Forecasting Daily Dynamic Hedge Ratios of Indian Index Future Contracts

Anuja Gupta, Manoj Jha, Namita Srivastava

Abstract*: Forecasting plays a crucial role in determining the direction of future trends and in making necessary investment decisions. This research presents the forecasting performance of three multivariate GARCH models: SGARCH, EGARCH, and GJR-GARCH based on Gaussian and Student's t-distribution. The forecasting ability of the models is evaluated on the basis of forecasting performance measures: MAE, SSE, MSE, and RMSE. This is done by examining the hedged portfolios of three indices of NSE: NIFTY50, BANKNIFTY, and NIFTYIT. Daily data from Jan 2006 to Dec 2017 is taken and forecasts are conducted using out of sample data from Jan 2016-Dec 2017. Minimum mean square error (MMSE) forecasting method is used to generate conditional variance and covariance forecasts which in turn generate hedge ratios and corresponding hedged portfolio. Minimum variance hedge ratio framework of Ederington (1979) is used for hedging. The in-sample analysis shows that SGARCH with both the distribution performed better than the other models while out-of-sample analysis provides mixed results. EGARCH model assigns the lowest hedge ratio to NIFTY50 and BANKNIFTY while SGARCH model assigns the lowest hedge ratio to NIFTYIT. Forecasting performance measures show the least value for SGARCH and EGARCH model. In future these models are able to reduce maximum risk from the spot market. The results of this research has important implications for financial decision and policy makers.

***Index Terms*: forecasting measures, hedge ratio, hedging effectiveness, multivariate GARCH models.**

I. INTRODUCTION

Price discovery, hedging, gambling, and risk distribution are the main functions of future markets. Investors and hedgers use them to mitigate the risk of spot market by taking the equal and opposite position in future markets. Due to these different hedging theories have been proposed in the literature. The naïve hedging theory assumed an optimal hedge ratio of one for a spot position. But this theory failed to deliver due to the lack of coordination between spot and future price movements [1]. Working's hedging theory (1953) is an improvement over naive hedging theory [2]. According to this theory, the hedger act as a speculator. This theory is based on the unrealistic assumption of maximizing returns by speculating on basis [1]. This theory is also criticized due to maximizing returns at any level of risk. Thereafter, portfolio approach to hedging given by Johnson (1960) and Stein (1961) came into being ([3],[4]). It assumed that the objective of hedger is neither minimization of risk nor maximization of returns. The main objective of hedger is the optimization of risk-return trade-off in the portfolio ([3], [4]). Ederington (1979) extended the work of Johnson and Stein and proposed the concept of MVHR and hedging effectiveness ([5], [6]). Portfolio hedging theory gained much importance as compared to other theories. It helps in estimating both static and dynamic hedge ratios [1].

Various studies estimated optimal hedge ratio from Ordinary Least Squares (OLS) method. Also hedging effectiveness has been measured using R-square. Hedge ratio obtained through OLS is criticized due to its biased nature. This biasedness occurs when the returns of cash and future markets are co-integrated. Another difficulty with this hedge ratio is the autocorrelation in OLS residuals. There is always present some heteroscedasticity in cash and future price [7]. Financial time series have fat tails, heteroscedasticity, leverage effects, autocorrelation and co-movements in volatilities. GARCH models can easily capture these features of financial time series [8]. Due to these difficulties GARCH models are proposed in the literature.

Forecasting from any method depends on some reasonable assumptions. First, spot and future prices are properly correlated to obtain reliable hedge ratios. Second, time horizons should be as long as possible. Longer time horizon increases forecasting accuracy due to inclusion of more data points and provides more accurate results than shorter horizon. Third, the activities of competitors and decision makers also affect forecasting to a great extent [8]. Therefore, forecasting of hedge ratios is essential for traders and investors.

A. *Research motivation*

Though, there exists a plethora of research works in estimating hedge ratios and hedging effectiveness. There is a scarcity of literature in forecasting hedge ratio for Indian index future markets. Also, a very few research work available for estimating forecasting capacity and exactness of models [7]. This research tries to fill the above research gap. Forecasting enables an investor to understand the performance of future markets. It also assists portfolio managers in making ideal hedging and trading decisions [8].

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It helps an investor in selecting and adjusting a suitable portfolio in dynamic hedging. This is done by investigating ex-post and ex-ante portfolio performance in spot and futures markets. On the basis of hedge ratios of portfolios, an investor select a portfolio with minimum risk.

B. Focus of the present study

This research investigates forecasting performance of SGARCH, EGARCH and GJR-GARCH models using daily data of CNXNIFTY50, BANKNIFTY, and NIFTYIT index futures. Forecasting is done using MMSE forecasting method in MATLAB software. To our knowledge these models and methods have never been considered for forecasting using the Indian index future contracts. MMSE is a versatile measure of forecasting which is easy to calculate and has not been considered before for forecasting hedge ratios. Forecasting performance is compared through MAE, MSE, SSE and RMSE which is of great help to investors and traders in selecting a particular hedging strategy.

C. Novelty of the proposed approach and research contribution

Forecasting done in this research offers many new dimensions when compared with the existing literature on forecasting using GARCH models. The data taken in this research is large enough to include all properties of time series data. The models used in this research are known to remove computational problems arising due to other GARCH models and the ordinary least square (OLS) model. Therefore, this research will provide substantial contribution to the literature and will help researchers and financial analysts in proper decision-making.

D. Organisation of the paper

Rest of the paper is structured as follows: next section contains the literature review. Section 3 describes some preliminaries and the models used in hedging. Section 4 describes numerical illustration with data description, methodology and results. The last section 5 concludes the paper.

II. LITERATURE REVIEW

Floros and Vougas evaluated out-of-sample hedging effectiveness of Greek stock index futures contracts using OLS, ECM, VECM and BGARCH (1,1) [9]. Hsu, Tseng, and Wang estimated hedge ratio and hedging effectiveness using copula-based GARCH models. Comparison is done with the conventional static, CCC-GARCH, and the DCC-GARCH models [10]. Bhaduri and Durai evaluated hedge ratios using OLS, VECM, VAR and DVEC-GARCH models [7]. Hedged and unhedged portfolios are compared using mean and variance for different hedging horizons. Kenourgios, Samitas, and Drosos forecasted hedge ratio and hedging effectiveness using OLS, ECM, GARCH, and EGARCH models [6]. Zhou and Wu forecasted hedge ratios and hedging effectiveness using OLS, naïve, and SGARCH hedging strategy [11]. Choudhry and Wu investigated forecasting ability of bivariate GARCH, BEKK GARCH, GJR-GARCH and Kalman filter [12]. Yang and Lai forecasted hedge ratios and hedging effectiveness using GJR-EC-GARCH and EC-OLS models [13]. Iqbal proposed an approach for forecasting conditional correlation and VaR using QMLE, LAD, and B-estimator of SGARCH and other multivariate GARCH models [14]. Ramasamy and Munisamy investigated forecasting ability of GARCH, GJR-GARCH, and EGARCH models [15]. Salvador and Arago estimated optimal hedge ratio using linear and non-linear GARCH models. Conclusion derived is that non-linear GARCH models provide better hedge ratios for in and out sample data [16]. Choudhary and Zhang forecasted hedge ratios using GARCH, BEKK GARCH, GARCH-X, BEKK-X, Q-GARCH, and GJR-GARCH. The estimation is done on the basis of normal and Student's t-distribution [8]. Singh suggested an optimal hedge ratio by analyzing NIFTY, BANKNIFTY, and NIFTYIT using OLS, GARCH, EGARCH, TARCH, VAR, and VECM [1]. Lai forecasted hedge ratios using dynamic copula GARCH models [17]. From the literature review, it is concluded that majority of the investigations on index futures are associated with developed nations. There are a very few studies in the context of emerging nations like India. The results obtained through dynamic hedge ratios are more efficient than static hedge ratios [18]. In this research, the models used for forecasting and hedging are used in very few studies and the results obtained from them are more reliable. So an attempt is made to compare their hedging and forecasting performance by taking a sufficiently large hedging horizon.

III. PRELIMINARIES

The basic concepts used in this paper are briefly discussed in this section.

A. Hedge ratio

To hedge with future contracts, an investor should select an appropriate hedge ratio. It is determined by the number of future contracts required to reduce the risk of spot portfolio ([6]-[8]). Minimum value of hedge ratio should be taken into account. Higher hedge ratios require more future contracts which in turn demand higher investment. The hedge ratio which minimizes the variance of hedged portfolio is called optimal hedge ratio (OHR). It is also called the minimum variance hedge ratio (MVHR). Suppose the log difference of spot and future price generates spot and future portfolio returns. Let $r_{s,t}$ and $r_{f,t}$ are the returns of spot and future portfolio at any time t based on the information up to time t-1. According to Johnson (1960), Baillie, and Myers (1991) ([2], [19]), the minimum variance hedge ratio β ([6-8],[11],[16]) is given by

$$\beta = \frac{\operatorname{cov}\left(r_{s,t}, r_{f,t}\right)}{\operatorname{var}\left(r_{f,t}\right)} \tag{1}$$

Where numerator in (1) is the covariance between spot and future portfolio. Denominator is the variance of future portfolio.

The return r_h of hedged portfolio ([6-8]) can be calculated as

$$r_h = r_{s,t} - \beta r_{f,t} \tag{2}$$

And the variance of the hedged portfolio ([6-8]) can be calculated as:

$$\operatorname{Var}(H) = \operatorname{Var}\left(r_{s,t}\right) + \beta^2 \operatorname{Var}\left(r_{f,t}\right) - 2\beta \operatorname{cov}\left(r_{s,t}, r_{f,t}\right) \tag{3}$$

B. Hedging effectiveness

The performance and usefulness of hedging strategies can be examined by finding the hedging effectiveness of each strategy. The concept was given by Ederington in 1979.

It may be defined as the percentage decline in the variance of hedged portfolio relative to the unhedged portfolio [5]. Hedging effectiveness ([6-8]) is given by

$$HE = \frac{Var(U) - Var(H)}{Var(U)} \tag{4}$$

Where Var(U) =variance of unhedged portfolio i.e., variance of spot portfolio

Var(H) = variance of hedged portfolio given by (3).

C. Models used in hedging

In this research, the data taken is large enough to cover all properties of time series data. According to central limit theorem large sample sizes tend towards normal distribution [20].

The residual term (ε_t) in the mean equation of GARCH models is supposed to be conditionally normal. It has mean 0 and variance σ^2. For large sample sizes financial time series tends to be non-normal. It has leptokurtic fatter tail distribution. Non-normality, skewness and kurtosis are mostly found in the returns of financial time series. This makes the conditional normal distribution of the error term unfit with the observations. In this case Student's t-distribution is more advantageous than normal distribution. This distribution is similar to normal distribution for large degrees of freedom. In this research, both the normal (Gaussian) and Student t-distribution are used for forecasting hedge ratios.

1. *Multivariate GARCH Models*

To model ARCH terms in return series, GARCH models are proposed in the literature. These models are applied to the residual of the return series. Let the daily spot and future prices are denoted by p_t, t=1, 2,…, T and their logarithmic returns at time t be

$$r_t = \ln \frac{p_t}{p_{t-1}} \quad t=1,2,\ldots,T \quad [7] \tag{5}$$

Model 1: SGARCH (Simplified multivariate Generalised Auto Regressive Conditional Heteroscedastic) model: This model is given by Harris, Stoja and Tucker in 2007 [21]. In this model, conditional variance is estimated using GARCH (1,1) model. Conditional variance of spot and future return series is estimated. Also the same is estimated for the sum and difference of both series. The covariance is estimated using the variance estimates of sum and difference of return series. According to Harris, Stoja and Tucker, the model for spot returns based on GARCH (1, 1) model [21] is:

$$\left.\begin{aligned} r_{s,t} &= \mu_s + \varepsilon_{s,t}, \varepsilon_{s,t} = \sigma_{s,t} z_t \\ \sigma^2_{s,t} &= a + b\sigma^2_{s,t-1} + c\epsilon^2_{s,t-1} \end{aligned}\right\} \tag{6}$$

Where z_t~iid N (0, 1) a random number drawn from the standard normal distribution. μ_s is the conditional mean return. $\varepsilon_{s,t}$ is the residual term. ω is the constant or the intercept term and ω>0. α is the coefficient of GARCH term and α ≥0. It is the forecasted volatility from the past period. γ is the coefficient of ARCH term and γ ≥0. $\sigma^2_{s,t}$ is the conditional variance of $r_{s,t}$ and is set on the information known at the time t-1. The covariance-stationary condition for GARCH (1,1) process is α + γ <1. α + γ gives the level of persistence.

Similarly, the model for future return [21] is

$$\left.\begin{aligned} r_{f,t} &= \mu_f + \varepsilon_{f,t}, \varepsilon_{f,t} = \sigma_{f,t} z_t \\ \sigma^2_{f,t} &= a + b\sigma^2_{f,t-1} + c\epsilon^2_{f,t-1} \end{aligned}\right\} \tag{7}$$

Using GARCH (1, 1) model $\sigma^2_{s,t}$ and $\sigma^2_{f,t}$ can be found out. After that two series $r_{+,t} = r_{s,t} + r_{f,t}$ and $r_{-,t} = r_{s,t} - r_{f,t}$ are created and GARCH (1,1) is used to find the conditional variance ($\sigma^2_{+,t}$ and $\sigma^2_{-,t}$) of these two series similar to model (6) and (7). Then the covariance between spot and future return [21] can be calculated as

$$\sigma_{sf,t} = \frac{1}{4}(\sigma^2_{+,t} - \sigma^2_{-,t}) \tag{8}$$

The minimum variance hedge ratio ([6-8],[11],[16],[21]) is then obtained as

$$\beta = \frac{\operatorname{cov}(r_{s,t}, r_{f,t})}{\operatorname{var}(r_{f,t})} = \frac{\sigma_{sf,t}}{\sigma^2_{f,t}} \tag{9}$$

The measure of hedging effectiveness is then given by (4)

Model 2: GJR GARCH (Glosten-Jagannathan-Runkle GARCH): Apart from non-normality and leptokurtosis, one more feature exists in the return series called the 'leverage effect'. It is defined as the equity's volatility negatively correlated with equity's returns. GARCH models are based on square of the past observations and past variances. These observations and variances estimate current variance. Thus the sign of return does not affect volatilities. Glosten, Jagannathan and Runkle (1993) introduced a modified GARCH model. Different impacts on conditional variance can be modeled through positive and negative innovations to returns GJR-GARCH approach. Asymmetric effect can be modeled by adding a dummy variable in the GARCH (1, 1) model [22]. GJR-GARCH model ([15],[22]) is represented by the expression:

$$\sigma^2_{s,t} = a + b\sigma^2_{s,t-1} + c\epsilon^2_{s,t-1} + \epsilon^2_{s,t-1} I_{t-1} \tag{10a}$$

$$\sigma^2_{f,t} = a + b\sigma^2_{f,t-1} + c\epsilon^2_{f,t-1} + \epsilon^2_{f,t-1} I_{t-1} \tag{10b}$$

$$\sigma_{sf,t} = a + b\sigma_{sf,t-1} + c(\epsilon_{s,t-1}\epsilon_{f,t-1}) \tag{10c}$$

Where I_{t-1} is a dummy variable and I_{t-1}=1 if ε_{t-1} <0 otherwise I_{t-1}= 0. This helps in modeling negative returns more accurately and will help in hedging decisions. The minimum variance hedge ratio is then obtained by (9) and the corresponding hedging effectiveness is given by (4).

Model 3: EGARCH (Exponential GARCH): This model was given by Nelson (1991). The asymmetric relationship between conditional volatility and conditional mean can be modeled through this GARCH technique [23]. It is based on the logarithmic conditional volatility of cash and future returns. The estimated hedge ratio is better than other GARCH models. In this model, the conditional variance ([15],[23]) may be expressed as follows:

$$\varepsilon_t = \sigma_t z_t$$

$$\ln(\sigma_t^2) = \omega + \alpha \ln(\sigma^2_{t-1}) + \gamma\left(\left|\frac{\epsilon_{t-1}}{\sigma_{t-1}}\right| - \frac{\sqrt{2}}{\pi}\right) + \xi\left(\frac{\epsilon_{t-1}}{\sigma_{t-1}}\right) \tag{11}$$

The conditional covariance is given by 10(c). In this model, conditional variance is an exponential function of the variables under study. This preserves the positive character of conditional variance.

The exponential nature also indicates that external shocks strongly influence the forecasted volatility. Here $\xi > 0$ indicates the asymmetric effect and $\xi < 0$ shows the presence of 'leverage effect'

Parameters of above GARCH models are estimated using maximum likelihood estimation technique in MATLAB software. Both the Guassian and Student t-distribution (with 118 degrees of freedom) are used for analysis.

D. MMSE (Minimum Mean Square Error) forecasting method

If $\sigma_1^2, \sigma_2^2, \dots, \sigma_t^2$ are the given conditional variances and h is the forecast horizon, then this method generate the predictions for $\sigma_{t+1}^2, \sigma_{t+2}^2, \dots, \sigma_{t+h}^2$.

Suppose at time t+1, $\hat{\sigma}_{t+1}^2$ be the forecasted conditional variance based on the history up to time t. The conditional expected square loss function is defined as

$$E(\sigma_{t+1}^2 - \hat{\sigma}_{t+1}^2 | H_t) \tag{12}$$

Minimizing this loss function yields the MMSE forecast [25]

$$\hat{\sigma}_{t+1}^2 = E(\sigma_{t+1}^2 | H_t) = E(\varepsilon_{t+1}^2 | H_t) \tag{13}$$

Where H_t is the information set up to time t.

The forecasted conditional variance and covariance are then used to estimate hedge ratios using (9). The corresponding hedged portfolios are constructed using (2).

Forecasting accuracy of GARCH models is examined on the basis of performance measures MAE, SSE, MSE and RMSE. If y_t is the actual value of an observation and y_t' is the forecasted value of that observation then the error

$$e_t = y_t - y_t' \tag{14}$$

If the length of out-of-sample data is n, then different forecasting measures based on the value of e_t can be defined as:

(a) $\text{MAE (mean absolute error)} = \frac{1}{n}\sum_t \text{abs}(e_t)$

t=1…n (15)

(b) $\text{MSE(mean square error)} = \frac{1}{n}\sum_t e_t^2$

t=1…n (16)

(c) $\text{SSE (sum of the sqaure error)} = \sum_t e_t^2$

t=1…n (17)

(d) $\text{RMSE(Root Mean Square Error)} = \sqrt{\text{MSE}}$

(18)

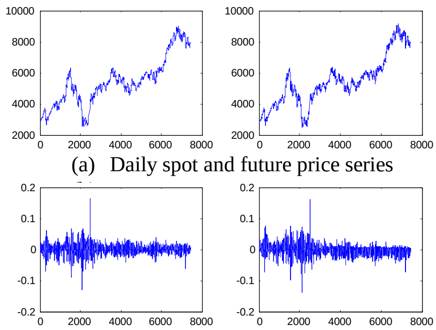
The value of these statistics should be as small as possible for a better forecast ([24], [25]).

IV. NUMERICAL ILLUSTRATION

To examine the efficiency of the forecasting models described in section 3, three main indices of NSE are selected.

A. Data description and methodology

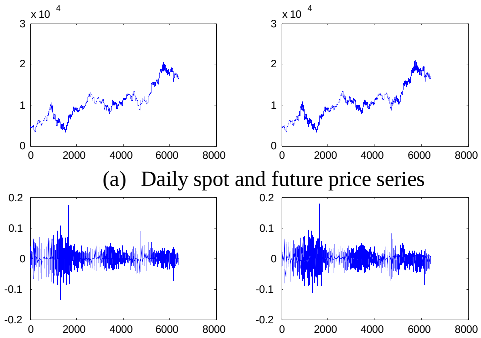
In this research data of NIFTY50, BANKNIFTY, and NIFTYIT is used to estimate dynamic hedge ratios. Daily closing spot and future price of NIFTY50, BANKNIFTY and NIFTYIT is downloaded from nseindia.com from 1 Jan 2006 to 31 Dec 2017. The data is divided into two parts. Training data set from 1-Jan-2006 to 31-Dec-2015 for estimating the parameters of models. Testing data set from 1-Jan-2016 to 31-Dec-2017 for assessing the predictive performance of models. A total of 7441 observations for NIFTY50, 6418 for BANKNIFTY and 3910 for NIFTYIT index are used for estimating the models. All futures price indices are continuous series. (Fig 1-3, where x axis represents time and y-axis price).



(a) Daily spot and future price series

(b) Daily spot and future returns

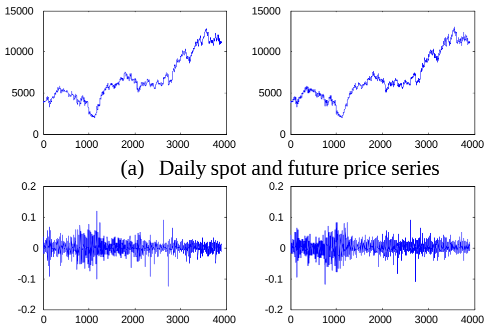
Figure 1: Daily spot and future price and return series of NIFTY50



(a) Daily spot and future price series

(b) Daily spot and future returns

Figure 2: Daily spot and future price and return series of BANKNIFTY



(a) Daily spot and future price series

(b) Daily spot and future returns

Figure 3: Daily spot and future price and return series of NIFTYIT

NIFTY50 is a broad market index consisting of the large, liquid stocks listed on nseindia.com. BANKNIFTY and NIFTYIT are the sectoral indices having the most liquid and large capitalized Indian Banking and IT sector stocks respectively. The models used for hedging are the three multivariate GARCH models SGARCH, EGARCH and GJR-GARCH. Forecasts are generated using MMSE method in MATLAB software. To assess the forecasting performance of GARCH models, forecasting measures discussed in section 3 are employed.

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In hedging, investments of cash and futures market are combined to form a new investment [11]. Also a hedger takes offsetting position in cash and futures market. Hedge ratio and hedging effectiveness are estimated statistically. Before doing any calculations, statistical properties of a time series under study should be considered. To test the stationarity of the data, ADF (Augmented Dickey Fuller) test is conducted in EVIEWS software (Table 1-3). At a certain significance level, the null hypothesis for ADF test is the variables contain a unit root or are non-stationary. The daily spot and future price series is shown in Fig 1-3(a) and stationary return series are shown in Fig 1-3 (b). After that monthly average of daily closing spot and future price is calculated. The reason behind this is that stock prices fluctuate daily due to supply and demand, market psychological effect and individual investor needs. Also just one day does not give an accurate view of the stock's price movement for investment decision. Spot and future returns are calculated using (5).

Descriptive statistics of spot and future returns of the indices (table 4) show excess kurtosis. It indicates the presence of fat tails and high peak values. These values also indicates non-normality in data. In table 4, the value of skewness is negative. It shows the existence of longer tail on the left-hand side for all price series.

Table 1: Stationarity test of NIFTY50 index

Null Hypothesis: SPOTPRICESERIES has a unit root
Exogenous: Constant
Lag Length: 3 (Automatic - based on SIC, maxlag=35)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.261192	0.6497
Test critical values:	1% level	-3.431055	
	5% level	-2.861736	
	10% level	-2.566916	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: FUTUREPRICESERIES has a unit root
Exogenous: Constant
Lag Length: 30 (Automatic - based on SIC, maxlag=35)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.321740	0.6217
Test critical values:	1% level	-3.431055	
	5% level	-2.861736	
	10% level	-2.566916	

*MacKinnon (1996) one-sided p-values.

Table 2: Stationarity test of BANKNIFTY index

Null Hypothesis: SPOTPRICESERIES has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=30)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.078047	0.7266
Test critical values:	1% level	-3.431656	
	5% level	-2.862002	
	10% level	-2.567059	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: FUTUREPRICESERIES has a unit root
Exogenous: Constant
Lag Length: 24 (Automatic - based on SIC, maxlag=30)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.057494	0.7344
Test critical values:	1% level	-3.431660	
	5% level	-2.862004	
	10% level	-2.567060	

*MacKinnon (1996) one-sided p-values.

Table 3: Stationarity test of NIFTYIT index

Null Hypothesis: SPOTPRICESERIES has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=29)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.434705	0.9009
Test critical values:	1% level	-3.431844	
	5% level	-2.862085	
	10% level	-2.567104	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: FUTUREPRICESERIES has a unit root
Exogenous: Constant
Lag Length: 3 (Automatic - based on SIC, maxlag=30)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.401251	0.9067
Test critical values:	1% level	-3.431839	
	5% level	-2.862083	
	10% level	-2.567103	

*MacKinnon (1996) one-sided p-values.

Table 4: Basic statistics for spot and future price of NIFTY50, BANKNIFTY and NIFTYIT

	NIFTY50		BANKNIFTY		NIFTYIT	
	Spot returns	Futures returns	Spot returns	Futures returns	Spot returns	Futures returns
Mean	0.00834	0.00845	0.01092	0.01093	0.00884	0.00886
Median	0.01885	0.01866	0.00978	0.00762	0.01179	0.01338
Std. Deviation	0.0605	0.06191	0.08237	0.08325	0.0675	0.06767
Kurtosis	3.99344	3.54815	4.20782	4.12542	4.14708	4.29127
Skewness	-0.85057	-0.77476	-0.20685	-0.22262	-1.0265	-1.06344
Minimum	-0.27033	-0.26776	-0.24001	-0.23966	-0.32242	-0.32674
Maximum	0.18146	0.18809	0.26485	0.26222	0.15917	0.15732
Sum sq.dev.	0.43192	0.45221	0.80055	0.81779	0.53757	0.5404
Count	119	119	119	119	119	119
Jarque-Bera	84.74522	67.27746	8.08191	7.26292	96.75454	103.7208
Probability	0.00000	0.00000	0.01758	0.02648	0.00000	0.00000

The Jarque-Bera (JB) test statistics shows the rejection of null hypothesis that the financial time series taken in this research is normal. This is because of large values or high heteroscedasticity in the sample. ARCH or heteroscedasticity test in performed in EVIEWS software using ARCH -LM test (table 5-7). The null hypothesis for ARCH test is that there are no ARCH effects present. For hedging, monthly hedging horizon is chosen and forecasting is done using MMSE forecasting technique for two years.

Table 5: NIFTY50 ARCH test

Heteroskedasticity Test: ARCH

F-statistic	0.152887	Prob. F(1,116)	0.6965
Obs*R-squared	0.155318	Prob. Chi-Square(1)	0.6935

Table 6: BANKNIFTY ARCH test

Heteroskedasticity Test: ARCH

F-statistic	8.964235	Prob. F(1,116)	0.0034
Obs*R-squared	8.464660	Prob. Chi-Square(1)	0.0036

Table 7: NIFTYIT ARCH test

Heteroskedasticity Test: ARCH

F-statistic	1.362472	Prob. F(1,116)	0.2455
Obs*R-squared	1.369873	Prob. Chi-Square(1)	0.2418

Parameters of GARCH models discussed in section 3 are estimated using maximum likelihood estimation in MATLAB software (table 8-10). After estimating parameters, forecasting is done using MMSE forecasting method discussed in section 3 to obtain hedge ratio and hedging effectiveness of in and out of sample data of the indices (table 11-14). Effectiveness of hedge ratios is judged using mean and variance of hedged and unhedged portfolios.

Table 8: Parameter estimation of the three GARCH models for the index NIFTY50. The values in parenthesis denote the corresponding standard error.

parameter	SGARCH		EGARCH		GJR-GARCH	
	Gaussian	t-dist.	Gaussian	t-dist.	Gaussian	t-dist.
Constant (ω1)	0.00006 (0.00010)	0.00006 (0.00010)	-6.22042 (2.65878)	-5.88268 (3.24923)	0.00006 (0.00009)	0.00007 (0.00010)
ω 2	0.00006 (0.00010)	0.00006 (0.00010)	-5.93732 (2.75136)	-0.20137 (0.2720)	0.00006 (0.00010)	0.00007 (0.00010)
ω 3	0.00025 (0.00041)	0.00025 (0.00041)	0 0	0 0	0 0	0 0
ω 4	0 0	0 0	--	--	--	--
ARCH term (λ1)	0.14804 (0.08620)	0.13622 (0.08193)	0.45167 (0.16408)	0.44043 (0.24104)	0.06901 (0.10239)	0.05765 (0.09734)
λ 2	0.14216 (0.08583)	0.13120 (0.08140)	0.43094 (0.16941)	0.23344 (0.16734)	0.06467 (0.10212)	0.05389 (0.09804)
λ 3	0.14491 (0.08594)	0.13352 (0.08156)	0.13829 (0.01428)	0.02403 (0.00721)	0.13829 (0.01428)	0.15002 (0.02083)
λ 4	0.10693 (0.05927)	0.08526 (0.04973)	--	--	--	--
GARCH term (α2)	0.83782 (0.09371)	0.84523 (0.09277)	-0.09687 (0.46362)	-0.00521 (0.55451)	0.83901 (0.09302)	0.84653 (0.09191)
α2	0.84408 (0.09337)	0.85153 (0.09174)	-0.05643 (0.48441)	0.9678 (0.04756)	0.83867 (0.09452)	0.84685 (0.09369)
α3	0.84115 (0.09352)	0.84861 (0.09220)	0.861712 (0.01515)	0.87961 (0.01709)	0.861712 (0.01515)	0.84998 (0.01360)
α4	0.83316 (0.05676)	0.87644 (0.04416)	--	--	--	--
Leverage term	--	--	-0.21594 (0.07936)	-0.01412 (0.14520)	0.13303 (0.08043)	0.13561 (0.08586)
	--	--	-0.20912 (0.08443)	-0.10285 (0.08192)	0.14632 (0.08901)	0.14780 (0.09358)

Table 9: Parameter estimation results of the three GARCH models for the index BANKNIFTY. The values in parenthesis denote the corresponding standard error.

parameter	SGARCH		EGARCH		GJR-GARCH	
	Gaussian	t-dist.	Gaussian	t-dist.	Gaussian	t-dist.
Constant (ω1)	0.00036 (0.00048)	0.00035 (0.00049)	-0.50114 (0.32714)	-0.35994 (0.37015)	0.00042 (0.00038)	(0.00038) (0.00049)
ω 2	0.00031 (0.00046)	0.00030 (0.00046)	-0.51188 (0.33268)	-0.37896 (0.38121)	0.00041 (0.00040)	0.00038 (0.00052)
ω 3	0.00134 (0.00190)	0.00132 (0.00191)	0 0	0 0	0.00001 (0.00000)	0 0
ω 4	0 0	0 0	--	--	--	--
ARCH term (λ1)	0.09929 (0.05368)	0.09965 (0.05653)	0.20056 (0.07478)	0.17581 (0.11688)	--	--
λ 2	0.08718 (0.05012)	0.08752 (0.05309)	0.19606 (0.07516)	0.17470 (0.11946)	--	--
λ 3	0.09338 (0.05197)	0.09375 (0.05489)	0.15228 (0.03735)	0.06910 (0.02143)	0.15228 (0.03735)	0.06910 (0.02143)
λ 4	0.13158 (0.06393)	0.13158 (0.06898)	--	--	--	--
GARCH term (α1)	0.8405 (0.10961)	0.84023 (0.11221)	0.90312 (0.06324)	0.93006 (0.07265)	0.81217 (0.08558)	0.83195 (0.13403)
α 2	0.8606 (0.10425)	0.86063 (0.10727)	0.90065 (0.06461)	0.9261 (0.07512)	0.82125 (0.08662)	0.83867 (0.13604)
α 3	0.84992 (0.10724)	0.84977 (0.11003)	0.78141 (0.03911)	0.75376 (0.05379)	0.78141 (0.03911)	0.75376 (0.05379)
α 4	0.79895 (0.05584)	0.79895 (0.06000)	--	--	--	--
Leverage term	--	--	-0.22299 (0.08122)	-0.19206 (0.08694)	(0.23886) (0.13098)	0.22358 (0.14294)
	--	--	-0.22518 (0.07914)	-0.19531 (0.08823)	0.22163 (0.12712)	0.21128 (0.14070)

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Table 10: Parameter estimation results of the three GARCH models for the index NIFTYIT.The values in parenthesis denote

parameter	SGARCH		EGARCH		GJR-GARCH	
	Gaussian	t-dist.	Gaussian	t-dist.	Gaussian	t-dist.
Constant (ω1)	0.00036 (0.00044)	0.00036 (0.00045)	-0.08556 (0.00718)	-0.07447 (0.01133)	0.00022 (0.00025)	0.00020 (0.00030)
ω 2	0.00033 (0.00044)	0.00034 (0.00046)	-0.06666 (0.0050)	-0.00882 (0.01712)	0.00021 (0.00025)	0.00020 (0.00031)
ω 3	0.00138 (0.00177)	0.00140 (0.00180)	0.00001 (0.00000)	0 0	0.00001 (0.00000)	0 0
ω 4	0 0	0 0	--	--	--	--
ARCH term (λ1)	0.151932 (0.12761)	0.14516 (0.12580)	-0.16206 (0.07177)	-0.17392 (0.12521)	--	--
λ 2	0.14663 (0.12649)	0.13890 (0.12519)	(-0.15751) (0.05449)	-0.12882 (0.09647)	--	--
λ 3	0.14944 (0.12723)	0.14218 (0.12564)	0.31391 (0.18749)	0.13574 (0.04848)	--	0.13574 (0.04848)
λ 4	0.19254 (0.13408)	0.20800 (0.14929)	--	--	--	--
GARCH term (α1)	0.77283 (0.20389)	0.77319 (0.20792)	0.98511 (0.00086)	0.98595 (0.00101)	0.82366 (0.15942)	0.85605 (0.15676)
α 2	0.78344 (0.20276)	0.78376 (0.20870)	0.98783 (0.00033)	0.99780 (0.00162)	0.82875 (0.16190)	0.86172 (0.16170)
α 3	0.77799 (0.20346)	0.77833 (0.20840)	0.68609 (0.14135)	0.63639 (0.06332)	0.68609 (0.14135)	0.636394 (0.06332)
α 4	0.66028 (0.13172)	0.65821 (0.13901)	--	--	--	--
Leverage term	--	--	-0.19310 (0.04372)	-0.20105 (0.05402)	0.23414 (0.10781)	0.16096 (0.12832)
--	--	--	-0.15482 (0.05221)	-0.152 (0.05916)	0.22740 (0.10912)	0.15105 (0.13028)

the corresponding standard error.

Table 11: In sample analysis of NIFTY50, BANKNIFTY and NIFTYIT using GARCH models with Guassian distribution.

In sample analysis (Guassian distribution)			
	SGARCH	EGARCH	GJRGARCH
NIFTY50			
Return (unhedged)	0.00834	0.00834	0.00834
Return (hedged)	0.00007	-0.00086	-0.00028
variance (unhedged)	0.00366	0.00366	0.00366
variance (hedged)	0.00007	0.00012	0.00008
hedge ratio	0.97811	1.08893	1.02170
hedging effectiveness	98%	97%	98%
BANKNIFTY			
Return (unhedged)	0.01092	0.01092	0.01092
Return (hedged)	0.00012	-0.00175	-0.00110
variance (unhedged)	0.00678	0.00678	0.00678
variance (hedged)	0.00012	0.00035	0.00022
hedge ratio	0.98900	1.16180	1.09797
hedging effectiveness	98%	95%	97%
NIFTYIT			
Return (unhedged)	0.00884	0.00884	0.00884
Return (hedged)	0.00005	-0.00519	-0.00099
variance (unhedged)	0.00456	0.00456	0.00456
variance (hedged)	0.00008	0.00171	0.00015
hedge ratio	0.99277	1.58428	1.10641
hedging effectiveness	98%	62%	97%

Table 12: Out of sample analysis of NIFTY50, BANKNIFTY and NIFTYIT using GARCH models with Guassian distribution.

Out sample analysis (Guassian distribution)			
	SGARCH	EGARCH	GJRGARCH
NIFTY50			
Return (unhedged)	0.01166	0.01166	0.01166
Return (hedged)	-0.00007	0.00533	0.00058
variance (unhedged)	0.04889	0.04889	0.04889
variance (hedged)	0.00412	0.01235	0.00399
hedge ratio	1.01023	0.54531	0.95435
hedging effectiveness	92%	75%	92%
BANKNIFTY			
Return (unhedged)	0.01712	0.01712	0.01712
Return (hedged)	-0.00008	0.00372	0.00181
variance (unhedged)	0.00172	0.00172	0.00172
variance (hedged)	0.00015	0.00019	0.00015
hedge ratio	1.01293	0.78725	0.89906
hedging effectiveness	91%	89%	91%
NIFTYIT			
Return (unhedged)	0.00063	0.00063	0.00063
Return (hedged)	0.00010	-0.00004	-0.00002
variance (unhedged)	0.00083	0.00083	0.00083
variance (hedged)	0.00007	0.00015	0.00013
hedge ratio	1.00091	1.27036	1.22692
hedging effectiveness	91%	82%	85%

Table 13: In sample analysis of NIFTY50, BANKNIFTY and NIFTYIT using GARCH models with Student's t-distribution.

In sample analysis (Student's t- distribution)			
	SGARCH	EGARCH	GJRGARCH
NIFTY50			
Return (unhedged)	0.00834	0.00834	0.00834
Return (hedged)	0.00014	-0.00020	-0.00005
variance (unhedged)	0.00366	0.00366	0.00366
variance (hedged)	0.00007	0.00007	0.00007
hedge ratio	0.96995	1.00950	0.99285
hedging effectiveness	98%	98%	98%
BANKNIFTY			
Return (unhedged)	0.01092	0.01092	0.01092
Return (hedged)	0.00015	-0.00099	-0.00066
variance (unhedged)	0.00678	0.00678	0.00678
variance (hedged)	0.00012	0.00020	0.00016
hedge ratio	0.98620	1.08925	1.05919
hedging effectiveness	98%	97%	98%
NIFTYIT			
Return (unhedged)	0.00884	0.00884	0.00884
Return (hedged)	0.00003	-0.00347	-0.00028
variance (unhedged)	0.00456	0.00456	0.00456
variance (hedged)	0.00008	0.00081	0.00009
hedge ratio	0.99467	1.38611	1.03116
hedging effectiveness	98%	82%	98%

Table 14: Out of sample analysis of NIFTY50, BANKNIFTY and NIFTYIT using GARCH models with Student's t-distribution distribution

Out sample analysis (Student's t- distribution)			
	SGARCH	EGARCH	GJRGARCH
NIFTY50			
Return (unhedged)	0.01166	0.01166	0.01166
Return (hedged)	0.00020	0.00122	0.00060
variance (unhedged)	0.04889	0.04889	0.04889
variance (hedged)	0.00403	0.00417	0.00400
hedge ratio	0.98710	0.89927	0.95204
hedging effectiveness	92%	91%	92%
BANKNIFTY			
Return (unhedged)	0.01712	0.01712	0.01712
Return (hedged)	-0.00268	0.00135	0.00019
variance (unhedged)	0.00172	0.00172	0.00172
variance (hedged)	0.00015	0.00015	0.00015
hedge ratio	0.99648	0.92617	0.99381
hedging effectiveness	91%	92%	91%
NIFTYIT			
Return (unhedged)	0.00063	0.00063	0.00063
Return (hedged)	0.00010	-0.00008	-0.00001
variance (unhedged)	0.00083	0.00083	0.00083
variance (hedged)	0.00007	0.00019	0.00012
hedge ratio	0.99865	1.34364	1.21560
hedging effectiveness	91%	77%	85%

Forecasting accuracy between the observed and the forecasted value is evaluated using forecasting performance measures discussed in section 3 (table 15). The best forecasting model is one which has the smallest value of these forecasting measures.

Table 15: Forecast accuracy test results NIFTY50 and BANKNIFTY

Model	Index	Distribution	Forecasting Measures			
			MAE	MSE	SSE	RMSE
SGARCH	NIFTY50	GUASSIAN	0.09224	0.04787	1.14894	0.21880
	BANKNIFTY		0.03872	0.00203	0.04865	0.04503
	NIFTYIT		0.02392	0.00078	0.01866	0.02788
	NIFTY50	T- DIST	0.09014	0.04572	1.09720	0.21381
	BANKNIFTY		0.03535	0.00178	0.04267	0.04217
	NIFTYIT		0.02376	0.00077	0.01844	0.02772
EGARCH	NIFTY50	GUASSIAN	0.06849	0.02640	0.63354	0.16247
	BANKNIFTY		0.03017	0.00123	0.02954	0.03508

	NIFTYIT		0.03038	0.00125	0.03011	0.03542
	NIFTY50	T- DIST	0.08210	0.03793	0.91027	0.19475
	BANKNIFTY		0.03550	0.00170	0.04090	0.04128
	NIFTYIT		0.03205	0.00140	0.03350	0.03736
GJR GARCH	NIFTY50	GUASSIAN	0.08712	0.04271	1.02506	0.20667
	BANKNIFTY		0.03447	0.00161	0.03855	0.04008
	NIFTYIT		0.02942	0.00118	0.02822	0.03429
	NIFTY50	T- DIST	0.08694	0.04253	1.02077	0.20623
	BANKNIFTY		0.03811	0.00196	0.04713	0.04431
	NIFTYIT		0.02918	0.00116	0.02777	0.03401

B. Results and discussion

Results from the proposed methodology are quite standard. Guassian and Student's t-distribution are used for estimating GARCH models. All future price indices are continuous settlement prices. Returns are the log difference of spot (cash) and future price (fig 1-3). Results from ADF unit root test are given in Table 1-3. The null hypothesis of having unit root in cash and future price series is rejected. The stationary cash and future price series enables one to predict the returns significantly. Descriptive statistics of spot and future portfolio is given in table 4. Results from JB test statistics indicates non-normality in return series. Due to non-normality, returns are not equally distributed among buyers and sellers [1]. ARCH-LM test results indicates significant ARCH effects in the data (table 5-7).

The estimated parameters of the proposed models using both the distributions are provided in table 8-10. All the three models show significant ARCH and GARCH effects. EGARCH and GJR-GARCH models express considerable leverage effects. MMSE forecasting method is applied in MATLAB software to forecast conditional variances and covariance. This in turn generate hedge ratios and hedging effectiveness using (4) and (7) (table 11-14). The hedged portfolio is thus formed on the basis of forecasted hedge ratios. The mean and variance of hedged portfolio is estimated using (2) and (3). An in-sample analysis helps an investor to have an idea what the model performed in the past. An out-of-sample analysis assists in evaluating the predictive performance of strategies. Comparison between the forecasted and the actual returns thus provide relative accuracy of dynamic hedge ratios.

The results of forecasting performance measures for the GARCH models is given in table 15. For NIFTY50 and BANKNIFTY, EGARCH with both distributions has the least value for all the performance measures. While for NIFTYIT, SGARCH with both distribution has the least value of performance measures. This implies that investors will be benefitted on investing in NIFTY50 and BANKNIFTY using EGARCH hedging strategy. Also NIFTYIT will give better returns in future on using SGARCH hedging strategy. On comparing both the Guassian and Student t-distribution, the results from the latter are more consistent than the former.

C. Comparative analysis

Hedging effectiveness is an important component of hedging which helps in deciding a proper hedging strategy. Higher the value of hedging effectiveness, more effective will be the strategy. The proposed methodology provides better results as compared to the results of Bhaduri and Durai [7] and Singh [1]. The high value of hedging effectiveness in this research indicates removal of maximum risk of spot portfolio. The results will be beneficial for the market players to select the best hedging strategy.

V. CONCLUSION

This research attempts to estimate and forecast hedge ratios of NIFTY50, BANKNIFTY and NIFTYIT index futures. Proposed methodology is conducted using SGARCH, EGARCH and GJR-GARCH model with Guassian and Student t-distribution. The stationarity of cash and future market returns indicates proper distribution of information in both markets and so returns can be predicted significantly. Forecasting is conducted by means of MMSE forecasting method taking two year forecasting horizon. Performance measures are applied to test the forecasting ability of GARCH models. The outcome of performance measures illustrates mixed results. EGARCH model dominates other models for NIFTY50 and BANKNIFTY. Value of hedge ratio is the least for NIFTY50 and BANKNIFTY using EGARCH model SGARCH model has the best forecasting performance for NIFTYIT. SGARCH model gives the minimum value of hedge ratio for NIFTYIT.

This research will help the financial decision makers in making proper investment decisions. The proposed methodology will help the investors to understand the role of future markets in trading and fund management. This will enable them to select the appropriate portfolio and make necessary adjustments in it.

The proposed methodology with Student's t-distribution offers better forecasting performance than that of Guassian distribution. There is no particular model to provide better forecasting performance for all index futures. Different models have different forecasting performance due to various distributions and set of data points. This research can be extended to include more complex dynamics of financial markets using various GARCH models and distributions.

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