A novel framework for Time Dependent Availability Analysis of Degraded Systems

Srinivasa Rao Meesala, Naikan V.N.A

Abstract: During the continuous production process, the machines and their components will degrade after several cycles of operations. So there is a need to evaluate these systems for their availability with respect to time to predict their multistage degradation process. The solution procedures for the evaluation of availability of modern complex systems adopted earlier in the literature are very complicated and mathematical cumbersome. Simulation approach for availability analysis of systems has been considered in the literature to get realistic solutions. This work is aimed to buildup a new system dynamics simulation for the time dependent availability analysis of a multistage degraded system. In this a three state degraded system with repair is analyzed and validated. It is proved that that this analysis is an substitute method for transient availability analysis.

Index Terms: Degraded systems, Markov approach, MSD approach, Time Dependent availability.

I. INTRODUCTION

Engineering systems will degrade with respect to time in their performance due to applied loads and other operating conditions. So there is an urgent need to study and analysis of multistage degraded systems in assessing their reliability. In general, the systems are continuously deteriorating by many causes. The techniques used for the analysis for modern complex systems are system specific and unrealistic in practical use. In the earlier research work about degradation of systems, a common assumption is that the system after repair is as good as new. However, it is not always true for a deteriorating system.

With increase in usage time, in case of mechanical systems, failure rate increases. Misra [1] developed simulation and modeling on the basis of Markov process using probabilistic approach for estimating time dependent availability. However the method is an analytical conventional approach. Gilseung et al.[2] described that existing network analysis methods such as the Markov network does not consider the time dependency of the unreliable edges. Arash *et al.*[3] presented maintenance time interval guidelines by considering both cost and system availability on a system that exhibits a linearly increasing hazard rate and a constant repair rate.

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Generally, the breakdown of the system depends on the condition of the system with respect to time. So, for the availability analysis of such systems, several levels of degradation must be considered. In fact, availability of the system decreases due to the degradation. The degradation process might take faster rate when the system reaches its last stages [4]. So that the state transition rates of these systems for the degradation process can be considered.

A. Related work

Several authors studied non repairable components with degradation in the literature. Girish et al [5] described a methodology for availability evaluation of manufacturing systems using semi markov model for each element at various hierarchical levels of the system by taking a Vertical Machining Center and analyzed the availability of the system by solving the mathematical equations using a computer program.

Hong et al [6] presented procedure for parameter estimation for positioning accuracy and output power of heavy machine tools analysis using a bivariate degradation model.

Kwok et al [7] proposed a two stage strategy to predict the condition of a bearing, where the degradation information was estimated by calculating the deviation of multiple statistics of vibration signals of a bearing from a known state. However, this method is system specific for condition monitoring of the systems.

Ramesh et al [8] studied a sugar industry to find its reliability by analyzing all its subsystems using supplementary variable technique for a feeding system. However, the solution procedure of this technique is mathematically cumbersome.

Takashi et al [9] analysed an aircraft using a failure prognostics methodology based on degradation messages with a particle filter framework.

However, when the system becomes very complex, these techniques become less amenable and the solution is difficult as described by several authors.

In this work an attempt is made by using the proposed MSD approach to solve either discrete or continuous Markov chains which are having more number of states. A stage wise procedure has been proposed by taking a degraded system. In this work time dependent availability analysis has been performed by the proposed method and the results obtained are compared with the conventional method.

The remaining sections of this work are organized as follows: The Markov system dynamics method for a degraded system is presented initially with conventional Markov analysis in section 2. Section 3 deals with the proposed MSD analysis for time dependent availability analysis of a degraded system. Results and analysis have given in Section4.The conclusions are presented in section 5.

II. TIME DEPENDENT AVAILABILITY ANALYSIS OF A DEGRADED SYSTEM

To mitigate the limitations of conventional Markov analysis for degraded systems, simulation has been used as a tool in the literature for various specific cases. The present authors have been proved that the continuous time Markov models are equivalent to system dynamics models ([10]-[11]). However, this analysis procedure has not used so far for degradation of systems in the literature. By taking this analysis procedure, in this section a Markov system dynamics model has been proposed for availability analysis of a degraded system.

Proposed Model

This work consists two distinct methods namely Markov analysis and the proposed MSD method which are constructed to model the availability of the system. To alleviate the limitation of the Markov approach, the MSD model has simulated and compared the results for availability modeling. Thereafter, time dependent availability analysis is performed on the same system.

A. Markov analysis for a degraded system

A degraded system with repair is considered in this work. In this analysis the assumptions made are as follows.

* The system may have functioning, degraded, or not worked states.

* The system can be repaired only once it has stopped.

Figure 1 shows the state transition diagram of this system. State1 represents that the system is functional; state 2 represents a degraded state and state represents not working state. According to Markov process, the related differential equations are as follows.

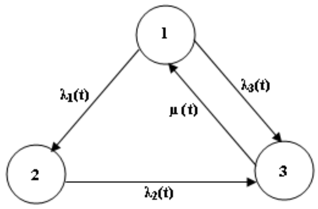

Fig1: State transition diagram of a three state degraded system with repair

Solution by Markov Analysis:

Markov Analysis for constant failure and repair rate case is presented in this section. The failure rates are λ_1 , λ_2, λ_3 indicated with z_1,z_2,$z_3$and the repair rate is *μ indicated with r*. State probabilities(P_1,P_2,P_3) are indicated with Q_1, Q_2,Q_3. The resulting Kolmogorov systems of differential equations are presented below.

$$\frac{dQ_1(t)}{dt} = rQ_3(t) - (z_1 + z_3)Q_1(t) \quad (1)$$

$$\frac{dQ_2(t)}{dt} = z_1Q_1(t) - (z_2Q_2(t)) \quad (2)$$

$$\frac{dQ_3(t)}{dt} = z_3Q_1(t) + (z_2Q_2(t)) - rQ_3(t) \quad (3)$$

Since sum of all system state probabilities is unity, we have;

$$Q_1(t) + Q_2(t) + Q_3(t) = 1 \quad (4)$$

The above Kolmogorov system of differential equations (1), (2) and (3) represent the rate of change of system up and down state probabilities. The point availability *A(t)* is the probability that this system is operating at time t. It is given by

$$A(t) = Q_1(t) + Q_2(t) \quad (5)$$

In general, to solve these Kolmogorov systems of differential equations, Laplace transforms technique has been used in the literature by the several authors. However, this solution procedure is very difficult and tedious.

In general, due to the difficulty in solving the Kolmogorov system of differential equations, steady state solution has been carried out by the several authors in the literature. This method is presented as follows.

1) Markov analysis for steady state availability:

According to the steady state solution (by assuming $t = \infty$) for this system, the system of differential equations (1), (2), (3), (4) become,

$$rQ_3 - (z_1 + z_3)Q_1 = 0 \quad (6)$$

$$z_1Q_1 - z_2Q_2 = 0 \quad (7)$$

$$z_3Q_1 + z_2Q_2 - rQ_3 = 0 \quad (8)$$

$$Q_1 + Q_2 + Q_3 = 1 \quad (9)$$

By solving the above steady state equations recursively, the steady state probabilities can be obtained as follows.

$$Q_1 = \left[1 + \frac{z_1}{z_2} + \frac{z_1+z_3}{r}\right]^{-1} \quad (10)$$

$$Q_2 = \frac{z_1}{z_2}Q_1 \quad (11)$$

$$Q_3 = \frac{z_1+z_3}{r}Q_1 \quad (12)$$

Steady state availability $A_s = Q_1 + Q_2$ (13)

In many practical situations it is very much required to find the steady state time for production planning and control. But this task is very difficult by using the aforementioned conventional Markov method. To mitigate this limitation, in this work MSD method has been proposed. The time dependent or transient availability *A(t)* of the system can also be evaluated very easily. The following sections would elaborate this process.

III. MSD ANALYSIS

The authors of this work established this MSD analysis [10]-[11]. However, the application of this analysis for degraded systems has not yet performed. The present work proposes MSD analysis for availability analysis of a three state degraded system as shown below.

A. Availability analysis of a degraded system using MSD method

This method consists following steps.

1*: Construction of State transition diagram*

This step has been described through Markov analysis in the section2. The remaining steps of MSD approach are as fallows.

2*: Data collection and analysis*

The data for this type of systems are generally available from the resources like data banks, lab testing and from the maintenance logbooks. A hypothetical example has been considered to analyze the degraded system which has any of the states like functioning, degraded, or not functioned. When functioning, it fails at the constant rate of 2 failures per day and becomes degraded at the rate of 3 failures per day. If degraded, its failure rate increases 1 failure per day.

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Repair occurs only in the failed mode and is to the functioning state with a repair rate of 10 repairs per day. If the functioning and degraded states are considered the available states, the steady state availability and at what time it is achieved need to be established.

3*: Developing the MSD simulation model*

Now it is required to convert the state transition diagram of system into the rate and level diagram as a system dynamic model. Figure2 shows a comprehensive system dynamic model .

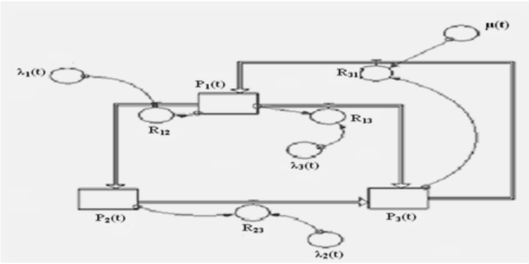

Fig2: A comprehensive Markov system dynamics model for a three state degraded system with repair

The system states are indicated with level variables $Q_1(t)$, $Q_2(t)$, $Q_3(t)$ and the state transitions are indicated with rate variables R_{12}, R_{13}, R_{23}, R_{31} with the corresponding transition rates z_1 (failure rate of system when functioning), z_2 (degraded failure rate of the system), z_3 (increased failure rate of the system after degradation) and r indicates its repair rate in the above figure. In this proposed approach the rate variables R_{ij} are equal to the product of level variables and the transition rates. The initial value of system availability is assumed as unity. In system dynamics analysis, the level of system availability decreases by the failure rate of the system and it recovers with the repair.

4: MSD simulation

Thereafter it is required to simulate the MSD model of the system by using the proposed algorithm. The state probabilities of the system have been calculated for availability analysis as described below.

MSD algorithm:

1:Inputs are $z_1(t)$, $z_2(t)$, $z_3(t)$, $r(t)$, *total time T and dt.*
2: Set $Q_1(t)$ *equal to one.*
3: Put $Q_i(t)$ *equal to zero for i= 2, 3.*
4: , t<T , A conditional loop is formed.
5: In each run, t is incremented by dt, i.e. t =t + dt.
6: When $Q_1(t)$ *becomes zero, the system fails. Run a conditional loop as* $Q_1(t)>0$ *is satisfied.*
7: The state probabilities are as follows.

$$Q_1(t) = Q_1(t\text{-}dt) + (R_{31}\text{-}R_{12}\text{-}R_{13})*dt \quad (1)$$

$$Q_2(t) = Q_2(t\text{-}dt) + (R_{12}\text{--}R_{23})*dt \quad (2)$$

$$Q_3(t) = Q_3(t\text{-}dt) + (R_{13}+R_{23}\text{-}R_{31})*dt \quad (3)$$

8: All the required results are displayed

5: The Model Experimentation

The model analysis can be performed as given below.

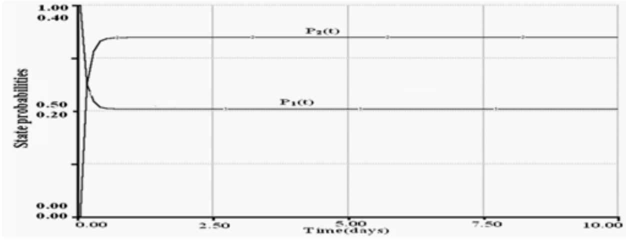

Fig3: Probabilities of operating, degraded states

In this experimentation, within the simulation of 10 days, the steady state obtained at 1.375 days . The simulation results in Figure 4 indicate that due to degradation, the operating state decreases and reaches its steady state therafter to the failed state. $Q_1(t)$, $Q_2(t)$, $Q_3(t)$ curves indicates these scenarios in Figure4. The system time dependent availability can be obtained by the sum of $Q_1(t)$, $Q_2(t)$ state probabilities.

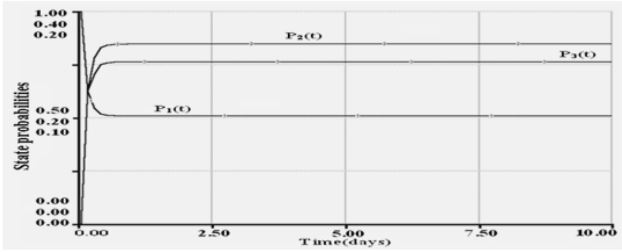

Fig4: Probabilities of operating, degraded, failed states

IV. RESULTS AND ANALYSIS

Markov system dynamics simulation results are presented in this section. Constant failure and repair rates are considered as a first case. Time varying ailure and repair rates are incorporated in the later case.

*1: **Availability Analysis with Constant Parameters***

The simulation results obtained are presented in Table1, Figure4 and Figure5. The steady state obtained at 1.375 days and its availability is 0.847458. The steady state availability of the same system evaluated by conventional Markov analysis (equation 21) is 0.847457. This is closely matching with the results of simulation.

Table1: System state probabilities and its time dependent availability values (Case 1)

Time (days)	System state probabilities			Transient availability from MSD approach
	$P_1(t)$	$P_2(t)$	$P_3(t)$	$A_s(t) = P_1(t) + P_2(t)$
0	1.000000	0.000000	0.000000	1.000000
0.5	0.511475	0.336914	0.151611	0.848389
1	0.508493	0.338970	0.152537	0.847463
1.375	**0.508475**	**0.338983**	**0.152542**	**0.847458***
2	0.508475	0.338983	0.152542	0.847458
3	0.508475	0.338983	0.152542	0.847458
4	0.508475	0.338983	0.152542	0.847458
5	0.508475	0.338983	0.152542	0.847458
6	0.508475	0.338983	0.152542	0.847458
7	0.508475	0.338983	0.152542	0.847458
8	0.508475	0.338983	0.152542	0.847458
9	0.508475	0.338983	0.152542	0.847458
10	0.508475	0.338983	0.152542	0.847458

* Steady state availability from the proposed approach is 0.847458
Steady state availability from conventional approach (A_c) is 0.847457

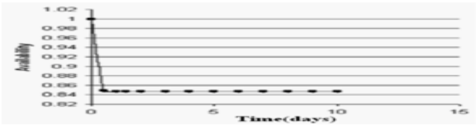

Fig5: Transient availability Vs Time (Case 1)

2: Time dependent Availability Analysis with Time Varying Parameters

In this section, the failure and repair rates which are not constant have been considered in the model. In this analysis, failure rates follows Weibull distribution with parameters $\beta_1 = 2.3, \theta_1 = 26, \beta_2 = 2.2, \theta_2 = 21, \beta_3 = 2.6, \theta = 29$ and repair rate follows Weibull distribution with parameters $\beta_4 = 3.0, \theta_4 = 18$. The simulation has been done for 300 days. The results are presented in Table2 and Figure 6. The Table 2 indicates the time dependent probabilities. From this it is clear that system time dependent availability declining from 1 to 0.660731.

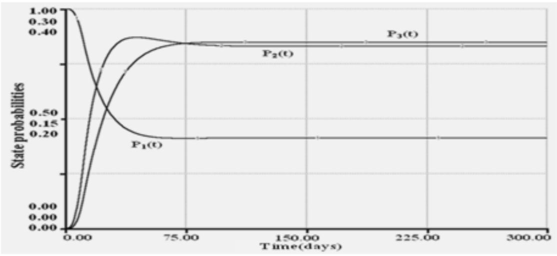

Fig6: Probabilities of operating, degraded, failed states with time varying failure rates (Case 2)

Table2: System state probabilities and its transient availability with Time Varying Failure and Repair Rates

Time(days)	System state probabilities			Transient availability from MSD approach
	$P_1(t)$	$P_2(t)$	$P_3(t)$	$A_s(t) = P_1(t) + P_2(t)$
0	1.000000	0.000000	0.000000	1.000000
30	0.493835	0.250911	0.255254	0.744746
60	0.411681	0.256815	0.331504	0.668496
90	0.410438	0.250384	0.339178	**0.660822**
120	0.411583	0.249130	0.339288	0.660714
123.125	**0.411612**	**0.249099**	**0.339269**	**0.660711***
150	0.411612	0.249099	0.339269	0.660711
180	0.411612	0.249099	0.339269	0.660711
210	0.411612	0.249099	0.339269	0.660711
240	0.411612	0.249099	0.339269	0.660711
270	0.411612	0.249099	0.339269	0.660711
300	0.411612	0.249099	0.339269	0.660711

* Steady state availability

From the above results it is proved that Markov System Dynamics analysis as an alternative method for time varying availability analysis of degraded systems.

V. CONCLUSIONS

To evaluate time dependent availability analysis of the degraded systems, a new System Dynamics based Markov method has been implemented in this work. A multi state degraded system has been modeled and analyzed. The results obtained by the MSD method are compared with the conventional Markov method. It is proved that the proposed MSD analysis can be applied for modeling of degraded systems.

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