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Pythagorean Fuzzy Labeling Graph

P. Ashwini Sibiya Rani, T. Bharathi

***Abstract*: The objective of this paper is to introduce a new concept of Pythagorean Fuzzy Labeling. If a fuzzy labeling is a Pythagorean Fuzzy Labeling, then the membership value of the edges is strictly less than the corresponding membership value of its incident vertices and the non - membership value of the edges is strictly greater than the corresponding non - membership value of the incident vertices of a graph. The sub graphs and union of Pythagorean Fuzzy Labeling Graphs are defined.**

***Index Terms*: Pythagorean fuzzy graph, Pythagorean fuzzy labeling graph, Pythagorean fuzzy labeling sub graph.**

I. INTRODUCTION

The idea of fuzzy set was originated by Lotfi A Zadeh [6] in 1965 where the membership degree μ of an element in a set is discussed. In 1983, Krassimir T Atanassov [4] extended the concept by adding the degree of non membership and called it as Intuitionistic Fuzzy Set (IFS) such that the membership degree μ and the non membership degree ν of an element satisfies the condition $0 \le \mu + \nu \le 1$ and the hesitancy grade is given as $\pi = \sqrt{1-\mu-\nu}$. In 1973, the concept of fuzzy graph was defined by Arnold Kaufmann [1] and it was extended by Azriel Rosenfeld [2] in 1975. The idea of fuzzy labeling was introduced by Nagoor A Gani and Rajalaxmi [3] and they discussed the properties of fuzzy labeling cycle. Krassimir T Atanassov [5] proposed the concept of Intuitionistic Fuzzy Graph (IFG) and discussed the Intuitionistic Fuzzy Relation (IFR). Intuitionistic Fuzzy Labeling Graph (IFLG) was introduced by Sahoo S and Pal M [12] in which they defined intutitionistic fuzzy labeling tree and its properties.

The limitation $\mu + \nu \le 1$ in IFS restrict the choice of membership and non membership degree. To overcome this situation, Ronald R Yager [10], introduced a new class of non standard fuzzy sets named Pythagorean Fuzzy Set (PFS) where the membership and non membership degree satisfies the condition $0 \le \mu^2 + \nu^2 \le 1$ and the degree of hesitancy is given as $\pi = \sqrt{1-\mu^2-\nu^2}$. The negation and basic set operations of Pythagorean membership grades had been discussed by Ronald R Yager and Ali M Abbasov [9]. Naz et.al [11] introduced the concept of Pythagorean Fuzzy Graph (PFG) and developed a series of operational laws for PFGs. They also investigated their properties and determined the degree and total degree of vertices in PFGs. The rejection and symmetric difference of PFGs has been given by Muhammad Akram et.al [7] and they also discussed the application of PFGs in decision making. The maximal product and residue product of PFGs were discussed by Muhammad Akram et.al [8].

II. PRELIMINARIES

Let X be a non empty set. A *Pythagorean Fuzzy Set* (PFS) [8] $\underset{\sim}{A}$ in X is given by

$$\underset{\sim}{A} = \left\{\left(a, \mu_{\underset{\sim}{A}}(a), \nu_{\underset{\sim}{A}}(a)\right) \text{such that } a \in X\right\}$$

where $\mu_{\underset{\sim}{A}} : X \to [0,1]$ and $\nu_{\underset{\sim}{A}} : X \to [0,1]$ represent the *degree of membership* (*d.m*) and *degree of non membership* (*d.n.m*) of $\underset{\sim}{A}$ respectively. Also, $\mu_{\underset{\sim}{A}}$ and $\nu_{\underset{\sim}{A}}$ satisfies the condition $0 \le \mu_{\underset{\sim}{A}}^2(a) + \nu_{\underset{\sim}{A}}^2(a) \le 1$ for all $a \in X$.

A *Pythagorean Fuzzy Relation (PFR)* [11] of the Pythagorean Fuzzy Set $\underset{\sim}{B}$ in $X * X$ is given by

$$\underset{\sim}{B} = \{((a_i, a_{i+1}), \mu_{\underset{\sim}{B}}(a_i, a_{i+1}), \nu_{\underset{\sim}{B}}(a_i, a_{i+1})) \text{ such that } (a_i, a_{i+1}) \in X * X\}$$

where $\mu_{\underset{\sim}{B}} : X * X \to [0,1]$ and $\nu_{\underset{\sim}{B}} : X * X \to [0,1]$ represent the *d.m* and *d.n.m* of *A* respectively. Also, $\mu_{\underset{\sim}{B}}$ and $\nu_{\underset{\sim}{B}}$ satisfies the condition

$$0 \le \mu_{\underset{\sim}{B}}^2(a_i, a_{i+1}) + \nu_{\underset{\sim}{B}}^2(a_i, a_{i+1}) \le 1 \text{ for all } (a_i, a_{i+1}) \in X * X$$

Let $G = (V, E)$ be a *Pythagorean Fuzzy Graph* (PFG) [11] with Pythagorean Fuzzy Set V on X and Pythagorean Fuzzy Relation E on X such that

$$\mu_2(a_i, a_{i+1}) \le \min(\mu_1(a_i), \mu_1(a_{i+1})),$$

$$\nu_2(a_i, a_{i+1}) \ge \max(\nu_1(a_i), \nu_1(a_{i+1})) \text{ and}$$

$$0 \le \mu_2^2(a_i, a_{i+1}) + \nu_2^2(a_i, a_{i+1}) \le 1$$

for all $(a_i, a_{i+1}) \in X * X$. The *d.m* and *d.n.m* of the vertices are represented as $\mu_1 : V \to [0,1]$ and $\nu_1 : V \to [0,1]$ and the *d.m* and *d.n.m* of the edges are represented as $\mu_2 : V * V \to [0,1]$ and $\nu_2 : V * V \to [0,1]$.

Let $G = (V, E)$ be a PFG. The *strength of a path* [8] (a_i, a_j) in G is denoted as (μ^∞, ν^∞) and defined as $\mu^\infty = \min(\mu_2(a_i, a_j))$ and $\nu^\infty = \max(\nu_2(a_i, a_j))$.

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* Correspondence Author
P. Ashwini Sibiya Rani*, P.G and Research Department of Mathematics, Loyola College, University of Madras, Chennai – 600034, Tamil Nadu, India.
T. Bharathi, P.G and Research Department of Mathematics, Loyola College, University of Madras, Chennai – 600034, Tamil Nadu, India.

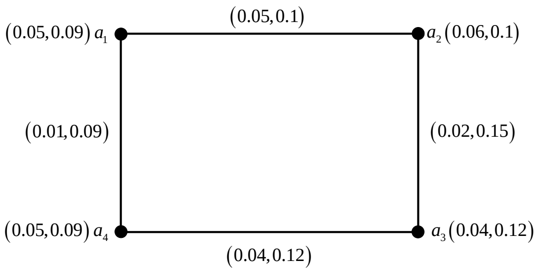

Fig. 1. Pythagorean Fuzzy Graph

The *strength of connectedness* [8] of a pair of vertices $a_i, a_j \in V$ in a PFG is defined as

$$CONN_G(a_i,a_j) = (\mu - CONN_G(a_i,a_j), v - CONN_G(a_i,a_j)) \quad (1)$$

such that

$$\mu - CONN_G(a_i,a_j) = \max(\mu^{\infty}(a_i,a_j)) \text{ and}$$
$$v - CONN_G(a_i,a_j) = \min(v^{\infty}(a_i,a_j))$$

where $\mu - CONN_G(a_i,a_j)$ represent the μ *strength of connectedness* and $v - CONN_G(a_i,a_j)$ represent the v *strength of connectedness* of the path (a_i,a_j) in G.

An edge (a_i,a_{i+1}) is called a *strong edge* [8] if

$$\mu(a_i,a_{i+1}) \geq \mu - CONN_G(a_i,a_j) \text{ and}$$
$$v(a_i,a_{i+1}) \leq v - CONN_G(a_i,a_j).$$

The union [11] of two PFG $G_1 = (V_1,E_1)$ and $G_2 = (V_2,E_2)$ is defined as follows

$$\left(\mu_{V_1} \cup \mu_{V_2}\right)(a_i) = \begin{cases} \mu_{V_1}(a_i) & if\ a_i \in V_1 - V_2 \\ \mu_{V_2}(a_i) & if\ a_i \in V_2 - V_1 \\ \max(\mu_{V_1}(a_i), \mu_{V_2}(a_i)) & if\ a_i \in V_1 \cap V_2 \end{cases}$$

$$\left(v_{V_1} \cup v_{V_2}\right)(a_i) = \begin{cases} v_{V_1}(a_i) & if\ a_i \in V_1 - V_2 \\ v_{V_2}(a_i) & if\ a_i \in V_2 - V_1 \\ \min(v_{V_1}(a_i), v_{V_2}(a_i)) & if\ a_i \in V_1 \cap V_2 \end{cases}$$

$$\left(\mu_{E_1} \cup \mu_{E_2}\right)(a_i,a_{i+1}) = \begin{cases} \mu_{E_1}(a_i,a_{i+1}) & if\ (a_i,a_{i+1}) \in E_1 - E_2 \\ \mu_{E_2}(a_i,a_{i+1}) & if\ (a_i,a_{i+1}) \in E_2 - E_1 \\ \max(\mu_{E_1}(a_i,a_{i+1}), \mu_{E_2}(a_i,a_{i+1})) & \\ & if\ (a_i,a_{i+1}) \in E_1 \cap E_2 \end{cases}$$

$$\left(v_{E_1} \cup v_{E_2}\right)(a_i,a_{i+1}) = \begin{cases} v_{E_1}(a_i,a_{i+1}) & if\ (a_i,a_{i+1}) \in E_1 - E_2 \\ v_{E_2}(a_i,a_{i+1}) & if\ (a_i,a_{i+1}) \in E_2 - E_1 \\ \min(v_{E_1}(a_i,a_{i+1}), v_{E_2}(a_i,a_{i+1})) & \\ & if\ (a_i,a_{i+1}) \in E_1 \cap E_2 \end{cases}$$

III. PYTHAGOREAN FUZZY LABELING GRAPH

A Pythagorean Fuzzy Graph $G = (V,E)$ is said to be a *Pythagorean Fuzzy Labeling Graph,* Ω_{PFLG} if $\mu_1 : V \to [0,1]$, $v_1 : V \to [0,1]$ and $\mu_2 : V * V \to [0,1]$, $v_2 : V * V \to [0,1]$ are bijective such that the degree of membership and degree of non membership of the vertices and edges are distinct.

For every edge (a_i,a_{i+1}),

$$\mu_2(a_i,a_{i+1}) < \min(\mu_1(a_i), \mu_1(a_{i+1})),$$
$$v_2(a_i,a_{i+1}) > \max(v_1(a_i), v_1(a_{i+1}))$$

such that

$$0 \leq \mu_1^{\,2}(a_i) + v_1^{\,2}(a_i) \leq 1 \text{ and}$$
$$0 \leq \mu_2^{\,2}(a_i,a_{i+1}) + v_2^{\,2}(a_i,a_{i+1}) \leq 1.$$

The Pythagorean Fuzzy Labeling Graph, $G' = (V',E')$ is called a *Pythagorean Fuzzy Labeling Subgraph*, Ω'_{PFLG} of $G = (V,E)$, if $\mu_1' : V' \to [0,1]$, $v_1' : V' \to [0,1]$ and $\mu_2' : V' * V' \to [0,1]$, $v_2' : V' * V' \to [0,1]$ are the *d.m* and *d.n.m* of the vertices and edges respectively. And for every $a_i \in V$, $\mu_1'(a_i) \leq \mu_1(a_i)$, $v_1'(a_i) \geq v_1(a_i)$ such that $0 \leq {\mu_1'}^2(a_i) + {v_1'}^2(a_i) \leq 1$. For every $(a_i,a_{i+1}) \in E$,

$$\mu_2'(a_i,a_{i+1}) \leq \mu_2(a_i,a_{i+1}),$$
$$v_2'(a_i,a_{i+1}) \geq v_2(a_i,a_{i+1})$$

such that

$$0 \leq {\mu_2'}^2(a_i,a_{i+1}) + {v_2'}^2(a_i,a_{i+1}) \leq 1.$$

The *strength of a path* (a_i,a_j) in Ω'_{PFLG}, $G' = (V',E')$ is denoted as $({\mu'}^{\infty}, {v'}^{\infty})$ and defined as ${\mu'}^{\infty} = \min(\mu_2'(a_i,a_j))$ and ${v'}^{\infty} = \max(v_2'(a_i,a_j))$.

The *strength of connectedness* of a pair of vertices $a_i, a_j \in V'$ in a Ω'_{PFLG} is defined as

$$CONN_{G'}(a_i,a_j) = (\mu - CONN_{G'}(a_i,a_j), v - CONN_{G'}(a_i,a_j)) \quad (2)$$

such that

$$\mu - CONN_{G'}(a_i,a_j) = \max({\mu'}^{\infty}(a_i,a_j)) \text{ and}$$
$$v - CONN_{G'}(a_i,a_j) = \min({v'}^{\infty}(a_i,a_j)).$$

where $\mu - CONN_{G'}(a_i,a_j)$ represent the μ *strength of connectedness* and $v - CONN_{G'}(a_i,a_j)$ represent the v *strength of connectedness* of the path (a_i,a_j) in G'.

Proposition 3.1. If $G' = (V',E')$ is a Pythagorean Fuzzy Labeling Subgraph of $G = (V,E)$ then ${\mu'}^{\infty}(a_i,a_j) \leq \mu^{\infty}(a_i,a_j)$ and ${v'}^{\infty}(a_i,a_j) \geq v^{\infty}(a_i,a_j)$ for all $(a_i,a_j) \in E$.

Proof:

Let $G = (V,E)$ be any Ω_{PFLG} and let (a_i,a_j) be any path in *G*, then its strength is given by

$$(\mu^{\infty}(a_i,a_j), v^{\infty}(a_i,a_j)).$$

Let $G' = (V',E')$ be Ω'_{PFLG} of G. For any path (a_i,a_j) in G', its strength is given by

$$({\mu'}^{\infty}(a_i,a_j), {v'}^{\infty}(a_i,a_j)).$$

Since G' is a subgraph of G, by definition of Ω'_{PFLG}, $\mu_1'(a_i) \le \mu_1(a_i)$, $\nu_1'(a_i) \ge \nu_1(a_i)$ for all $a_i \in V$ and for all $(a_i, a_{i+1}) \in E$,

$$\mu_2'(a_i, a_{i+1}) \le \mu_2(a_i, a_{i+1}) \text{ and}$$
$$\nu_2'(a_i, a_{i+1}) \ge \nu_2(a_i, a_{i+1}).$$

$\Rightarrow \mu'^{\infty}(a_i, a_j) \le \mu^{\infty}(a_i, a_j)$, $\nu'^{\infty}(a_i, a_j) \ge \nu^{\infty}(a_i, a_j)$ for all $(a_i, a_j) \in E$.

Proposition 3.2. If $G' = (V', E')$ is a Pythagorean Fuzzy Labeling Subgraph of $G = (V, E)$ then $\mu - CONN_{G'}(a_i, a_j) \le \mu - CONN_G(a_i, a_j)$, $\nu - CONN_{G'}(a_i, a_j) \ge \nu - CONN_G(a_i, a_j)$ for all $(a_i, a_j) \in E$.

Proof:

Let $G = (V, E)$ be any Ω_{PFLG}.

For any path (a_i, a_j) in G, the strength of connectedness is given by

$$(\mu - CONN_G(a_i, a_j), \nu - CONN_G(a_i, a_j)).$$

Let $G' = (V', E')$ be Ω'_{PFLG} of G. For any path (a_i, a_j) in G, the strength of connectedness is given by

$$(\mu - CONN_{G'}(a_i, a_j), \nu - CONN_{G'}(a_i, a_j)).$$

By proposition 3.1,

$\mu'^{\infty}(a_i, a_j) \le \mu^{\infty}(a_i, a_j)$, $\nu'^{\infty}(a_i, a_j) \ge \nu^{\infty}(a_i, a_j)$ for all $(a_i, a_j) \in E$.

$$\Rightarrow \mu - CONN_{G'}(a_i, a_j) \le \mu - CONN_G(a_i, a_j),$$
$$\nu - CONN_{G'}(a_i, a_j) \ge \nu - CONN_G(a_i, a_j)$$

for all $(a_i, a_j) \in E$.

Example:

Consider a graph $G = (V, E)$ such that $V = \{a_1, a_2, a_3, a_4\}$ and $E = \{a_1a_2, a_2a_3, a_3a_4, a_4a_1, a_1a_3\}$. Let $\underset{\sim}{A}$ and $\underset{\sim}{B}$ be the Pythagorean fuzzy vertex set and Pythagorean fuzzy edge set defined on V and E respectively.

$$\underset{\sim}{A} = \left\{ \left(\frac{a_1}{0.05}, \frac{a_2}{0.06}, \frac{a_3}{0.07}, \frac{a_4}{0.1} \right), \left(\frac{a_1}{0.09}, \frac{a_2}{0.1}, \frac{a_3}{0.09}, \frac{a_4}{0.06} \right) \right\} \text{ and}$$

$$\underset{\sim}{B} = \left\{ \left(\frac{a_1a_2}{0.04}, \frac{a_2a_3}{0.02}, \frac{a_3a_4}{0.03}, \frac{a_4a_1}{0.04}, \frac{a_1a_3}{0.01} \right), \left(\frac{a_1a_2}{0.13}, \frac{a_2a_3}{0.14}, \frac{a_3a_4}{0.15}, \frac{a_4a_1}{0.16}, \frac{a_1a_3}{0.9} \right) \right\}$$

Consider the path $a_1 - a_3$ in G.

The strength of the path $a_1 - a_3$ in G is denoted by $(\mu^{\infty}, \nu^{\infty})$.

Let the strength of the path a_1, a_2, a_3 be denoted as $(\mu_1^{\infty}, \nu_1^{\infty})$.

$$\mu_1^{\infty} = \min(\mu_2(a_1, a_2), \mu_2(a_2, a_3)) = \min(0.04, 0.02) = 0.02$$
$$\nu_1^{\infty} = \max(\nu_2(a_1, a_2), \nu_2(a_2, a_3)) = \max(0.13, 0.14) = 0.14$$
$$\Rightarrow (\mu_1^{\infty}, \nu_1^{\infty}) = (0.02, 0.14).$$

Let the strength of the path a_1, a_4, a_3 be denoted as $(\mu_2^{\infty}, \nu_2^{\infty})$.

$$\mu_2^{\infty} = \min(\mu_2(a_1, a_4), \mu_2(a_4, a_3)) = \min(0.04, 0.03) = 0.03$$
$$\nu_2^{\infty} = \max(\nu_2(a_1, a_4), \nu_2(a_4, a_3)) = \max(0.16, 0.15) = 0.16$$
$$\Rightarrow (\mu_2^{\infty}, \nu_2^{\infty}) = (0.03, 0.16).$$

Let the strength of the path a_1, a_3 be denoted as $(\mu_3^{\infty}, \nu_3^{\infty})$

$$\Rightarrow (\mu_3^{\infty}, \nu_3^{\infty}) = (0.01, 0.9).$$

By (1),

$$CONN_G(a_1, a_3) = (\mu - CONN_G(a_1, a_3), \nu - CONN_G(a_1, a_3))$$
$$\mu - CONN_G(a_1, a_3) = \max(\mu_1^{\infty}, \mu_2^{\infty}, \mu_3^{\infty}) = \max(0.02, 0.03, 0.01) = 0.03$$
$$\nu - CONN_G(a_1, a_3)) = \min(\nu_1^{\infty}, \nu_2^{\infty}, \nu_3^{\infty}) = \min(0.14, 0.16, 0.9) = 0.14$$
$$\Rightarrow CONN_G(a_1, a_3) = (0.03, 0.14).$$

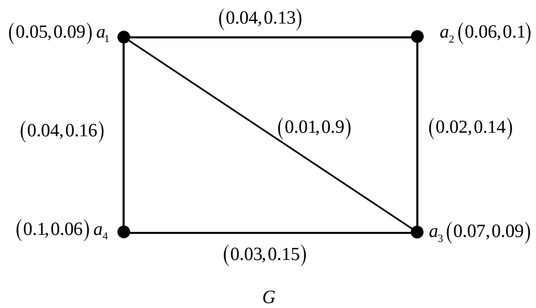

Fig. 2. Pythagorean Fuzzy Labeling Graph

Consider a sub graph $G' = (V', E')$ of graph G such that $V' = \{a_1, a_2, a_3, a_4\}$ and $E' = \{a_1a_2, a_2a_3, a_3a_4, a_4a_1\}$. Let $\underset{\sim}{A}'$ and $\underset{\sim}{B}'$ be the Pythagorean fuzzy vertex set and Pythagorean fuzzy edge set defined on V' and E' respectively.

$$\underset{\sim}{A}' = \left\{ \left(\frac{a_1}{0.03}, \frac{a_2}{0.04}, \frac{a_3}{0.02}, \frac{a_4}{0.07} \right), \left(\frac{a_1}{0.15}, \frac{a_2}{0.12}, \frac{a_3}{0.8}, \frac{a_4}{0.08} \right) \right\} \text{ and}$$

$$\underset{\sim}{B}' = \left\{ \left(\frac{a_1a_2}{0.01}, \frac{a_2a_3}{0.01}, \frac{a_3a_4}{0.015}, \frac{a_4a_1}{0.02} \right), \left(\frac{a_1a_2}{0.18}, \frac{a_2a_3}{0.9}, \frac{a_3a_4}{0.85}, \frac{a_4a_1}{0.17} \right) \right\}$$

Consider the path $a_1 - a_3$ in G'.

The strength of the path $a_1 - a_3$ in G' is denoted by $(\mu'^{\infty}, \nu'^{\infty})$.

Let the strength of the path a_1, a_2, a_3 be denoted as $(\mu_1'^{\infty}, \nu_1'^{\infty})$.

$$\mu_1'^{\infty} = \min(\mu_2(a_1, a_2), \mu_2(a_2, a_3)) = \min(0.01, 0.01) = 0.01$$
$$\nu_1'^{\infty} = \max(\nu_2(a_1, a_2), \nu_2(a_2, a_3)) = \max(0.18, 0.9) = 0.9$$
$$\Rightarrow (\mu_1'^{\infty}, \nu_1'^{\infty}) = (0.01, 0.9).$$

Let the strength of the path a_1, a_4, a_3 be denoted as $(\mu_2'^{\infty}, \nu_2'^{\infty})$.

$$\mu_2'^{\infty} = \min(\mu_2(a_1, a_4), \mu_2(a_4, a_3)) = \min(0.02, 0.015) = 0.015$$
$$\nu_2'^{\infty} = \max(\nu_2(a_1, a_4), \nu_2(a_4, a_3)) = \max(0.17, 0.85) = 0.85$$

$\Rightarrow (\mu_2'^{\infty}, \nu_2'^{\infty}) = (0.015, 0.85)$.

By (2),

$$CONN_{G'}(a_1,a_3) = (\mu - CONN_{G'}(a_1,a_3), \nu - CONN_{G'}(a_1,a_3))$$

$$\mu - CONN_{G'}(a_1,a_3) = \max(\mu_1'^{\infty}, \mu_2'^{\infty}) = \max(0.01, 0.015) = 0.015$$

$$\nu - CONN_{G'}(a_1,a_3)) = \min(\nu_1'^{\infty}, \nu_2'^{\infty}) = \min(0.9, 0.85) = 0.85$$

$$\Rightarrow CONN_{G'}(a_1,a_3) = (0.015, 0.85).$$

Then, $\mu_1'^{\infty}(a_1,a_3) = 0.01 < 0.02 = \mu_1^{\infty}(a_1,a_3)$

$\nu_2'^{\infty}(a_1,a_3) = 0.9 > 0.14 = \nu_1^{\infty}(a_1,a_3)$

Thus, $\mu_2'^{\infty}(a_1,a_3) = 0.015 < 0.03 = \mu_2^{\infty}(a_1,a_3)$

$\nu_2'^{\infty}(a_1,a_3) = 0.85 > 0.16 = \nu_2^{\infty}(a_1,a_3)$

$$\Rightarrow \mu - CONN_{G'}(a_1,a_3) = 0.015 < 0.03 = \mu - CONN_G(a_1,a_3)$$

$$\Rightarrow \nu - CONN_{G'}(a_1,a_3)) = 0.85 > 0.14 = \nu - CONN_G(a_1,a_3))$$

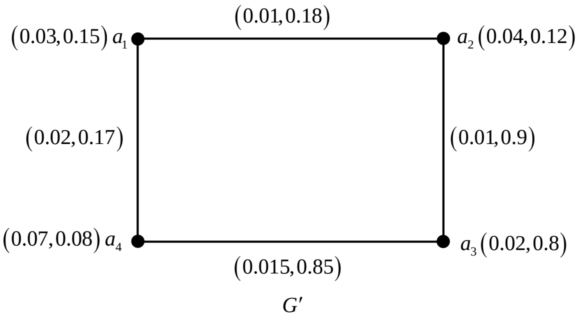

Fig. 3. Pythagorean Fuzzy Labeling Subgraph

Proposition 3.3. If G_1 and G_2 are Ω_{PFLG} then their union $G_1 \cup G_2$ is also Ω_{PFLG} if their *d.m* and *d.n.m* of the vertices and edges are distinct.

Proof:

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two Ω_{PFLG}.

By definition of Ω_{PFLG}, the *d.m* and *d.n.m* of the vertices and edges in G_1 and G_2 are distinct.

Let $G_3 = (V_3, E_3)$ be the union of G_1 and G_2. Then the *d.m* of the vertices and edges in G_3 are given as

$$\mu_{V_3}(a_i) = \begin{cases} \mu_{V_1}(a_i) & if\ a_i \in V_1 - V_2 \\ \mu_{V_2}(a_i) & if\ a_i \in V_2 - V_1 \\ \max(\mu_{V_1}(a_i), \mu_{V_2}(a_i)) & if\ a_i \in V_1 \cap V_2 \end{cases}$$

$$\mu_{E_3}(a_i,a_{i+1}) = \begin{cases} \mu_{E_1}(a_i,a_{i+1}) & if\ (a_i,a_{i+1}) \in E_1 - E_2 \\ \mu_{E_2}(a_i,a_{i+1}) & if\ (a_i,a_{i+1}) \in E_2 - E_1 \\ \max(\mu_{E_1}(a_i,a_{i+1}), \mu_{E_2}(a_i,a_{i+1})) & \\ & if\ (a_i,a_{i+1}) \in E_1 \cap E_2 \end{cases}$$

The *d.n.m* of the vertices and edges in G_3 are given as

$$\nu_{V_3}(a_i) = \begin{cases} \nu_{V_1}(a_i) & if\ a_i \in V_1 - V_2 \\ \nu_{V_2}(a_i) & if\ a_i \in V_2 - V_1 \\ \min(\nu_{V_1}(a_i), \nu_{V_2}(a_i)) & if\ a_i \in V_1 \cap V_2 \end{cases}$$

$$\nu_{E_3}(a_i,a_{i+1}) = \begin{cases} \nu_{E_1}(a_i,a_{i+1}) & if\ (a_i,a_{i+1}) \in E_1 - E_2 \\ \nu_{E_2}(a_i,a_{i+1}) & if\ (a_i,a_{i+1}) \in E_2 - E_1 \\ \min(\nu_{E_1}(a_i,a_{i+1}), \nu_{E_2}(a_i,a_{i+1})) & \\ & if\ (a_i,a_{i+1}) \in E_1 \cap E_2 \end{cases}$$

Thus the *d.m* and *d.n.m* of the vertices and edges of G_3 are distinct. This proves that the union G_3 of G_1 and G_2 is a pythagorean fuzzy labeling graph Ω_{PFLG}.

Example:

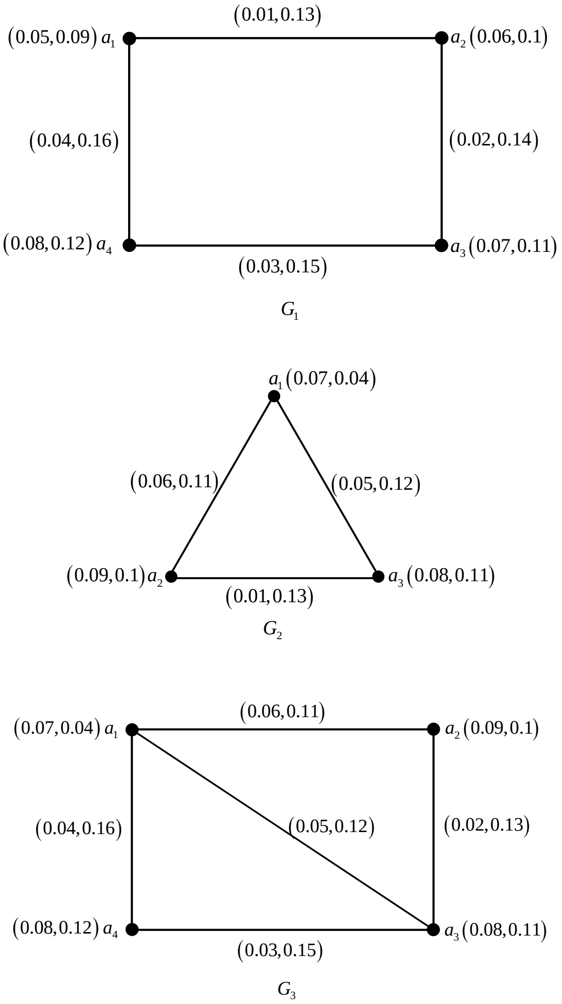

Fig. 4. Union of Pythagorean fuzzy labeling graph

IV. CONCLUSION

A new concept of Pythagorean fuzzy labeling graph, Ω_{PFLG} have been introduced. The subgraphs and union of Pythagorean fuzzy labeling graphs are newly defined.

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AUTHORS PROFILE

P. Ashwini Sibiya Rani is a Research Scholar in Loyola College, University of Madras, Chennai. Area of interest is fuzzy sets, fuzzy logic, fuzzy graphs, fuzzy labeling, Pythagorean fuzzy set, Pythagorean fuzzy graph.
e-mail: ashwinifiona@gmail.com

Dr. T. Bharathi ia an Assistant Professor in Loyola College, University of Madras, Chennai. She have a teaching experience over 10 years and published more than 15 papers. Area of interest is fuzzy logic, fuzzy clustering, fuzzy analysis, fuzzy graph, fuzzy labeling.
e- mail: prith.bharu23@gmail.com