_3}^{0,n_1+1;m_2,n_2;m_3,n_3}\left[\begin{matrix}\frac{z_1}{4^{\sigma}}\\ \frac{z_2}{4^{\eta}}\end{matrix}\middle|\begin{matrix}(-n-2pk-2g(\rho,s);2\sigma,2\eta;1);\\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1};\end{matrix}\right.$$

$$(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}$$
$$\left(\frac{-n}{2}-pk-g(\rho,s)-m;\sigma,\eta;1\right);$$

$$\left.\begin{matrix}:A,B\\ \left(\frac{-n}{2}-pk-g(\rho,s)+m;\sigma,\eta;1\right):C,D\end{matrix}\right]$$

IV. EXPANSION FORMULA

Take $y=b/2$ in (3.5) we get expansion formula for $f(x)$ as

$$f(x)=\sum_{k=0}^{[l/r]}\frac{(-l)_{r,k}}{k!}A_{l+t,k}$$

$$\sum_{s=0}^{\infty}\frac{(-1)^{r}\left(\frac{b}{2^{2\rho+1}}\right)^{v+2s+1}}{\Gamma\left(\omega s+\frac{b}{2}\right)\Gamma(v+\lambda s+u)}$$

$$\left[\frac{1}{2^{n}}I(\theta_1)+\sum_{m=0}^{\infty}\frac{\cos\left(\frac{2\pi mx}{a}\right)}{2^{n-1}}I(\theta_2)\right]$$

where $I(\theta_1)$ and $I(\theta_2)$ are given by (3.6).

V. CONCLUSION

Since the I-function of two variables discussed in this paper is more general and has special cases as most of the special functions. We can express the results in terms of other special functions. Also, the same results can be extended for I-function of several variables.

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