Dualities between Mohand Transform and Some Useful Integral Transforms

Sudhanshu Aggarwal, Anjana Rani Gupta

Abstract: Integral transforms have wide applications in the different areas of engineering and science to solve the problems of springs, Newton’s law of cooling, electrical networks, bending of beams, mixing problems, signal processing, carbon dating problems, Newton’s second law of motion, exponential growth and decay problems. In this paper, we will discuss the dualities of some useful integral transforms namely Laplace transform, Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform, Mahgoub (Laplace- Carson) transform and Sawi transform with Mohand transform. To visualize the importance of dualities between Mohand transform and mention integral transforms, we give tabular presentation of the integral transforms (Laplace transform, Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform, Mahgoub transform and Sawi transform) of mostly used basic functions by using mention dualities relations.

Keywords: Mohand; Laplace; Kamal; Elzaki; Aboodh; Sumudu; Mahgoub (Laplace- Carson); Sawi transforms.

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I. INTRODUCTION

Many process and phenomenon of science, engineering and real life that can be expressed mathematically solved by using integral transforms. The problems arise in the field of statistics, thermal science, medicine, aerodynamics, civil engineering, control theory, cardiology, mechanics, space science, marine science, biology, gravitation, heat conduction, economics, telecommunications, nuclear reactors, detection of diabetes, chemistry, stress analysis, electricity, physics, potential theory, mathematics, deflection of beams, vibration of plates, defence, Brownian motion and many other fields can be easily handle with the help of integral transforms by converting them into mathematical form. In the advanced time, researchers are interested in solving the advance problems of research, science, space, engineering and real life by introducing new integral transforms. Recently many scholars [1-6] used different integral transforms namely Mohand transform, Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform, Mahgoub (Laplace- Carson) transform for evaluating improper integrals which contains error function in the integrand. In 2019, Aggarwal and Chaudhary [7] discussed Mohand and Laplace transforms comparatively by solving system of differential equations using both integral transforms. Mahgoub [8] gave Sawi transform which is a new integral transform.

The aim of this study is to establish dualities between Mohand transform and some useful integral transforms namely Laplace transform, Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform, Mahgoub (Laplace- Carson) transform and Sawi transform.

II. MOHAND TRANSFORM

Mohand transform of the function $Z(\gamma)$, $\gamma \geq 0$ is given by [1]

$$M\{Z(\gamma)\} = e^{2 \int_{0}^{\infty} Z(\gamma)e^{-\gamma} d\gamma} = B(\epsilon),$$

$$0 < k_1 \leq \epsilon \leq k_2$$

III. LAPLACE TRANSFORM

The Laplace transform of the function $Z(\gamma)$, $\gamma \geq 0$ is given by [7]

$$L\{Z(\gamma)\} = \int_{0}^{\infty} Z(\gamma)e^{-\gamma} d\gamma = C(\epsilon),$$

IV. KAMAL TRANSFORM

Kamal transform of the function $Z(\gamma)$, $\gamma \geq 0$ is given by [2]

$$K\{Z(\gamma)\} = \int_{0}^{\infty} Z(\gamma)e^{\frac{\gamma}{\epsilon}} d\gamma = D(\epsilon),$$

$$0 < k_1 \leq \epsilon \leq k_2$$

V. ELZAKI TRANSFORM

Elzaki transform of the function $Z(\gamma)$, $\gamma \geq 0$ is given by [3]

$$E\{Z(\gamma)\} = \int_{0}^{\infty} Z(\gamma)e^{-\epsilon\gamma} d\gamma = G(\epsilon),$$

$$0 < k_1 \leq \epsilon \leq k_2$$

VI. ABOODH TRANSFORM

Aboodh transform of the function $Z(\gamma), \gamma \geq 0$ is given by [4]

$$A\{Z(\gamma)\} = \int_{0}^{\infty} Z(\gamma)e^{-\epsilon\gamma} d\gamma = H(\epsilon),$$

$$0 < k_1 \leq \epsilon \leq k_2$$

VII. SUMUDU TRANSFORM

Sumudu transform of the function $Z(\gamma), \gamma \geq 0$ is given by [5]

$$S\{Z(\gamma)\} = \int_{0}^{\infty} Z(\gamma)e^{-\gamma} d\gamma = I(\epsilon),$$

$$0 < k_1 \leq \epsilon \leq k_2$$

VIII. MAHGOUB (LAPLACE – CARSON) TRANSFORM

Mahgoub (Laplace- Carson) transform of the function $Z(\gamma), \gamma \geq 0$ is given by [6]

$$M_{c}\{Z(\gamma)\} = \int_{0}^{\infty} Z(\gamma)e^{-\epsilon\gamma} d\gamma = J(\epsilon),$$

$$0 < k_1 \leq \epsilon \leq k_2$$
IX. SAWI TRANSFORM

Sawt transform of the function $Z(y), \gamma \geq 0$ is given by [8]

$$
S_\epsilon(Z(y)) = \frac{1}{\epsilon^2} \int_0^\infty Z(y)e^{-\gamma y} dy = L(\epsilon),
$$

(8)

$$
0 < k_1 \leq \epsilon \leq k_2
$$

X. DUALITIES OF MOHAND TRANSFORM WITH SOME USEFUL INTEGRAL TRANSFORMS

In this section, we define the dualities of Mohand transform with some useful integral transforms namely Laplace transform, Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform, Mahgoub (Laplace-Carson) transform and Sawi transform.

A. Mohand – Laplace Duality

If Mohand and Laplace transforms of $Z(y)$ are $B(\epsilon)$ and $C(\epsilon)$ respectively then

$$
B(\epsilon) = \epsilon^2 C(\epsilon),
$$

(9)

and

$$
C(\epsilon) = \frac{1}{\epsilon^2} B(\epsilon),
$$

(10)

**Proof:** From (1),

$$
B(\epsilon) = \epsilon^2 \int_0^\infty Z(y)e^{-\gamma y} dy
$$

Now, using (2) in above Equation, we obtain

$$
B(\epsilon) = \epsilon^2 \int_0^\infty Z(y)e^{-\gamma y} dy = \epsilon^2 C(\epsilon)
$$

$$
\Rightarrow B(\epsilon) = \epsilon^2 C(\epsilon).
$$

To drive (9), we use (2)

$$
C(\epsilon) = \frac{1}{\epsilon^2} \int_0^\infty Z(y)e^{-\gamma y} dy
$$

$$
\Rightarrow C(\epsilon) = \frac{1}{\epsilon^2} \left[ \epsilon^2 \int_0^\infty Z(y)e^{-\gamma y} dy \right]
$$

(11)

It is immediately concluded using (1) in (11),

$$
\Rightarrow C(\epsilon) = \frac{1}{\epsilon^2} B(\epsilon).
$$

B. Mohand – Kamal Duality

If Mohand and Kamal transforms of $Z(y)$ are $B(\epsilon)$ and $D(\epsilon)$ respectively then

$$
B(\epsilon) = \epsilon^2 D\left(\frac{1}{\epsilon}\right),
$$

(12)

and

$$
D(\epsilon) = \epsilon^2 B\left(\frac{1}{\epsilon}\right)
$$

(13)

**Proof:** Using (1) follows

$$
B(\epsilon) = \epsilon^2 \int_0^\infty Z(y)e^{-\gamma y} dy
$$

$$
\Rightarrow B(\epsilon) = \epsilon^2 \int_0^\infty Z(y)e^{-\gamma y} dy
$$

(14)

Now, using (3) in above equation, we obtain

$$
B(\epsilon) = \epsilon^2 D\left(\frac{1}{\epsilon}\right)
$$

To drive (12), we use (3)

$$
D(\epsilon) = \int_0^\infty Z(y)e^{-\gamma y} dy
$$

$$
\Rightarrow D(\epsilon) = \epsilon^2 \int_0^\infty Z(y)e^{-\gamma y} dy
$$

$$
\Rightarrow D(\epsilon) = \epsilon^2 B\left(\frac{1}{\epsilon}\right)
$$

C. Mohand – Elzaki Duality

If Mohand and Elzaki transforms of $Z(y)$ are $B(\epsilon)$ and $G(\epsilon)$ respectively then

$$
B(\epsilon) = \epsilon^2 G\left(\frac{1}{\epsilon}\right),
$$

(15)

and

$$
G(\epsilon) = \epsilon^2 B\left(\frac{1}{\epsilon}\right)
$$

(16)

**Proof:** It is immediately concluded from (1)

$$
B(\epsilon) = \epsilon^2 \int_0^\infty Z(y)e^{-\gamma y} dy
$$

$$
\Rightarrow B(\epsilon) = \epsilon^2 \int_0^\infty Z(y)e^{-\gamma y} dy
$$

Now, using (4) in above Equation, we have

$$
B(\epsilon) = \epsilon^3 G\left(\frac{1}{\epsilon}\right).
$$

To drive (15), we use (4)

$$
G(\epsilon) = \epsilon \int_0^\infty Z(y)e^{-\gamma y} dy
$$

$$
\Rightarrow G(\epsilon) = \epsilon \int_0^\infty Z(y)e^{-\gamma y} dy
$$

$$
\Rightarrow G(\epsilon) = \epsilon^3 B\left(\frac{1}{\epsilon}\right).
$$

D. Mohand – Aboodh Duality

If Mohand and Aboodh transforms of $Z(y)$ are $B(\epsilon)$ and $H(\epsilon)$ respectively then

$$
B(\epsilon) = \epsilon^3 H(\epsilon)
$$

(17)

and

$$
H(\epsilon) = \frac{1}{\epsilon^3} B(\epsilon)
$$

(18)

**Proof:** From (1), we have

$$
B(\epsilon) = \epsilon^2 \int_0^\infty Z(y)e^{-\gamma y} dy
$$

$$
\Rightarrow B(\epsilon) = \epsilon^3 \int_0^\infty Z(y)e^{-\gamma y} dy
$$

Now, using (5) in above equation, we have

$$
B(\epsilon) = \epsilon^3 H(\epsilon).
$$

To drive (17), we use (5)

$$
H(\epsilon) = \int_0^\infty Z(y)e^{-\gamma y} dy
$$

$$
\Rightarrow H(\epsilon) = \int_0^\infty Z(y)e^{-\gamma y} dy
$$

$$
\Rightarrow H(\epsilon) = \int_0^\infty Z(y)e^{-\gamma y} dy
$$

$$
\Rightarrow H(\epsilon) = \epsilon H(\epsilon).
$$

E. Mohand – Sumudu Duality

If Mohand and Sumudu transforms of $Z(y)$ are $B(\epsilon)$ and $I(\epsilon)$ respectively then

$$
B(\epsilon) = \epsilon I\left(\frac{1}{\epsilon}\right)
$$

(19)

and

$$
I(\epsilon) = \epsilon B\left(\frac{1}{\epsilon}\right)
$$

(20)

**Proof:** From (1), we have

$$
B(\epsilon) = \epsilon^2 \int_0^\infty Z(y)e^{-\gamma y} dy
$$

Put $\gamma = u \Rightarrow dy = \frac{du}{\epsilon}$ in above equation, we have

$$
B(\epsilon) = \epsilon^2 \int_0^\infty Z\left(\frac{u}{\epsilon}\right)e^{-u} du
$$

$$
\Rightarrow B(\epsilon) = \epsilon \int_0^\infty Z\left(\frac{\epsilon u}{\epsilon}\right)e^{-u} du
$$

Now, using (6) in above equation, we have

$$
B(\epsilon) = \epsilon I\left(\frac{1}{\epsilon}\right).
$$

To drive (19), we use (6)

$$
I(\epsilon) = \int_0^\infty Z(\epsilon y)e^{-\gamma y} dy
$$

Put $\gamma = u \Rightarrow dy = \frac{du}{\epsilon}$ in above equation, we have

$$
I(\epsilon) = \int_0^\infty Z(u)e^{-\epsilon u} du
$$

$$
\Rightarrow I(\epsilon) = \int_0^\infty Z(u)e^{-\epsilon u} du
$$
\[ I(e) = e \left[ \frac{1}{e^2} \int_0^{\infty} Z(u)e^{-u/e} du \right] \]
\[ = I(e) = eB \left( \frac{1}{e} \right) \]

**F. Mohand – Mahgoub (Laplace – Carson) Duality**

If Mohand and Mahgoub transforms of \( Z(y) \) are \( B(e) \) and \( f(e) \) respectively then
\[ B(e) = e f(e) \quad (21) \]
and \( f(e) = \frac{1}{e} B(e) \quad (22) \)

**Proof:** From (1), we have
\[ B(e) = e^2 \int_0^{\infty} Z(y)e^{-y/e} dy \]
\[ B(e) = e \left[ \frac{1}{e} \int_0^{\infty} Z(y)e^{-y/e} dy \right] \]
Now, using (7) in above equation, we have
\[ B(e) = e f(e). \]
To drive (22), we use (7)
\[ f(e) = e \int_0^{\infty} Z(y)e^{-y/e} dy \]
\[ \Rightarrow f(e) = \frac{1}{e} \left[ \int_0^{\infty} Z(y)e^{-y/e} dy \right] \]
\[ \Rightarrow f(e) = \frac{1}{e} B(e). \]

**G. Mohand – Sawi Duality**

If Mohand and Sawi transforms of \( Z(y) \) are \( B(e) \) and \( L(e) \) respectively then
\[ B(e) = L \left( \frac{1}{e} \right) \quad (23) \]
and \( L(e) = B \left( \frac{1}{e} \right) \quad (24) \)

**Proof:** Using (1) follows
\[ B(e) = e^2 \int_0^{\infty} Z(y)e^{-y/e} dy \]
\[ \Rightarrow B(e) = \frac{1}{(1/e^2)} \int_0^{\infty} Z(y)e^{-y/e} dy \]
Now, using (8) in above equation, we obtain
\[ B(e) = L \left( \frac{1}{e} \right). \]
To drive (24), we use (8)
\[ L(e) = \frac{1}{e^2} \int_0^{\infty} Z(y)e^{-y/e} dy \]
\[ \Rightarrow L(e) = \left[ \frac{1}{e} \right] \int_0^{\infty} Z(y)e^{-y/e} dy \]
Now, using (1) in above equation, we obtain
\[ L(e) = B \left( \frac{1}{e} \right). \]

**XI. APPLICATIONS OF MENTION DUALITY RELATIONS FOR FINDING INTEGRAL TRANSFORMS (LAPLACE TRANSFORM, KAMAL TRANSFORM, ELZAKI TRANSFORM, ABOODH TRANSFORM, SUMUDU TRANSFORM, MAHGOUB TRANSFORM AND SAWI TRANSFORM) OF USEFUL BASIC FUNCTIONS**

We are giving tabular presentation of the integral transforms of mostly used basic functions by using mention dualities relations to visualize the usefulness of dualities between Mohand transform and mention integral transforms in the application field.

**Table-I: Laplace transform of useful basic functions with the help of Mohand – Laplace duality relation**

<table>
<thead>
<tr>
<th>S. N.</th>
<th>( Z(y) )</th>
<th>( M(Z(y)) = B(e) )</th>
<th>( L(Z(y)) = C(e) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1</td>
<td>( e )</td>
<td>( \frac{1}{e} )</td>
</tr>
<tr>
<td>2.</td>
<td>( y )</td>
<td>1</td>
<td>( \frac{1}{e} )</td>
</tr>
<tr>
<td>3.</td>
<td>( y^2 )</td>
<td>( \frac{2!}{e} )</td>
<td>( \frac{2!}{e^2} )</td>
</tr>
<tr>
<td>4.</td>
<td>( y^n )</td>
<td>( \frac{n!}{e^n} )</td>
<td>( \frac{n!}{e^{n+1}} )</td>
</tr>
<tr>
<td>5.</td>
<td>( y^n ), ( n \in \mathbb{N} )</td>
<td>( \frac{\Gamma(n+1)}{e^{n+1}} )</td>
<td>( \frac{\Gamma(n+1)}{e^{n+1}} )</td>
</tr>
<tr>
<td>6.</td>
<td>( e^y )</td>
<td>( \frac{e^y}{e-a} )</td>
<td>( \frac{1}{1-a} )</td>
</tr>
<tr>
<td>7.</td>
<td>( \sin ay )</td>
<td>( \frac{ae^y}{(e^2 + a^2)} )</td>
<td>( \frac{ae^y}{(e^2 + a^2)} )</td>
</tr>
<tr>
<td>8.</td>
<td>( \cos ay )</td>
<td>( \frac{e^y}{(e^2 + a^2)} )</td>
<td>( \frac{e^y}{(e^2 + a^2)} )</td>
</tr>
<tr>
<td>9.</td>
<td>( \tan ay )</td>
<td>( \frac{ae^y}{(e^2 - a^2)} )</td>
<td>( \frac{ae^y}{(e^2 - a^2)} )</td>
</tr>
<tr>
<td>10.</td>
<td>( \cot ay )</td>
<td>( \frac{e^y}{(e^2 - a^2)} )</td>
<td>( \frac{e^y}{(e^2 - a^2)} )</td>
</tr>
</tbody>
</table>

**Table-II: Kamal transform of useful basic functions with the help of Mohand – Kamal duality relation**

<table>
<thead>
<tr>
<th>S.N.</th>
<th>( Z(y) )</th>
<th>( M(Z(y)) = B(e) )</th>
<th>( K(Z(y)) = D(e) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1</td>
<td>( e )</td>
<td>( \frac{1}{e} )</td>
</tr>
<tr>
<td>2.</td>
<td>( y )</td>
<td>( \frac{1}{e} )</td>
<td>( e^2 )</td>
</tr>
<tr>
<td>3.</td>
<td>( y^2 )</td>
<td>( \frac{2!}{e} )</td>
<td>( 2!e^3 )</td>
</tr>
<tr>
<td>4.</td>
<td>( y^n ), ( n \in \mathbb{N} )</td>
<td>( \frac{n!}{e^{n+1}} )</td>
<td>( n!e^{n+1} )</td>
</tr>
<tr>
<td>5.</td>
<td>( y^n ), ( n \in \mathbb{N} )</td>
<td>( \frac{\Gamma(n+1)}{e^{n+1}} )</td>
<td>( \Gamma(n+1) )</td>
</tr>
<tr>
<td>6.</td>
<td>( e^y )</td>
<td>( \frac{e^y}{e-a} )</td>
<td>( \frac{1}{1-a} )</td>
</tr>
<tr>
<td>7.</td>
<td>( \sin ay )</td>
<td>( \frac{ae^y}{(e^2 + a^2)} )</td>
<td>( \frac{ae^y}{(1+a^2e^2)} )</td>
</tr>
<tr>
<td>8.</td>
<td>( \cos ay )</td>
<td>( \frac{e^y}{(e^2 + a^2)} )</td>
<td>( \frac{e^y}{(1+a^2e^2)} )</td>
</tr>
<tr>
<td>9.</td>
<td>( \tan ay )</td>
<td>( \frac{ae^y}{(e^2 - a^2)} )</td>
<td>( \frac{ae^y}{(1-a^2e^2)} )</td>
</tr>
<tr>
<td>10.</td>
<td>( \cot ay )</td>
<td>( \frac{e^y}{(e^2 - a^2)} )</td>
<td>( \frac{e^y}{(1-a^2e^2)} )</td>
</tr>
</tbody>
</table>
### Dualities between Mohand Transform and Some Useful Integral Transforms

#### Table-III: Elzaki transform of useful basic functions with the help of Mohand – Elzaki duality relation

<table>
<thead>
<tr>
<th>S.N.</th>
<th>( Z(y) )</th>
<th>( M(Z(y)) = B(e) )</th>
<th>( E(Z(y)) = G(e) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1</td>
<td>( e )</td>
<td>( e^2 )</td>
</tr>
<tr>
<td>2.</td>
<td>( y )</td>
<td>1</td>
<td>( e^3 )</td>
</tr>
<tr>
<td>3.</td>
<td>( y^2 )</td>
<td>( \frac{2!}{e} )</td>
<td>( 2!e^4 )</td>
</tr>
<tr>
<td>4.</td>
<td>( y^n, n \in N )</td>
<td>( \frac{n!}{e^n} )</td>
<td>( n!e^{n+2} )</td>
</tr>
<tr>
<td>5.</td>
<td>( y^n, n &gt; -1 )</td>
<td>( \frac{1}{e^{n-1}} )</td>
<td>( \frac{1}{e^{n+2}} )</td>
</tr>
<tr>
<td>6.</td>
<td>( e^{ar} )</td>
<td>( \frac{e^2}{(e-a)} )</td>
<td>( \frac{1}{1-ea} )</td>
</tr>
<tr>
<td>7.</td>
<td>( \sin ay )</td>
<td>( \frac{ae^2}{(e^2+a^2)} )</td>
<td>( \frac{ae^2}{1+a^2e^2} )</td>
</tr>
<tr>
<td>8.</td>
<td>( \cos ay )</td>
<td>( \frac{e^2}{(e^2+a^2)} )</td>
<td>( \frac{e^2}{1+a^2e^2} )</td>
</tr>
<tr>
<td>9.</td>
<td>( \sinhay )</td>
<td>( \frac{ae^2}{(e^2-a^2)} )</td>
<td>( \frac{ae^2}{1-a^2e^2} )</td>
</tr>
<tr>
<td>10.</td>
<td>( \coshay )</td>
<td>( \frac{e^2}{(e^2-a^2)} )</td>
<td>( \frac{e^2}{1-a^2e^2} )</td>
</tr>
</tbody>
</table>

#### Table-IV: Aboodh transform of useful basic functions with the help of Mohand – Aboodh duality relation

<table>
<thead>
<tr>
<th>S.N.</th>
<th>( Z(y) )</th>
<th>( M(Z(y)) = B(e) )</th>
<th>( A(Z(y)) = H(e) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1</td>
<td>( e )</td>
<td>( \frac{1}{e^2} )</td>
</tr>
<tr>
<td>2.</td>
<td>( y )</td>
<td>1</td>
<td>( \frac{1}{e^2} )</td>
</tr>
<tr>
<td>3.</td>
<td>( y^2 )</td>
<td>( \frac{2!}{e} )</td>
<td>( 2!e^4 )</td>
</tr>
<tr>
<td>4.</td>
<td>( y^n, n \in N )</td>
<td>( \frac{n!}{e^n} )</td>
<td>( n!e^{n+2} )</td>
</tr>
<tr>
<td>5.</td>
<td>( y^n, n &gt; -1 )</td>
<td>( \frac{1}{e^{n-1}} )</td>
<td>( \frac{1}{e^{n+2}} )</td>
</tr>
<tr>
<td>6.</td>
<td>( e^{ar} )</td>
<td>( \frac{e^2}{(e-a)} )</td>
<td>( \frac{1}{e(e-a)} )</td>
</tr>
<tr>
<td>7.</td>
<td>( \sin ay )</td>
<td>( \frac{ae^2}{(e^2+a^2)} )</td>
<td>( \frac{a}{e(e^2+a^2)} )</td>
</tr>
<tr>
<td>8.</td>
<td>( \cos ay )</td>
<td>( \frac{e^2}{(e^2+a^2)} )</td>
<td>( \frac{e^2}{e(e^2+a^2)} )</td>
</tr>
<tr>
<td>9.</td>
<td>( \sinhay )</td>
<td>( \frac{ae^2}{(e^2-a^2)} )</td>
<td>( \frac{ae^2}{e(e^2-a^2)} )</td>
</tr>
<tr>
<td>10.</td>
<td>( \coshay )</td>
<td>( \frac{e^2}{(e^2-a^2)} )</td>
<td>( \frac{e^2}{e(e^2-a^2)} )</td>
</tr>
</tbody>
</table>

#### Table-V: Sumudu transform of useful basic functions with the help of Mohand – Sumudu duality relation

<table>
<thead>
<tr>
<th>S.N.</th>
<th>( Z(y) )</th>
<th>( M(Z(y)) = B(e) )</th>
<th>( S(Z(y)) = I(e) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1</td>
<td>( e )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>2.</td>
<td>( y )</td>
<td>1</td>
<td>( e )</td>
</tr>
<tr>
<td>3.</td>
<td>( y^2 )</td>
<td>( \frac{2!}{e} )</td>
<td>( 2!e^2 )</td>
</tr>
<tr>
<td>4.</td>
<td>( y^n, n \in N )</td>
<td>( \frac{n!}{e^n} )</td>
<td>( n!e^{n} )</td>
</tr>
<tr>
<td>5.</td>
<td>( y^n, n &gt; -1 )</td>
<td>( \frac{1}{e^{n-1}} )</td>
<td>( \frac{1}{e^{n}} )</td>
</tr>
<tr>
<td>6.</td>
<td>( e^{ar} )</td>
<td>( \frac{e^2}{(e-a)} )</td>
<td>( \frac{1}{1-ea} )</td>
</tr>
<tr>
<td>7.</td>
<td>( \sin ay )</td>
<td>( \frac{ae^2}{(e^2+a^2)} )</td>
<td>( \frac{ae^2}{1+a^2e^2} )</td>
</tr>
<tr>
<td>8.</td>
<td>( \cos ay )</td>
<td>( \frac{e^2}{(e^2+a^2)} )</td>
<td>( \frac{e^2}{1+a^2e^2} )</td>
</tr>
<tr>
<td>9.</td>
<td>( \sinhay )</td>
<td>( \frac{ae^2}{(e^2-a^2)} )</td>
<td>( \frac{ae^2}{1-a^2e^2} )</td>
</tr>
<tr>
<td>10.</td>
<td>( \coshay )</td>
<td>( \frac{e^2}{(e^2-a^2)} )</td>
<td>( \frac{e^2}{1-a^2e^2} )</td>
</tr>
</tbody>
</table>

#### Table-VI: Mahgoub (Laplace-Carson) transform of useful basic functions with the help of Mohand – Mahgoub (Laplace-Carson) duality relation

<table>
<thead>
<tr>
<th>S.N.</th>
<th>( Z(y) )</th>
<th>( M(Z(y)) = B(e) )</th>
<th>( M'(Z(y)) = f(e) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1</td>
<td>( e )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>2.</td>
<td>( y )</td>
<td>1</td>
<td>( \frac{1}{e} )</td>
</tr>
<tr>
<td>3.</td>
<td>( y^2 )</td>
<td>( \frac{2!}{e} )</td>
<td>( 2! \frac{1}{e^2} )</td>
</tr>
<tr>
<td>4.</td>
<td>( y^n, n \in N )</td>
<td>( \frac{n!}{e^n} )</td>
<td>( n! \frac{1}{e^n} )</td>
</tr>
<tr>
<td>5.</td>
<td>( y^n, n &gt; -1 )</td>
<td>( \frac{1}{e^{n-1}} )</td>
<td>( \frac{1}{e^{n}} )</td>
</tr>
<tr>
<td>6.</td>
<td>( e^{ar} )</td>
<td>( \frac{e^2}{(e-a)} )</td>
<td>( \frac{1}{e(e-a)} )</td>
</tr>
<tr>
<td>7.</td>
<td>( \sin ay )</td>
<td>( \frac{ae^2}{(e^2+a^2)} )</td>
<td>( \frac{a}{e(e^2+a^2)} )</td>
</tr>
<tr>
<td>8.</td>
<td>( \cos ay )</td>
<td>( \frac{e^2}{(e^2+a^2)} )</td>
<td>( \frac{e^2}{e(e^2+a^2)} )</td>
</tr>
<tr>
<td>9.</td>
<td>( \sinhay )</td>
<td>( \frac{ae^2}{(e^2-a^2)} )</td>
<td>( \frac{ae^2}{e(e^2-a^2)} )</td>
</tr>
<tr>
<td>10.</td>
<td>( \coshay )</td>
<td>( \frac{e^2}{(e^2-a^2)} )</td>
<td>( \frac{e^2}{e(e^2-a^2)} )</td>
</tr>
</tbody>
</table>
Table VII: Sawi transform of useful basic functions with the help of Mohand – Sawi duality relation

<table>
<thead>
<tr>
<th>S.N.</th>
<th>$Z(\gamma)$</th>
<th>$M(Z(\gamma)) = B(\epsilon)$</th>
<th>$S(Z(\gamma)) = L(\epsilon)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$\gamma$</td>
<td>$1$</td>
<td>$\frac{\epsilon}{\gamma}$</td>
</tr>
<tr>
<td>2.</td>
<td>$\gamma^2$</td>
<td>$1$</td>
<td>$2!\frac{\epsilon}{\gamma}$</td>
</tr>
<tr>
<td>3.</td>
<td>$\gamma^n, n \in N$</td>
<td>$\frac{n!}{\epsilon^{n-1}}$</td>
<td>$n!\frac{\epsilon^{n-1}}{\gamma}$</td>
</tr>
<tr>
<td>4.</td>
<td>$\gamma^n, n &gt; -1$</td>
<td>$\frac{\Gamma(n+1)}{\epsilon^{n-1}}$</td>
<td>$\Gamma(n+1)\epsilon^{n-1}$</td>
</tr>
<tr>
<td>5.</td>
<td>$e^{-\gamma}$</td>
<td>$\frac{\epsilon}{\epsilon - a}$</td>
<td>$\frac{1}{\epsilon(1 - ae)}$</td>
</tr>
<tr>
<td>6.</td>
<td>$\sin\gamma$</td>
<td>$\frac{ae^2}{(\epsilon^2 + a^2)}$</td>
<td>$\frac{a}{(1 + a^2\epsilon^2)}$</td>
</tr>
<tr>
<td>7.</td>
<td>$\cos\gamma$</td>
<td>$\frac{\epsilon^3}{(\epsilon^2 + a^2)}$</td>
<td>$\frac{1}{\epsilon(1 + a^2\epsilon^2)}$</td>
</tr>
</tbody>
</table>

XII. CONCLUSIONS

In the present paper, duality relations between Mohand transform and some useful integral transforms namely Laplace transform, Kamal transform, Mahgoub transform, Elzaki transform, Aboodh transform, Sumudu transform, Mahgoub (Laplace-Carson) transform and Sawi transform are established successfully. Tabular presentation of the integral transforms (Laplace transform, Kamal transform, Mahgoub transform, Elzaki transform, Aboodh transform, Sumudu transform, Mahgoub transform and Sawi transform) of mostly used basic functions are given with the help of mention dualities relations to visualize the importance of dualities between Mohand transform and mention integral transforms. In future using these duality relations, we can easily solved many advanced problems of modern era such as motion of coupled harmonic oscillators, drug distribution in the body, arms race models, Brownian motion and the common health problem detection of diabetes.

REFERENCES


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