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Inventory Model with Demand Dependent on Unit Price under Fuzzy Parameters and Decision Variables

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***Abstract:** An EOQ model with demand dependent on unit price is considered and a new approach of finding optimal demand value is done from the optimal unit cost price after defuzzification. Here the cost parameters like setup cost, holding cost and shortage cost and also the decision variables like unit price, lot size and the maximum inventory are taken under fuzzy environment. Triangular fuzzy numbers are used to fuzzify these input parameters and unknown variables. For the proposed model an optimal solution has been determined using Karush Kuhn-Tucker conditions method. Graded Mean Integration (GMI) method is used for defuzzification. Numerical solutions are obtained and sensitivity analysis is done for the chosen model.*

***Keywords:** Crisp and Fuzzy total cost, Demand dependent on unit price, Graded Mean Integration, Triangular fuzzy numbers.*

I. INTRODUCTION

The main objective of inventory control is to increase the profit and decrease the investment without affecting the manufacturing process. The growth of an organization depends on a proper inventory control.

Inventory problems helps to make decisions like minimize the total cost or maximize the profit. Therefore the variables that minimize the total cost or maximize the profit are obtained by solving the inventory problem. Most of the inventory problems deal with minimizing the inventory carrying cost (3). Harris (4) first determined an inventory model with fixed demand rate. An inventory model with increasing demand under inflation was developed by Sarkar and Sana (9). In the above mentioned works the parameters are considered as crisp values. We can also find some of the Works in inventory management where the input parameters and unknown variables are taken in fuzzy environment.

Fuzzy set theory was first introduced by Zadeh in the year 1965. Later on Bellman and Zadeh (1) used the fuzzy set theory to the decision making problems. Further Tanaka et al (10) considered the objectives as fuzzy goals over the α-cuts of a fuzzy constraint set. Cheng (2) applied geometric programming method to solve an EOQ model with unit price dependent on demand. Roy and Maiti (8) used both fuzzy linear and geometric programming techniques to solve inventory model with fuzzy objective function and storage area. Kazemi et al (6) developed an EOQ model with fuzzy parameters and decision variables and used the GMI method for defuzzification.

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II. ASSUMPTIONS AND NOTATIONS FOR THE INVENTORY MODEL

a. Assumptions

The following assumptions have been made to solve the proposed inventory problem.

- Demand is deterministic
- Shortages are allowed and fully backlogged.
- The models a single item and warehouse space.
- At the beginning of each cycle, one placed order is manufactured and delivered in each cycle.

b. Notations

The following notations are made to develop the mentioned EOQ problem.

TC – Total cost
Q – Lot size or batch size
I – Maximum inventory level
S – Order cost or set up cost per order
D – Demand rate units per unit of time
P – Unit price per item
H – Unit holding cost per unit of item
M – Penalty cost or shortage cost per unit of time

Here S, D, H and m are input parameters and p, Q, I are decision variables.
Demand D is dependent on unit price p, given by the relation $D = Ap^{-\beta}$, where A and β are constants.

III. FORMULATION OF EOQ MODEL IN VARIOUS ENVIRONMENT

A. EOQ model in Crisp environment

In general, the classical inventory models are designed by assuming that the demand and unit costs are constant and independent. But conversely they are related to each other. That is the unit cost of an item is inversely proportional to the demand of that item.

Hence $D = Ap^{-\beta}$, where A (>0) and $0<\beta<1$ are real numbers selected to provide the best fit of the estimated price function. Therefore the annual total cost of the crisp model is the total cost = Production cost + Setup cost + Holding cost + Shortage cost

$$TC = pD + \frac{SD}{Q} + \frac{I^2H}{2Q} + \frac{(Q-I)^2 m}{2Q} \quad (1)$$

$$D = Ap^{-\beta} \quad (2)$$

$$TC = Ap^{1-\beta} + \frac{SAp^{-\beta}}{Q} + \frac{I^2H}{2Q} + \frac{(Q-I)^2 m}{2Q} \quad (3)$$

Where p, Q and I are the decision variables.

The crisp values of the unit cost p, lot size Q and the maximum inventory I that minimizes the total cost function are given by

$$p = \left[\frac{mHS\beta^2}{2A(H+m)(1-\beta)^2}\right]^{\frac{1}{2-\beta}} \quad (4)$$

$$Q = \frac{S\beta}{p(1-\beta)} \quad (5)$$

$$I = \frac{Qm}{H+m} \quad (6)$$

IV. EOQ MODEL IN FUZZY ENVIRONMENT

An EOQ model with input parameters and decision variables under fuzzy environment is considered in this section.

Since the holding cost, setup cost and shortage cost and also the unit price, lot size and maximum inventory are fuzzy in nature, they are taken as triangular fuzzy numbers. The Triangular Fuzzy Numbers are represented by

Order cost: $S = (S-\delta_1, S, S+\delta_2), S > \delta_1$

Holding cost: $H = (H-\delta_3, H, H+\delta_4), H > \delta_3$

Penalty cost: $m = (m-\delta_5, m, m+\delta_6), m > \delta_5$

Decision Variables

Maximum Inventory level: $\tilde{I} = (I-\delta_7, I, I+\delta_8), I > \delta_7$

Lot size: $Q = (Q-\delta_9, Q, Q+\delta_{10}), Q > \varphi_9$

Unite Price: $p = (p-\delta_{11}, p, p+\delta_{12}), p > \delta_{11}$

In this section, we apply GMI method to defuzzify the objective function and solved for the decision variables p, Q and I using Karush Kuhn-Tucker conditions technique.

Hence the total cost under fuzzy environment is given by

$$TC = (C_1, C_2, C_3) \quad (7)$$

Where

$$C_1 = A(p-\delta_{11})(p+\delta_{12})^{-\beta} + \frac{A(S-\delta_1)(p+\delta_{12})^{-\beta}}{Q+\delta_{10}} + \frac{(H-\delta_3+m-\delta_5)(I-\delta_7)^2}{2(Q+\delta_{10})} + \frac{(Q-\delta_9)(m-\delta_5)}{2} - (I+\delta_8)(m+\delta_6) \quad (8)$$

$$C_2 = Ap^{1-\beta} + \frac{ASp^{-\beta}}{Q} + \frac{(H+m)I^2}{2Q} + \frac{Qm}{2} - \mathrm{Im} \quad (9)$$

$$C_3 = A(p+\delta_{12})(p-\delta_{11})^{-\beta} + \frac{A(S+\delta_2)(p-\delta_{11})^{-\beta}}{Q-\delta_9} + \frac{(H+\delta_4+m+\delta_6)(I+\delta_8)^2}{2(Q-\delta_9)} + \frac{(Q+\delta_{10})(m+\delta_6)}{2} - (I-\delta_7)(m-\delta_5) \quad (10)$$

Defuzzification of equation (7) by using Graded Mean Integration method gives

$$\theta(TC) = \frac{1}{6}\left[C_1 + 4C_2 + C_3\right] \quad (11)$$

$$\theta(\tilde{T}C(\tilde{p},\tilde{Q},\tilde{I})) = \frac{1}{6}\left[A(p-\delta_{11})(p+\delta_{12})^{-\beta} + \frac{A(S-\delta_1)(p+\delta_{12})^{-\beta}}{Q+\delta_{10}} + \frac{(H-\delta_3+m-\delta_5)(I-\delta_7)^2}{2(Q+\delta_{10})} + \frac{(Q-\delta_9)(m-\delta_5)}{2} - (I+\delta_8)(m+\delta_6)\right]$$
$$+ \frac{2}{3}\left[Ap^{1-\beta} + \frac{ASp^{-\beta}}{Q} + \frac{(H+m)I^2}{2Q} + \frac{Qm}{2} - \mathrm{Im}\right]$$
$$+ \frac{1}{6}\left[A(p+\delta_{12})(p-\delta_{11})^{-\beta} + \frac{A(S+\delta_2)(p-\delta_{11})^{-\beta}}{Q-\delta_9} + \frac{(H+\delta_4+m+\delta_6)(I+\delta_8)^2}{2(Q-\delta_9)} + \frac{(Q+\delta_{10})(m+\delta_6)}{2} - (I-\delta_7)(m-\delta_5)\right] \quad (12)$$

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V. COMPUTATION OF P,Q AND I USING KKT CONDITIONS APPROACH

Let $I_1=I-\delta_7$, $I_2=I$, $I_3=I+\delta_8$

$Q_1=Q-\delta_9$, $Q_2=Q$, $Q_3=Q+\delta_{10}$

$p_1=p-\delta_{11}$, $p_2=p$, $p_3=p+\delta_{12}$

Substituting the above equations in (12), we get

$$\theta(\tilde{T}C(\tilde{p},\tilde{Q},\tilde{I}))=\frac{1}{6}\left[\begin{array}{l} Ap_1p_3^{-\beta}+\dfrac{A(S-\delta_1)p_3^{-\beta}}{Q_3} \\ +\dfrac{(H-\delta_3+m-\delta_5)I_1^2}{2Q_3}+ \\ \dfrac{Q_1(m-\delta_5)}{2}-I_3(m+\delta_6) \end{array}\right] +\frac{2}{3}\left[\begin{array}{l} Ap_2^{1-\beta}+\dfrac{ASp_2^{-\beta}}{Q_2}+\dfrac{(H+m)I_2^2}{2Q_2} \\ +\dfrac{Q_2m}{2}-I_2m \end{array}\right] +\frac{1}{6}\left[\begin{array}{l} Ap_3p_1^{-\beta}+\dfrac{A(S+\delta_2)p_1^{-\beta}}{Q_1}+ \\ \dfrac{(H+\delta_4+m+\delta_6)I_3^2}{2Q_1} \\ +\dfrac{Q_3(m+\delta_6)}{2} \\ -I_1(m-\delta_5) \end{array}\right] \quad (13)$$

Where

$0\le I_1\le I_2\le I_3$, $0\le p_1\le p_2\le p_3$ and

$0\le Q_1\le Q_2\le Q_3$

The constraints are

$I_1-I_2\le 0$, $I_2-I_3\le 0$, $-I_1<0$,

$p_1-p_2\le 0,\ p_2-p_3\le 0, -p_1<0$

$Q_1-Q_2\le 0$, $Q_2-Q_3\le 0$, $-Q_1<0$

By KKT conditions approach,

$$\frac{1}{6}\left[\begin{array}{l}\dfrac{(H-\delta_3+m-\delta_5)I_1}{Q_3} \\ -(m-\delta_5)\end{array}\right]-\lambda_1+\lambda_3\le 0 \quad (14.1)$$

$$\frac{2}{3}\left[\frac{(H+m)I_2}{Q_2}-m\right]+\lambda_1-\lambda_2\le 0 \quad (14.2)$$

$$\frac{1}{6}\left[\begin{array}{l}-(m+\delta_6) \\ +\dfrac{(H+\delta_4+m+\delta_6)I_3}{Q_1}\end{array}\right]+\lambda_2\le 0 \quad (14.3)$$

$$\frac{1}{6}\left[\begin{array}{l}\dfrac{(m-\delta_5)}{2}-\dfrac{A(S+\delta_2)p_1^{-\beta}}{Q_1^2} \\ -\dfrac{(H+\delta_4+m+\delta_6)I_3^2}{2Q_1^2}\end{array}\right]-\lambda_4+\lambda_6\le 0 \quad (14.4)$$

$$\frac{2}{3}\left[\begin{array}{l}-\dfrac{ASp_2^{-\beta}}{Q_2^2}-\dfrac{(H+m)I_2^2}{2Q_2^2} \\ +\dfrac{m}{2}\end{array}\right]+\lambda_4-\lambda_5\le 0 \quad (14.5)$$

$$\frac{1}{6}\left[\begin{array}{l}-\dfrac{A(S-\delta_1)(p_3)^{-\beta}}{Q_3^2} \\ -\dfrac{(H-\delta_3+m-\delta_5)I_1^2}{2Q_3} \\ +\dfrac{m+\delta_6}{2}\end{array}\right]+\lambda_5\le 0 \quad (14.6)$$

$$\frac{1}{6}\left[\begin{array}{l}Ap_3^{-\beta}-A\beta p_3p_1^{-\beta-1} \\ -\dfrac{A\beta(S+\delta_2)p_1^{-\beta-1}}{Q_1}\end{array}\right]-\lambda_7+\lambda_9\le 0 \quad (14.7)$$

$$\frac{2}{3}\left[\begin{array}{l}A(1-\beta)p_2^{-\beta} \\ -\dfrac{AS\beta p_2^{-\beta-1}}{Q_2}\end{array}\right]+\lambda_7-\lambda_8\le 0 \quad (14.8)$$

$$\frac{1}{6}\left[\begin{array}{l}-Ap_1\beta)p_3^{-\beta-1} \\ -\dfrac{A\beta(S-\delta_1)p_3^{-\beta-1}}{Q_3} \\ +Ap_1^{-\beta}\end{array}\right]+\lambda_8\le 0 \quad (14.9)$$

Also

$$I_1\frac{\partial g}{\partial I_1}=0 \quad (14.10)$$

$$Q_1\frac{\partial g}{\partial Q_1}=0 \quad (14.11)$$

$$p_1\frac{\partial g}{\partial p_1}=0 \quad (14.12)$$

$$I_2\frac{\partial g}{\partial I_2}=0 \quad (14.13)$$

$$Q_2\frac{\partial g}{\partial Q_2}=0 \quad (14.14)$$

$$p_2\frac{\partial g}{\partial p_2}=0 \quad (14.15)$$

$$I_3\frac{\partial g}{\partial I_3}=0$$

(14.16)

$$Q_3 \frac{\partial g}{\partial Q_3} = 0 \quad (14.17)$$

$$p_3 \frac{\partial g}{\partial p_3} = 0 \quad (14.18)$$

$$I_1 - I_2 \le 0, I_2 - I_3 \le 0 \quad (14.19)$$

$$-I_1 \le 0 \quad (14.20)$$

$$Q_1 - Q_2 \le 0, Q_2 - Q_3 \le 0 \quad (14.21)$$

$$-Q_1 \le 0 \quad (14.22)$$

$$p_1 - p_2 \le 0, p_2 - p_3 \le 0 \quad (14.23)$$

$$-p_1 \le 0 \quad (14.24)$$

$$\lambda_1(I_1 - I_2) = 0, \lambda_2(I_2 - I_3) = 0 \quad (14.25)$$

$$-\lambda_3 I_1 = 0 \quad (14.26)$$

$$\lambda_4(Q_1 - Q_2) = 0, \lambda_5(Q_2 - Q_3) = 0 \quad (14.27)$$

$$-\lambda_6 Q_1 = 0 \quad (14.28)$$

$$\lambda_7(p_1 - p_2) = 0, \lambda_8(p_2 - p_3) = 0 \quad (14.29)$$

$$-\lambda_9 p_1 = 0 \quad (14.30)$$

Solving the equations (14.1) to (14.30), the optimum values of p,Q and I are as follows

$$p = \left[\frac{\beta^2 (H - \delta_3 + 4H + H + \delta_4)(S + \delta_2 + 4S + S - \delta_1)(m - \delta_5 + 4m + m + \delta_6)}{72A(1-\beta^2)(H - \delta_3 + m - \delta_5 + 4(H+m) + H + \delta_4 + m + \delta_6)} \right]^{\frac{1}{(2-\beta)}}$$

$$Q = \left[\frac{\beta(S + \delta_2 + 4S + S - \delta_1)}{6p(1-\beta)} \right]$$

$$I = \left[\frac{\beta(S + \delta_2 + 4S + S - \delta_1)(m - \delta_5 + 4m + m + \delta_6)}{6p(1-\beta)(H - \delta_3 + m - \delta_5 + 4(H+m) + H + \delta_4 + m + \delta_6)} \right]$$

VI. ALGORITHM FOR FINDING FUZZY DECISION VARIABLES AND FUZZY TOTAL COST

Step 1:

The crisp values of the decision variables p, Q and I are calculated from equations (4), (5) and (6) using input parameters S, H and m and hence the crisp total cost TC value is calculated from equation (1).

Step 2:

The fuzzy total cost is determined by the decision variables that are triangular fuzzy numbers.

Step 3:

Karush Kuhn-Tucker conditions Technique is used to obtain optimal values of the decision variables after applying the graded mean integration method for defuzzifying the total cost function.

VII. NUMERICAL EXAMPLE

The developed EOQ model under various environments is illustrated by assuming the following data

$\tilde{S} = (30,150,195)$, $\tilde{H} = (0.06, 0.25, 0.36)$, $\tilde{m} = (1,5,7.6)$

The demand D is dependent on unit price p is given by

$D = Ap^{-\beta}$

Where A = 100. The decision variables p, Q and I are calculated for different values of β and hence the corresponding demand and the total cost are also calculated (both crisp and fuzzy values).

VIII. SENSITIVITY ANALYSIS

Table 1:

Optimal policy for crisp and fuzzy by using triangular fuzzy number:

β	$\tilde{p}$	$\tilde{D}$	$\tilde{I}$	$\tilde{Q}$	TC (crisp)	$\tilde{TC}$ (Fuzzy)
0.86	4.71	26.38	170.85	179.33	167.52	164.66
0.87	5.56	22.48	157.67	165.50	164.95	162.31
0.88	6.64	18.90	144.67	151.86	162.18	159.73
0.89	8.07	15.59	131.33	137.85	159.14	156.92
0.90	9.97	12.62	118.25	124.12	155.80	153.82

The table shows that for different values of β, as the unit price value increases, values of lot size and maximum inventory decreases and hence the total cost value also decreases both in crisp and fuzzy environment.

IX. CONCLUSION

Our investigation deals with attaining both analytical solution and fuzzy optimal solution and a comparative study which meets the needs of the real world situation. It is concluded from this investigation that the impreciseness in inventory management can be overcome by using fuzzy input parameters and decision variables. Here the demand dependent on unit price is considered. The total annual cost for both crisp and fuzzy values are calculated which shows that the fuzzy values are better than the crisp one. Hence the conclusion drawn from this result is that the optimal solution is obtained under fuzzy environment compared to the crisp case.

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