Multivariate GARCH Model and Its Application to Bivariate Model

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***Abstract*: Multivariate GARCH model is a development of the univariate GARCH model. The multivariate GARCH model can be viewed as a conditional heteroskedasticity model in a multivariate time series. This paper discusses the parameterization of covariance matrices such as Vech model representation, BEEK model and Constant Correlation model. For parameter estimation the maximum likelihood method is used. Furthermore, multivariate GARCH model application is applied for bivariate model.**

***Index Terms*: Multivariate, GARCH, Maximum likelihood.**

I. INTRODUCTION

The conventional time series model assumes that the variance error is constant over time. The assumption of constant variance is an ideal assumption rarely encountered in real situations, especially related to the financial field [1]. Therefore a model developed with the assumption of nonconstant variance is known as heteroskedasticity model [2]. The heteroskedastic model is not only in univariate form but also in multivariate form [3]. The Generalized of space time autoregressive (GSTAR) model with ARCH error has also been developed [4]. In this paper we describe the parameterization of covariance matrices for the multivariate GARCH model.

II. UNIVARIATE ARCH AND GARCH MODEL

In conventional time series models such as the autoregressive moving average (ARMA) model it is assumed that the error variance (ε_t) is constant, ie $Var(\varepsilon_t) = \sigma^2$. Suppose the conditional variance of ε_t is not constant, then the variance of Y_t conditional on $Y_{t\text{-}1}$ is not constant $, Var(\varepsilon_t) = \sigma_t^2$. One strategy is to model conditional variance as AR(q) process through the preceding error square, ie:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \Lambda + \alpha_{t-q}\varepsilon_{t-q}^2 + \eta_t \quad (1)$$

With η_t is a white-noise process. For this reason, (1) is called an autoregressive conditional heteroscedastic (ARCH) model [2]. Engle then proposes a scheme in which heteroscedasticy depends on previous Y_t values, namely

$$Y_t = \eta_t \sqrt{h_t} \ \text{ and } \ h_t = \alpha_0 + \alpha_1 Y_{t-1}^2 \quad (2)$$

with η_t iid. N(0.1). The values of $\alpha_0 > 0$ and $\alpha_1 > 0$. Then (2) is called the ARCH(1)model. Note that the variance of Y_t conditional on $Y_{t\text{-}1}$ is

$$Var(Y_t|Y_{t-1}) = Var(\eta_t\sqrt{h_t}|Y_{t-1}) = h_t Var(\eta_t) = h_t$$

If it is related to the application of the model, let Y_t be inflation then a process ARCH(1) states that high inflation in the past period will result in great variance at the present time.

Furthermore, if the ARCH(q) process is included lag of σ_t^2 then obtained model GARCH (p, q) [5], namely

$$h_t = \alpha_0 + \beta_1 h_1 + \Lambda + \beta_p h_{t-p} + \alpha_1 Y_{t-1}^2 + \Lambda + \alpha_q Y_{t-9}^2 \quad (3)$$

where p denotes lag on σ_t^2 and q states lag on Y_t^2. Specifically for p = 1 and q = 1 obtained the GARCH model (1, 1) ie

$$h_t = \alpha_0 + \alpha_1 Y_{t-1}^2 + \beta_1 h_{t-1} \quad (4)$$

In this case

$E(Y_t|F_{t-1}) = 0$, and $Var(Y_t|F_{t-1}) = E(Y_{t-1}^2|F_{t-1}) = h_t$.

III. MULTIVARIATE ARCH/GARCH MODEL

The expansion of the ARCH/GARCH univariate model into the m-variate model requires the conditions that random variables $\boldsymbol{\varepsilon}_t$ have m-dimension, zero mean and conditional variance-covariance matrices of $\boldsymbol{\varepsilon}_t$ depend on the elements of the information set of historical data.

Let {$\boldsymbol{\eta}_t$} be a random variable vector i.i.d. sized (m x 1) with the following characteristics

$$E(\boldsymbol{\eta}_t) = 0$$
$$E(\boldsymbol{\eta}_t\boldsymbol{\eta}_t') = \mathbf{I}_m$$
$$\boldsymbol{\eta}_t \sim G(0,\mathbf{I}_m)$$

with G is a continuous density function. Suppose {$\boldsymbol{\varepsilon}_t$} is a randomly sized ($m$x1) random vector

$$\varepsilon_t = \eta_t\sqrt{H_t}\ .$$

where:

$$E_{t-1}(\varepsilon_t) = 0$$
$$E_{t-1}(\varepsilon_t\varepsilon_t') = H_t$$

and H_t is the positive definite matrix of size (mxm) and measured to the set of information $F_{t\text{-}1}$, ie σ-field generated by the past information: {$\varepsilon_{t\text{-}1}$, $\varepsilon_{t\text{-}2}$, ...}. Parameterization of H_t as an ARCH (GARCH) multivariate,

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as a function of the information set of F_{t-1}, then the elements of H_t are dependent on the lag-q of the ε_t and cross-products ε_t squares.

Thus the elements of the covariance matrix follow an ARMA process vector in the square and cross-products of disturbances (error). Parameterization of H_t as a multivariate ARCH (GARCH) is given in three forms, ie vech model, BEKK model and consonant correlation model [3].

A. Vech Representation

Vech is the vector-half operator (ie half-vector) which is piling the elements of the lower triangle of the $m\times m$ matrix into the vector sized $(m(m+1)/2)\times 1$. Vech representation is often called full parameterization. Since the covarians matrix H_t is a symmetric matrix, then vech (H_t) contains elements in H_t singly. Thus, the expansion of GARCH multivariate model (p,q) in vech representation [3] can be written as

$$vech(H_t) = W + \sum_{i=1}^{q} A_i vech(\varepsilon_{t-1}\varepsilon_{t-1}) + \sum_{j=1}^{p} B_j vech(H_{t-j}) \tag{5}$$

where: W is a vector of sizes $(m(m+1)/2)\times 1$, whereas A_i and B_j are matrices $(m(m+1)/2)\times(m(m+1)/2)$. The number of parameters in the general formulation of vech representation is as much as $\{m(m+1)/2+(p+q)(m(m+1)/2)^2\}$. For example, let $m = 2$, and $p = q = 1$, then the number of parameters is 21 pieces. The form of the *vech* (H_t) model for this example is as follows:

$$\begin{bmatrix} h_{11,t} \\ h_{21,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{21,t-1} \\ h_{22,t-1} \end{bmatrix}$$

In this case, the element to (i, j) in H_t depends on the (i, j) element corresponding in $\varepsilon_t\varepsilon_t'$ and H_{t-1}. To ensure that the H_t matrix is positive definite, the necessary conditions [3] are

$$\begin{aligned} & w_1 > 0,\ w_3 > 0,\ w_1w_3 - w_2^2 > 0, \\ & a_{11} \geq 0,\ a_{13} \geq 0,\ a_{31} \geq 0,\ a_{33} \geq 0,\ a_{11}a_{33} - a_{22}a_{22} \geq 0, \\ & a_{11}a_{13} - (1/4)\, a_{12}a_{12} \geq 0,\ a_{11}a_{31} - a_{21}a_{21} \geq 0, \\ & a_{31}a_{33} - (1/4)\, a_{32}a_{32} \geq 0,\ a_{13}a_{33} - a_{23}a_{23} \geq 0. \end{aligned} \tag{6}$$

To reduce the large number of parameters then simplified the vech representation. One way is to select the matrices A and B in diagonal form [3]. This is called the *vech diagonal* model. This simplification reduces the number of parameters where many parameters become $(m(m+1)/2)(1+p+q)$. Suppose that for m=2, and p=q=1, then the diagonal vech model [3] can be written as:

$$\begin{bmatrix} h_{11,t} \\ h_{21,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{21,t-1} \\ h_{22,t-1} \end{bmatrix} \tag{7}$$

By multiplying the above matrix is obtained

$$\begin{aligned} h_{11,t} &= w_1 + a_{11}\varepsilon_{1,t-1}^2 + b_{11}h_{11,t-1} \\ h_{21,t} &= w_2 + a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + b_{22}h_{21,t-1} \\ h_{22,t} &= w_3 + a_{33}\varepsilon_{2,t-1}^2 + b_{33}h_{22,t-1} \end{aligned} \tag{8}$$

In this case the number of parameters is reduced from 21 simplified to only 9. By subtracting this parameter, the conditions for the positive definite matrix H_t [3] are

$$\begin{aligned} & w_1 > 0,\ w_3 > 0,\ w_1w_3 - w_2^2 > 0, \\ & a_{11} \geq 0,\ a_{33} \geq 0,\ a_{11}a_{33} - a_{22}a_{22} \geq 0. \end{aligned} \tag{9}$$

B. Positive Definite Parameterization

This positive definite parameterization is also known as BEKK (Baba, Engle, Kraft and Kroner). The parameterization model from BEKK [3] is

$$h_t = CC' + \sum_{k=1}^{k}\sum_{i=1}^{q} A_{ik}\varepsilon_{t-1}\varepsilon_{t-1}'A_{ik}' + \sum_{k=1}^{k}\sum_{i=1}^{p} B_{ik}H_{t-i}B_{ik}' \tag{10}$$

where C is the lower triangular matrix, so CC' is the symmetric parameter matrix (mxm), whereas A_{ik} and B_{ik} are any parameter matrices (mxm). This model assures that the H_t matrix is a positive definite matrix. For example, take K=p=q=1 and N=2. Then the above BEKK model is

$$\begin{aligned} \begin{pmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{pmatrix} &= \begin{pmatrix} c_{11} & 0 \\ c_{21} & c_{22} \end{pmatrix}\begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix} \\ &+ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon_{2,t-1}^2 \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \\ &+ \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}\begin{pmatrix} h_{11,t-1} & h_{12,t-1} \\ h_{21,t-1} & h_{22,t-1} \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \end{aligned} \tag{11}$$

Then by multiplying the matrix is obtained

$$h_{11,t} = c_{11}^2 + (a_{11}\varepsilon_{1,t-1} + a_{12}\varepsilon_{2,t-1})^2 + (b_{11}h_{11,t-1} + b_{12}h_{22,t-1})^2$$

$$h_{22,t} = c_{21}^2c_{22}^2 + (a_{21}\varepsilon_{1,t-1} + a_{22}\varepsilon_{2,t-1})^2 + (b_{21}h_{12,t-1} + b_{22}h_{22,t-1})^2$$

$$\begin{aligned} h_{21,t} = h_{12,t} &= c_{11}c_{21} + a_{11}a_{21}\varepsilon_{1,t-1} \\ &+ (a_{11}a_{22} + a_{12}a_{21})\varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ &+ a_{12}a_{22}\varepsilon_{2,t-1}^2 + b_{11}b_{21}h_{11,t-1} \\ &+ (b_{11}b_{22} + b_{12}b_{21})h_{21,t-1} \\ &+ b_{21}b_{22}h_{22,t-1} \end{aligned} \tag{12}$$

C. Constant Correlation Model

In a constant correlation model, conditional covariance is proportional to the corresponding conditional standard deviation. In this case, it is assumed that the correlation matrix $\boldsymbol{\varepsilon}_t$, conditional to the past, is constant. This assumption simplifies the calculation load in the estimate and also the requirement that a positive definite H_t for each t be easily obtained. So the assumptions in this model [3] are

$$E_{t-1}[\varepsilon_t] = 0$$

$$E_{t-1}[\varepsilon_t\varepsilon_t'] = H_t$$

$$\{H_t\}_{ii} = h_{ii,t}$$

$$\{H_t\}_{ij} = h_{ij,t} = \rho_{ij}\sqrt{h_{ii,t}h_{jj,t}} \qquad if \quad i \neq j \qquad (13)$$

Let $\mathbf{D}_t$ denote the mxm diagonal matrix with its diagonal element is the conditional variance, ie $\{D_t\}_{ii} = h_{ii,t}$. Let $\mathbf{R}_t$ denote a constant correlation matrix in which the *ij*-element is

$$\{R_t\}_{ij} = \{H_t\}_{ij}(\{H_t\}_{ii}\{H_t\}_{jj})^{-1/2} \quad i, j = 1,2,\ldots,m \qquad (14)$$

Assuming the model $\mathbf{R}_t = \mathbf{R}$, then

$$\mathbf{H}_t = \mathbf{D}_t^{1/2}\,\mathbf{R}_t\mathbf{D}_t^{1/2}$$

$$H_t = diag(\sqrt{h_{11,t}},\ldots,\sqrt{h_{mm,t}})R_t diag(\sqrt{h_{11,t}},\ldots,\sqrt{h_{mm,t}})$$

For example, for $N = 2$, $p = q = 1$.Then

$$H_t = \begin{pmatrix}\sqrt{h_{11,t}} & 0\\ 0 & \sqrt{h_{22,t}}\end{pmatrix}\begin{pmatrix}1 & \rho_{12}\\ \rho_{21} & 1\end{pmatrix}\begin{pmatrix}\sqrt{h_{11,t}} & 0\\ 0 & \sqrt{h_{22,t}}\end{pmatrix} \qquad (15)$$

In this case,

$$D_t = \begin{pmatrix}h_{11,t} & 0\\ 0 & h_{22,t}\end{pmatrix}, \text{ and } R_t = \begin{pmatrix}1 & \rho_{12}\\ \rho_{21} & 1\end{pmatrix},$$

so

$$H_t = \begin{pmatrix}h_{11,t} & \rho\sqrt{h_{11,t}h_{22,t}}\\ \rho\sqrt{h_{11,t}h_{22,t}} & h_{22,t}\end{pmatrix}$$

where $\rho = \rho_{12} = \rho_{21}$ is the correlation coefficient between $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ and satisfies the correlation properties $|\rho| < 1$, while $h_{11,t}$ and $h_{22,t}$ are the variance for standard GARCH(p,q) processes. Suppose for $p = q = 1$, then

$$h_{11,t} = \alpha_{10} + a_{11}\varepsilon_{1,t-1}^2 + b_{11}h_{11,t-1}$$

$$h_{22,t} = \alpha_{20} + a_{21}\varepsilon_{2,t-1}^2 + b_{21}h_{22,t-1}$$

$$h_{12,t} = h_{21,t}$$

$$= \rho\sqrt{\alpha_{10} + a_{11}\varepsilon_{1,t-1}^2 + b_{11}h_{11,t-1}}\sqrt{\alpha_{20} + a_{21}\varepsilon_{2,t-1}^2 + b_{21}h_{22,t-1}}$$

The number of parameters to be estimated is 6. To qualify where the H_t matrix is positive definite then in this constant correlation model we need the following conditions [3]:

$$\alpha_{i0} > 0, \quad a_{ij} \geq 0 \text{ dan } b_{ik} \geq 0,$$
$$i = 1, \ldots, m, \quad j = 1, \ldots, q, \quad \text{and } k = 1, \ldots, p.$$

If the conditional variance along the diagonal of the matrix **D** is all positive, the correlation matrix R is a positive definite and this results in the sequence of conditional covariance matrices $\{H_t\}$ making sure the positively definite is almost certain for all t. Next inverse of H_t is:

$$\mathbf{H}_t^{-1} = \mathbf{D}_t^{-1/2}\,\mathbf{R}^{-1}\mathbf{D}_t^{-1/2}.$$

IV. PARAMETER ESTIMATION

Suppose $\{\mathbf{y}_t\}$, $t = 1, 2, \ldots, T$, is the realization of the stochastic process vector, with the mean vector $\boldsymbol{\mu}_t\mathbf{=0}$ and the conditional covariance matrix is $\mathbf{H}_t(\boldsymbol{\theta})$, where $\boldsymbol{\theta}$ is the parameter vector. Multivariate normal density of the process vector [1,6] is

$$f(y_t|F_{k-1}) = \frac{1}{\sqrt{(2\pi)^N}|H_t|^{1/2}}\exp\left\{\frac{-1}{2}y_t'H_t^{-1}y_t\right\}, \qquad (16)$$
$$t = 1,2,\ldots,T.$$

The multivariate ARCH(GARCH) model parameter estimation uses the likelihod maximum method. The log-likelihood function for normal multivariate combined density [7] is

$$L(\theta) = -\frac{TN}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}|H_t| - \frac{1}{2}\sum_{t=1}^{T}y_t'H_t^{-1}y_t \qquad (17)$$

Next we can estimate **θ** by completing the derivative of the log likelihood function $\partial L/\partial\theta = 0$. The parameter estimation for the diagonal vech model is as follows:

$$H_t = \begin{pmatrix}h_{11,t} & h_{12,t}\\ h_{21,t} & h_{22,t}\end{pmatrix}$$

$$\begin{bmatrix}h_{11,t}\\ h_{21,t}\\ h_{22,t}\end{bmatrix} = \begin{bmatrix}w_1\\ w_2\\ w_3\end{bmatrix} + \begin{bmatrix}a_{11} & 0 & 0\\ 0 & a_{22} & 0\\ 0 & 0 & a_{33}\end{bmatrix}\begin{bmatrix}\varepsilon_{1,t-1}^2\\ \varepsilon_{1,t-1}\varepsilon_{2,t-1}\\ \varepsilon_{2,t-1}^2\end{bmatrix} + \begin{bmatrix}b_{11} & 0 & 0\\ 0 & b_{22} & 0\\ 0 & 0 & b_{33}\end{bmatrix}\begin{bmatrix}h_{11,t-1}\\ h_{21,t-1}\\ h_{22,t-1}\end{bmatrix} \qquad (18)$$

Parameters $\boldsymbol{\theta} = (w_1, w_2, w_3, a_{11}, a_{22}, a_{33}, b_{11}, b_{22}, b_{33})'$, ie there are 9 parameters to be estimated. Solution $\partial L/\partial\theta = 0$ is solving simultaneously 9 equations, namely:

$$\frac{\partial L}{\partial w_1} = 0, \frac{\partial L}{\partial w_2} = 0, \frac{\partial L}{\partial w_3} = 0, \frac{\partial L}{\partial a_{11}} = 0, \frac{\partial L}{\partial a_{22}} = 0, \frac{\partial L}{\partial a_{33}} = 0, \qquad (19)$$
$$\frac{\partial L}{\partial b_{11}} = 0, \frac{\partial L}{\partial b_{22}} = 0, \frac{\partial L}{\partial b_{33}} = 0.$$

Compared with the constant correlation model, the parameter estimation is as follows

$$H_t = \begin{pmatrix}h_{11,t} & \rho\sqrt{h_{11,t}h_{22,t}}\\ \rho\sqrt{h_{11,t}h_{22,t}} & h_{22,t}\end{pmatrix}$$

$$h_{11,t} = a_{10} + a_{11}\varepsilon_{1,t-1}^2 + b_{11}h_{11,t-1}$$

$$h_{22,t} = a_{20} + a_{21}\varepsilon_{2,t-1}^2 + b_{21}h_{22,t-1}$$

$$h_{12,t} = h_{21,t}$$

$$= \rho\sqrt{a_{10} + a_{11}\varepsilon_{1,t-1}^2 + b_{11}h_{11,t-1}}\sqrt{a_{20} + a_{21}\varepsilon_{2,t-1}^2 + b_{21}h_{22,t-1}}$$

Parameter $\boldsymbol{\theta} = (a_{10}, a_{11}, b_{11}, a_{20}, a_{21}, b_{21})'$. Solution $\partial L / \partial\theta = 0$ is solving simultaneously 6 equations, namely

$$\frac{\partial L}{\partial a_{10}} = 0, \frac{\partial L}{\partial a_{11}} = 0, \frac{\partial L}{\partial a_{20}} = 0, \qquad (20)$$
$$\frac{\partial L}{\partial a_{21}} = 0, \frac{\partial L}{\partial b_{11}} = 0, \frac{\partial L}{\partial b_{21}} = 0$$

V. APPLICATION TO BIVARIATE MODEL

The application of the multivariate GARCH model in this paper is given for the bivariate model.

The data taken are weekly average price data of 2017 for chili and onion in Manado city. The amount of data is 52 data. The result of ARCH effect test showed that both data had significant ARCH effect with *p*-value of 0.0023 and 0.0005 respectively.

The result of the parameter estimation for the vech representation bivariate model with the help of computer softwear [8] is

```
> ca.ba.dvec = mgarch(ca.ba~1, ~dvec(1,1), trace=F)
Coefficients:
        C(1) 6.154e+004
        C(2) 3.664e+004
      A(1, 1) 3.742e+007
      A(2, 1) 8.265e+006
      A(2, 2) 4.086e+006
 ARCH(1; 1, 1) 1.000e-001
 ARCH(1; 2, 1) 1.000e-001
 ARCH(1; 2, 2) 1.000e-001
GARCH(1; 1, 1) 8.100e-001
GARCH(1; 2, 1) 8.100e-001
GARCH(1; 2, 2) 8.100e-001
```

Then the vech representation bivariate model (7) is obtained, namely

$$\begin{bmatrix} h_{11,t} \\ h_{21,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} 3.742e+007 \\ 8.265e+006 \\ 4.086e+006 \end{bmatrix} + \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix} \begin{bmatrix} \varepsilon^2_{1,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon^2_{2,t-1} \end{bmatrix} + \begin{bmatrix} 0.81 & 0 & 0 \\ 0 & 0.81 & 0 \\ 0 & 0 & 0.81 \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{21,t-1} \\ h_{22,t-1} \end{bmatrix}$$

$$\begin{bmatrix} h_{11,t} \\ h_{21,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} 3.742e+007 + 0.1\varepsilon^2_{1,t-1} + 0.81h_{11,t-1} \\ 8.265e+006 + 0.1\varepsilon_{1,t-1}\varepsilon_{2,t-1} + 0.81h_{21,t-1} \\ 4.086e+006 + 0.1\varepsilon^2_{2,t-1} + 0.81h_{22,t-1} \end{bmatrix} \quad (21)$$

VI. CONCLUSION

Representation of multivariate ARCH (GARCH) can be given in three forms, ie Vech, BEKK and Consonant Correlation model. The *Vech diagonal* model reduces the number of parameters thus simplifying the vech representation. The multivariate GARCH model can be applied to the data of chili and onion because the test of effect arch on the data is significant.

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