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Strong Split Litact Domination in Graphs

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Abstract: ***In this paper the strong split litact domination number of some standard graphs like Cycle graph C_p, wheel graph W_p, Complete graph K_p, Bi-partite graph $K_{m,n}$ etc., are calculated and tried to bring up the relationship between the γ_{ssm} with different limiting factors of G and also with other domination limiting factors of G.***

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Keywords: ***Litact graph, Domination number, Litact domination number, Strong split litact domination number.***

I. INTRODUCTION

The notations used in this paper by us are the notations used by F.Harary. Simple, finite, non-trivial, undirected and connected are used to depict the notations. All the notations and corresponding definitions can be found in F.Harary[1] and V.R.Kulli[8]

II. DEFINITIONS

A. Induced Subgraph:

A subgraph which is induced by vertices of a graph G is a subset having end points as vertices together with edges of a graph. It is denoted by $\langle x \rangle$.

B. Open and closed Neighbourhood:

The neighbourhood does not include v itself called open neighbourhood and is denoted by $N_G(V)$. Neighbourhood in which v is included called the closed neighbourhood and is denoted by $N_G[V]$.

C. Cut vertex

A vertex is said to be a cut vertex iff the removal of it disconnects the graph.

D. Complement

For any set of two vertices u, v in G, if uv is an edge in G which is not an edge in $\bar{G}$ then $\bar{G}$ is said to be a complement of G.

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E. Litact Graph

The litact graph $m(G)$ of a graph G is the graph whose vertex set is the union of the set of edges and the set of cut vertices of G in which two vertices are adjacent if and only if the corresponding edges and cut vertices are adjacent or incident in G.

F. Litact dominating set

A dominating set $D \subseteq V(m(G))$ is called litact dominating set of G, if every vertex in $V(m(G)) - D$ is adjacent to a vertex v in D. Litact domination number of G, is denoted by $\gamma_m(G)$ and $\gamma_m(G) = min|D|$.

G. Split Litact Dominating set

A litact dominating set $D \subseteq V(m(G))$ is a split litact dominating set, if the sub graph $\langle V(m(G)) - D \rangle$ is disconnected. Split litact domination number of $m(G)$, is denoted by $\gamma_{sm}(G)$ and $\gamma_{sm}(G) = min|D|$.

H. Strong Split Litact Dominating set

A dominating set $D \subseteq V(m(G))$ is a strong split litact dominating set, if the induced subgraph $\langle V(m(G)) - D \rangle$ is totally disconnected with atleast two vertices. Strong split litact domination number of G, is denoted by $\gamma_{ssm}(G)$ and $\gamma_{ssm}(G) = min|D|$.

III. RESULTS

For more distant results we require the succeeding theorems.

Theorem A [8]: For each graph G, $\gamma(G) \leq p - \Delta(G)$.
Theorem B [8]: For each graph G, $\left\lceil \frac{p}{1+\Delta(G)} \right\rceil \leq \gamma(G)$.
Theorem C [8]: For each graph G, $\gamma(G) \leq \beta_0(G)$.
Theorem D [8]: For each graph G, $\alpha_0(G) + \beta_0(G) = p$ and if
G has no disconnected vertices, then
$\alpha_1(G) + \beta_1(G) = p$.
Theorem E [8]: If G has p vertices and no isolates then
$\gamma_t(G) \leq p - \Delta(G) + 1$.
Theorem F[8]: For any graph G, $p - q \leq \gamma(G)$, Further more
$\gamma(G) = p - q$ if and only if each component
of G is star.
Theorem G[8]: If G is a connected graph, then
$\left\lceil \frac{diam(G)}{3} \right\rceil \leq \gamma(G)$.
Theorem H[8]: For any connected graph, $\gamma_c(G) \leq p - \Delta(G)$.

IV. THEOREMS

The strong split litact domination number for some standard graphs is given below.

Theorem 1:

(i) For every cycle graph C_p, with $p \geq 3$vertices, $\gamma_{ssm}(C_p) = \begin{cases} \left\lfloor \frac{p-1}{3} \right\rfloor if\ p = 3 \\ \left\lfloor \frac{2p-1}{3} \right\rfloor\ if\ p > 3 \end{cases}$

(ii) For any path graph $P_p, p \geq 3$ vertices $\gamma_{ssm}(P_p) = \begin{cases} \frac{p}{3} & if\ p \equiv 0\ (mod3) \\ \left\lfloor \frac{p+2}{3} \right\rfloor + 1 & if\ p \equiv 1(mod3) \\ p, & otherwise \end{cases}$

(iii) For every wheel graph W_p , $p \geq 4$ vertices $\gamma_{ssm}(W_p) = \begin{cases} 4 & if\ p = 4 \\ p + 1 & if\ p > 4 \end{cases}$

(iv) For every complete graph $K_p$with $p \geq 3$ vertices, $\gamma_{ssm}(K_p) = \begin{cases} \frac{p}{3}\ if\ p = 3n\ when\ n = 1 \\ 2\left\lfloor \frac{p}{2} \right\rfloor + 1\ if\ p > 3n + 1\ when\ n > 1 \end{cases}$

(v) For every star graph $K_{1,p}$,$p \geq 3$, $\gamma_{ssm}(K_{1,p}) = 0$.

(vi) For any complete bipartite graph K_{p_1,P_2}, $p = p_1 + p_2$, $\gamma_{ssm}(K_{p_1,P_2}) = \begin{cases} \frac{p}{3}\ if\ p \equiv 0(mod3) \\ \frac{p+2}{3} if p \equiv 1(mod3) \\ p - 1\ if\ p \equiv 2(mod3) \end{cases}$

In the next theorem, we relates the $\gamma_{ssm}(G), \gamma_m(G), q\ \&\ \gamma(G)$.

Theorem 2:
In a graph $G, \gamma_{ssm}(G) + \gamma_m(G) < 2q + \gamma(G)$.

Proof: Let $E_1 = \{e_1, e_2, e_3, \ldots \ldots \ldots e_m\} \subseteq E(G)$ and $E_2 = E(G) - E_1, V(m(G)) = E_1 \cup E_2 \cup S$ where S is the set of cut vertices of G. Then $\exists$ a minimal set $E_1' \subseteq E_1$ in G and its corresponding vertex set D in $m(G)$ such that it covers all vertices of $m(G)$. Clearly D forms a minimal $\gamma - set$ of $m(G)$ that is $|D| = \gamma_m(G)$. Now suppose that D' be the dominating set which is minimal in $m(G)$ whose vertex set $\langle V(m(G)) - D \rangle$ is disconnected totally with atleast two vertices. Then $|D'| = \gamma_{ssm}(G)$. Let $E = \{e_1, e_2, e_3, \ldots \ldots \ldots e_n\}$ be the set of edges of G that is $|E(G)| = q$. Let $D_1 = \{v_1, v_2, \ldots \ldots \ldots v_n\} \subseteq V(G)$ be a minimum number of vertices which dominates all other vertices $V(G) - D_1$ in G i.e, $|D_1| = \gamma(G)$. We have $|D| \cup |D'| \subset 2|E(G)| \cup |D_1|$. Hence $\gamma_{ssm}(G) + \gamma_m(G) < 2q + \gamma(G)$.

In the following corollary, we obtain the relation between $\gamma_{ssm}(G), \gamma_m(G), p, q, \gamma(G)$ of G

Corollary 1: **In a graph G,**

$$\boldsymbol{\gamma_{ssm}(G) + \gamma_m(G) + p \leq 3q + 2\gamma(G)}.$$

Proof: Adding Theorem F and Theorem 2, we get $\gamma_{ssm}(G) + \gamma_m(G) + p \leq 3q + 2\gamma(G)$.

In the succeeding theorem, we relates $\gamma_{ssm}(G), diam(G)$, $\Delta(G)$,$\alpha_0(G)$ of G.

Theorem 3: **In a graph G,**

$$\boldsymbol{\gamma_{ssm}(G) < diam(G) + \Delta(G) + \alpha_0(G)}.$$

Proof: Let D be a $\gamma - set$ in $m(G)$ and Let $D' \subseteq D, D'$ be a $\gamma - set$ of $m(G)$ which is minimal, whose subgraph induced by the vertex set $\langle V(m(G)) - D \rangle$ is totally disconnected with atleast two vertices. Then $|D'| = \gamma_{ssm}(G)$. Let the minimal set of vertices be $A \subseteq V(G)$ which covers all the edges in G with $|A| = \alpha_0(G)$ and there exists an edge set $E'(G) \subseteq E(G)$ where $E(G)$ is the set of all edges which are incident with the vertices constituting the longest path in G such that $|E'| = diam(G)$. For any graph G there exists atleast one vertex $v \in V(G)$ of utmost degree $\Delta(G)$. It follows that $\gamma_{ssm}(G) < diam(G) + \Delta(G) + \alpha_0(G)$.

In the next corollary, we obtain the relation between $\gamma_{ssm}(G), diam(G), \gamma_t(G), \alpha_0(G)$ of G.

Corollary 2: **In a graph G,**

$$\boldsymbol{\gamma_t(G) + \gamma_{ssm}(G) \leq diam(G) + p + \alpha_0(G) + 1}.$$

Proof: Adding Theorem 3 and Theorem E we get the result as $\gamma_t(G) + \gamma_{ssm}(G) \leq diam(G) + p + \alpha_0(G) + 1$.

In the following corollary, we obtain the relation between $\gamma_{ssm}(G), diam(G), \gamma_c(G), \alpha_0(G), p$ of G.

Corollary 3: **In a graph G ,**

$$\boldsymbol{\gamma_c(G) + \gamma_{ssm}(G) \leq diam(G) + p + \alpha_0(G)}.$$

Proof: Adding Theorem 3 and Theorem H we get $\gamma_c(G) + \gamma_{ssm}(G) \leq diam(G) + p + \alpha_0(G)$.

In the succeeding theorem, we relates the $\gamma_{ssm}(G), q, \delta(G)$, p of G.

Theorem 4: **If G has no isolated vertices, then $\gamma_{ssm}(G) < p + \left\lfloor \frac{2q}{\delta(G)} \right\rfloor$ where $\delta(G)$the minimum degree of G and $\lfloor x \rfloor$ is the least positive integer not less than the real number x.**

Proof: Let D be a dominating set in $m(G)$ and induced subgraph $D' = \langle V(m(G)) - D \rangle$ is totally disconnected with atleast two vertices. Clearly $min|D'| = \gamma_{ssm}(G)$. Each vertex in $V - D$ is neighbouring with atleast $\delta(G)$ vertices in D. This implies that $2q > |V - D|\delta(G)$

$$\Rightarrow \left\lfloor \frac{2q}{\delta(G)} \right\rfloor > |V - D|$$

$$\Rightarrow |V - D| < \left\lfloor \frac{2q}{\delta(G)} \right\rfloor$$

$$\Rightarrow p + |V - D| < p + \left\lfloor \frac{2q}{\delta(G)} \right\rfloor \ldots \ldots .. (1)$$

Clearly $|D'| < p + |V - D| \ldots \ldots \ldots (2)$

From (1) and (2) $|D'| < p + |V - D| < p + \left\lfloor \frac{2q}{\delta(G)} \right\rfloor$

$$|D'| < p + \left\lfloor \frac{2q}{\delta(G)} \right\rfloor$$

$$\gamma_{ssm}(G) < p + \left\lfloor \frac{2q}{\delta(G)} \right\rfloor$$

In the next theorem, we relates $\gamma_{ssm}(G), diam(G)$of G.

Theorem 5 :

Let the connected graph be G. Then$\left\lceil\frac{diam(G)+1}{3}\right\rceil < \gamma_{ssm}(G)$,

$if\ \gamma_{ssm}(G) \neq 0and\left\lceil\frac{diam(G)+1}{3}\right\rceil > \gamma_{ssm}(G), if \gamma_{ssm}(G) = 0$.

Proof: Suppose $E(G) = \{e_1, e_2, e_3, \dots \dots \dots e_n\}$ and $S(G) = \{c_1, c_2, \dots \dots c_n\}$ be the edge set and cut vertex set of G respectively. Then $V(m(G)) = E(G) \cup S(G)$.Let $C = \{e_1, e_2, e_3, \dots \dots \dots e_j\}$, $1 \leq j \leq n$ constitute the diametral path in G. Then $|C| = diam(G)$. Let D be a dominating set in $m(G)$ and $D_1 \subseteq V(m(G)) - D$ such that $D_1 \in N(D)$, again we take $D_1' \subseteq D_1$ such that $H = [V(m(G)) - (D \cup D_1')]$ and $\langle H \rangle$ is disconnected totally with minimum two vertices. Hence $D \cup D_1' = \gamma_{ssm}(G)$. Further since $C \subseteq V(m(G))$ along with $D \cup D_1'$ be $\gamma_{ssm} - set$, the diametrical path contains utmost $\gamma_{ssm} - 1$ vertices which belongs to neighbourhood of $D \cup D_1'$ in $m(G)$. Hence $diam(G) < 2\gamma_{ssm} + \gamma_{ssm} - 1$ and this follows $\left\lceil\frac{diam(G)+1}{3}\right\rceil < \gamma_{ssm}(G)$.

In the following corollary, we obtain the relation between $\gamma_{ssm}(G), diam(G), \gamma(G)$of G.

Corollary 4:

If G is a connected graph then $\left\lceil\frac{diam(G)+1}{3}\right\rceil < \left\lceil\frac{\gamma_{ssm}(G)+\gamma(G)}{2}\right\rceil$ if $\gamma_{ssm}(G) \neq 0$ & $\gamma_{ssm}(G) < \gamma(G)$ if $\gamma_{ssm}(G) = 0$.

Proof**:** By adding Theorem 5 and Theorem G we get this result.

In the next theorem, we relates $\gamma_{ssm}(G), \gamma'(G)\ and\ \Delta'$(G)

Theorem 6: In a graph G, $\left\lceil\frac{\gamma_{ssm}(G)}{2}\right\rceil < \gamma'(G) + \Delta'(G)$

Proof: Let $H = \{e_1', e_2' \dots \dots \dots e_n'\}$ be a edge set which are minimal such that $N[H] = E(G)$.Edge dominating set $H \subseteq E(G)$ is such that each edge in $(E(G) - H)$ must be adjacent to atleast one edge in H. Edge domination number of G, is denoted by $\gamma'(G)$ and $\gamma'(G) = min|D|$. Suppose $e_i \in E(G)$ is the edge maximum degree in G. Let $H = \{e_1, e_2 \dots \dots e_n\}$ be the set of edges such that $N(H') \subset H$ and $|H'| = \Delta'(G)$.Thus $|H| \leq |E(G) - \Delta'(G)|$. Let D and D'be a $\gamma - set$ and minimal $\gamma - set$ in $m(G)$ respectively whose vertex set is the induced subgraph $\langle V(m(G)) - D \rangle$ which is totally disconnected with atleast two vertices where $D' \subseteq D$.Then $|D'| = \gamma_{ssm}(G)$. It follows that $\left\lceil\frac{D'}{2}\right\rceil < |H| + \Delta'(G)$.

$\Rightarrow \left\lceil\frac{\gamma_{ssm}(G)}{2}\right\rceil < \gamma'(G) + \Delta'(G)$.

In the next theorem, we get a relation between $\gamma_{ssm}(G)$ and q

Theorem 7: For each graph G, $\gamma_{ssm}(G) < 2q$.

Proof **:** Let $E = \{e_1, e_2, \dots \dots \dots e_n\}$ be the edge set in G, that is $|E(G)| = q$. Let D be a $\gamma - set$ in $m(G)$ and D' be a minimal $\gamma - set$ of $m(G)$ and $D' \subseteq D$ whose vertex set is the induced subgraph $\langle V(m(G)) - D \rangle$ which is totally disconnected with atleast two vertices, that is min$|D'| = \gamma_{ssm}(G)$. It follows that $\gamma_{ssm}(G) < 2q$.

In the next theorem, we relates $\gamma_{ssm}(G), \gamma(G), \gamma'(G), S_0$.

Theorem 8: For every graph G,

$$\left\lceil\frac{\gamma_{ssm}(G)}{3}\right\rceil < \gamma(G) + \gamma'(G) + S_0$$

Proof: Let $D = \{v_1, v_2, \dots \dots v_n\}$ be a set of vertices which are minimal such that for each $v_i \in D$ and $N[v_i] = V(G)$.Then D is minimal $\gamma - set$ of $G, |D| = \gamma(G)$. Also in $m(G)$ for every $v_i \in D, [N(v_i) \cap D] = \emptyset$ & $N[D] = V(m(G))$. Thus D is a $\gamma - set$ in $m(G)$.Let $E = \{e_1, e_2, \dots \dots \dots e_j\}$ be a minimal edge set of G, $\forall e_i \in E, N(e_i) \cap E = \emptyset$. Thus $|E| = \gamma'(G)$. Also for each $e_i \in E, e_i = v_i v_j$ such that either of v_i or v_j or both dominate some vertices of G. Thus atleast one of v_i or $v_j \in D$. Now we consider a dominating set D_1 of $m(G)$ which is not minimal and vertex set $V' = \{v_1, v_2, \dots \dots v_n\}$ such that $u_i v_i = e_i$ and each e_i is in some $\gamma'(G)$of $m(G)$. Let $S = \{c_1, c_2, \dots \dots c_n\}$be the cut vertex set of $m(G)$, $|S| = S_0$ such that $\langle V(m(G)) - (D_1 \cup V' \cup S_1) \rangle$ is totally disconnected with atleast two vertices where S_1contains atleast half vertices of S. Thus $(D_1 \cup V' \cup S_1)$is a strong split litact dominating set of G. Clearly $\frac{(D_1 \cup V' \cup S_1)}{3} \subset |D| \cup |F| \cup |S_1|$which results $\left\lceil\frac{\gamma_{ssm}(G)}{3}\right\rceil < \gamma(G) + \gamma'(G) + S_0$.

In the following theorem, we get a relation $p, \Delta(G), \gamma_{ssm}(G)$.

Theorem 9: In a graph G, $\left\lceil\frac{p}{1+\Delta(G)}\right\rceil \leq \gamma_{ssm}(G)$.Except if $\gamma_{ssm}(G) = 0$.

Proof: Suppose there exists a vertex $v \in G$ such that $v_i \in V$ are adjacent to V, if $max|\deg(v)| = \Delta(G)$ and $N(v_i) > \Delta(G) + 1$. Let D be a dominating set in G. Let D' be the $\gamma_{ssm} - set$ of G that is $|D'| = \gamma_{ssm}(G)$. Then, $|D'| \geq \left\lceil\frac{V(G)}{1+\Delta(G)}\right\rceil$

$$\left\lceil\frac{p}{1+\Delta(G)}\right\rceil \leq \gamma_{ssm}(G).$$

In the following corollary, we get a relation $p, \Delta(G), \gamma_{ssm}(G)$ and $\gamma(G)$.

Corollary 5: For any connected graph G,

$$\left\lceil\frac{p}{1+\Delta(G)}\right\rceil \leq \left\lceil\frac{\gamma_{ssm}(G)+\gamma(G)}{2}\right\rceil$$

Proof: By adding Theorem B and Theorem 9 we get $\left\lceil\frac{p}{1+\Delta(G)}\right\rceil \leq \left\lceil\frac{\gamma_{ssm}(G)+\gamma(G)}{2}\right\rceil$

In the next theorem, we relates $\alpha_0(T)$ and $\gamma_{ssm}(T)$

Theorem 10: For any tree, $\alpha_0(T) - 1 < \gamma_{ssm}(T)$ where $\alpha_0(T)$ is the vertex covering number of T.

Proof: Consider a tree T with $V(T) = \{v_1, v_2, \dots \dots v_n\}$.Let $V_1(T) \subseteq V(T)$ be a set of vertices where every edge has atleast one end point in the vertex cover that is, $|V_1| = \alpha_0(T)$. Let D' be $\gamma_{ssm} - set$ in $m(T)$, that is $|D'| = \gamma_{ssm}(T)$.Then $|V_1 - 1| < |D'|$.Clearly $\alpha_0(T) - 1 < \gamma_{ssm}(T)$.

In the next theorem we relates $\gamma_{ssm}(G), \gamma(G), p$ and $\gamma_c(G)$

Theorem 11:
For any graph G, $\gamma_{ssm}(G)+\gamma(G)<p+\gamma_c(G)+2$.
Equality holds for K_5.
Proof: Let $V(G)=\{v_1,v_2,\ldots\ldots v_n\}$ be the set of all vertices in G. Suppose $\exists$ a minimal vertex set $D=\{v_1,v_2,\ldots\ldots v_i\}\subseteq V(G)$ such that $N[V_k]=V(G),\forall V_k\in D,1\le k\le n$. Then D forms a dominating set which is minimal of G. $|D|=\gamma(G)$. Further, if the subgraph $\langle D\rangle$ has exactly one component, then D itself is connected dominating set of G. Suppose D has more than one component then attach the minimal set of vertices $D_1$of $V(G)-D_1$which are every in $u-v$ path, $\forall u,v\in D$ gives a single component $D_2=D\cup D_1$. Clearly D_2 forms a minimal γ_c-set of G. Let D' be $\gamma-set$ of $m(G)$ whose vertex set is subgraph $\langle V(m(G))-D'$ is totally disconnected with atleast two vertices where $D''\subseteq D'$ which gives a $\gamma_{ssm}-set$ in $m(G)$. It gives $|D''|\cup|D|\subset|V(G)|\cup|D_2|+2$. Hence $\gamma_{ssm}(G)+\gamma(G)<p+\gamma_c(G)+2$.

In the following corollary, we obtain a relation between $\gamma_{ssm}(G),\gamma_c(G)$ & $\Delta(G)$

Corollary 6: **For any graph G,**

$$\gamma_{ssm}(G)\le\gamma_c(G)+\Delta(G)+2.$$

Proof: By subtracting Theorem A from Theorem 11 we get $\gamma_{ssm}(G)\le\gamma_c(G)+\Delta(G)+2$.

In the following theorem, $\gamma_{ssm}(T),q$ and $\Delta'(T)$.

Theorem 12: **For any tree T, $\gamma_{ssm}(T)<q+\Delta'(T)$.**

Proof: Let D be $\gamma_{ssm}-set$ of T. Let $E(G)=\{e_1,e_2,\ldots\ldots e_n\}$ be the edge set in T. Let $\Delta'(T)$ be the maximum degree among all edges. Clearly, $|D|\subset E(T)\cup\Delta'(T)$.Hence $\gamma_{ssm}(T)<q+\Delta'(T)$.

In the next theorem, we relates $\gamma_{ssm}(G),\alpha_0(G)$ and $\beta_0(G)$

Theorem 13:
For any graph G, $\gamma_{ssm}(G)\le\alpha_0(G)+\beta_0(G)+2$.
Proof: Let $A=\{v_1,v_2,\ldots\ldots v_n\}\subseteq V(G)$ be the vertex set in G and $V_1=V(G)-A$. Suppose there exists set of vertices $C\subseteq V_1\ \ni\ N(u)\cap N(w)\neq\emptyset,\forall u,w\in C$. Further $N(x)\cap N(y)\neq\emptyset,\forall x\in A,y\in C$. Then $C\cup A$ forms a minimal independent set of vertices. If $A=\emptyset$, then C itself forms a maximal independent set of vertices in G. Let $B=\{v_1,v_{2,}\ldots\ldots v_k\}\subseteq V(G)$ be the set of vertices with $dist(u_i,v_j)\ge 2,1\le i\le j\le k,\forall v_i,v_j\in B$, all edges covered in G. Clearly B forms a vertex covering set. Let D be a dominating set in $m(G)$ and D' be a minimal dominating set of $m(G)$ whose vertex set is induced subgraph $\langle V(m(G))-D\rangle$ is totally disconnected with atleast two vertices where $D'\subseteq D$. Clearly D' forms a $\gamma_{ssm}-set$ in $m(G)$. It follows that, $|D'|\subseteq|B|\cup|C|+2$. Hence, $\gamma_{ssm}(G)\le\alpha_0(G)+\beta_0(G)+2$.

In the following corollary, we get $\gamma_{ssm}(G)$ and p

Corollary 7: In a graph G, $\gamma_{ssm}(G)\le p+2$.

Proof: From Theorem D and Theorem 13 we get $\gamma_{ssm}(G)\le p+2$.

In the following corollary, we get
$\gamma_{ssm}(G)$,$\gamma(G),\Delta(G)$and p

Corollary 8: **In a graph G,**

$$\gamma_{ssm}(G)+\gamma(G)\le 2p-\Delta(G)+2.$$

Proof: Adding Corollary 7 and Theorem A we get $\gamma_{ssm}(G)+\gamma(G)\le 2p-\Delta(G)+2$.

In the next theorem, we get a relation $\gamma_{ssm}(G),\alpha_1(G),\beta_1(G)$.

Theorem 14:
For each graph G, $\gamma_{ssm}(G)<2(\alpha_1(G)+\beta_1(G))$.
Proof: A set $F\subseteq E$ is an edge dominating set if each in E is either in F or is adjacent to an edge in F. The maximum cardinality among the independent set of edges of G is $\beta_1(G)$. Let $F_1=\{e_1,e_2,\ldots\ldots e_n\}$ be the edge set in G and that cover all the vertices. The minimum cardinality among edge cover is $\alpha_1(G)$. Let D' be $\gamma_{ssm}-set$ of G. Clearly $|D'|\subseteq 2(|F_1|\cup|F|)$. Hence, $\gamma_{ssm}(G)<2(\alpha_1(G)+\beta_1(G))$.

In the following corollary, we obtain a relation $\gamma_{ssm}(G)$ & p

Corollary 9: **In a graph G,$\gamma_{ssm}(G)<2p$.**

Proof: From Theorem D and Theorem 14 we get $\gamma_{ssm}(G)<2p$.

V. CONCLUDING REMARKS

Graph properties has a significant role in encryption .A dominating set of a graph is used to break the code easily. The concept of Strong split litact domination is a variant of usual domination. In the information retrieval system, the dominating set and the elements of strong split litact dominating sets can stand alone to make the process of communication more easy. Also it has wide applications in coding theory, computer science, switching circuits, electrical networks etc.,

In this paper we extend litact dominating set to strong split litact dominating set of a graph. The accurate values of this new variant is calculated for different graphs and obtained the upper and lower bounds of it with different parameters of a graph. One can extend this work by studying their applications in a wider sense.

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