

# Some Degree-based Connectivity Indices of Tadpole Graph

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**Abstract:** Zagreb indices (thought this paper we denote ZbI) a well-known concept introduced in 1972 by I.Gutman and N.Trinajstic denoted as  $M_1(G)$  mathematically to study chemical compounds at molecular level. The first two hyper-ZbI of  $G$  denoted  $HM_1(G)$  and  $HM_2(G)$  are well established. In this paper we determine the first ZbI and second ZbI and their polynomial for the Tadpole graph.

**Index Terms:** Hyper-Zagreb indices, Tadpole graph.

## I. INTRODUCTION

Throughout this research article we are considering finite, connected, undirected simple graph  $G$ . We denote  $V(G)$  represent the set consists of vertices and  $E(G)$  represent the set consists of edges of respectively.  $d_G(v)$  is the number of vertices adjacent to  $v$ . We refer [16].

In theoretical chemistry distinctive atomic structures are frequently demonstrated utilizing sub-atomic graphs. Representation of atoms as vertices and bonds as edges respectively and named it as molecular graph.

A sub-atomic diagram is a graph in which, it compares the vertices and edges as to molecules and bonds, individually. In Chemical sciences, synthetic chart hypothesis made exceptionally powerful advancement, and furthermore a topological list for a diagram is utilized to decide some property of the chart sub-atomic by a solitary number. The first ZbI as well as second ZbI of a graph  $G$  represents topological indices on the basis of degree with respect to the vertices of  $G$ .

By definition the first ZbI of a graph  $G$  is denoted as  $M_1(G)$  and is given by  $M_1(G) = \sum_{v \in V(G)} d_G(v)^2$  or  $M_1(G) = \sum_{v_1, v_2 \in E(G)} [d_G(v_1) + d_G(v_2)]$  and second ZbI of a graph  $G$  is denoted as  $M_2(G)$  and is given by  $M_2(G) = \sum_{v_1, v_2 \in E(G)} [d_G(v_1)d_G(v_2)]$ . In [15] Gutman and Trinajstic introduced the concept of degree based on topological indices.

In the papers [4, 11, 12 & 19] we obtain the history, applications and characteristics of Zagreb indices.

Shirdel et al. [6] defined first hyper ZbI (HZbI) of  $G$  denoted by  $HM_1(G)$  and is given by  $HM_1(G) =$

$\sum_{v_1, v_2 \in E(G)} [d_G(v_1) + d_G(v_2)]^2$  and Farahani et. al. [10], introduced the concept of second HZbI of  $G$  denoted as  $HM_2(G)$  and is given by  $HM_2(G) = \sum_{v_1, v_2 \in E(G)} [d_G(v_1)d_G(v_2)]^2$ . Authors In [12],[16] and [19] studied these concepts for the comparative advantages of HZbI and other degree based topological indices.

The first Zagreb polynomials (Z Polynomial) of graph  $G$  is denoted and defined as  $M_1(G, x) = \sum_{v_1, v_2 \in E(G)} x^{d_G(v_1)+d_G(v_2)}$  and the second Z polynomials of  $G$  gives the equation  $M_2(G, x) = \sum_{v_1, v_2 \in E(G)} x^{d_G(v_1)d_G(v_2)}$  we refer [8]. For further information on ZbI and their polynomial we can go through [1].

The first HZb polynomial of  $G$  is well-defined as  $HM_1(G, x) = \sum_{v_1, v_2 \in E(G)} x^{[d_G(v_1)+d_G(v_2)]^2}$  and second HZb polynomial of  $G$  is represented as  $HM_2(G)$  and is given by  $HM_2(G) = \sum_{v_1, v_2 \in E(G)} x^{[d_G(v_1)d_G(v_2)]^2}$ .

The concept of Randic index in graph theory for any simple graph structure  $G$  can be represented as  $\mathfrak{R}(G)$  and is defined as  $\mathfrak{R}(G) = \sum_{v_1, v_2 \in E(G)} \frac{1}{\sqrt{d_G(v_1)d_G(v_2)}}$ . This index was proposed by Randic in [7].

Another interesting concept of graph theory named as the sum connectivity index (SCI) for any simple graph  $G$  is denoted and defined as  $X(G) = \sum_{v_1, v_2 \in E(G)} \frac{1}{\sqrt{d_G(v_1)+d_G(v_2)}}$ . This topological index was proposed by Zhou and Trinajstic in [20].

In [3], Estrada et.al. introduced concept of chemical graph theory called atom-bond connectivity index (ABCI), which is defined  $ABC(G) = \sum_{v_1, v_2 \in E(G)} \sqrt{\frac{d_G(v_1)+d_G(v_2)-2}{d_G(v_1)d_G(v_2)}}$ .

In this section, we conduct some calculations needed to calculate the indices of Zagreb and their polynomials  $G$ . We use the partition set by the vertex  $V_a = \{v \in V : d_G(v) = a\}$  and edge  $E_b = \{e = v_1v_2 \in E : d_G(v_1) + d_G(v_2) = b\}$  and  $E_c^* = \{e = v_1v_2 \in E : d_G(v_1)d_G(v_2) = c\}$ .

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II. DEFINITIONS

**Definition 2.1:**  $T_{m,n}$  denotes the tadpole graph constructed by joining a cycle  $C_m$  to a path  $P_n$ . Where  $m \geq 3$  and  $n \geq 1$ . Tadpole graph is also called as dragon graph.

Generally, in  $G$ , vertex set can be denoted as  $V(G)$  and set of all edges represents  $E(G)$  respectively. Also, the no. of vertices  $|V(G)| = m + n$  and no. of edges  $|E(G)| = m + n$ .

**Case 1:**

Vertex and edge partition of tadpole graphs  $T_{m,n}$  where  $m \geq 3$  and  $n = 1$

We split  $V(G)$  into three subsets  $V_1, V_2$  &  $V_3$  as the following three partitions.

$$V_1 = \{v \in V(G); d_1(v) = d_G(v) = 1\}; |V_1| = 1$$

$$V_2 = \{v \in V(G); d_2(v) = d_G(v) = 2\}; |V_2| = m - 1$$

$$V_3 = \{v \in V(G); d_3(v) = d_G(v) = 3\}; |V_3| = 1$$

Similarly, we consider edge set  $E(G)$  partition of  $G$  as  $E_{2,2}, E_{3,1}$  &  $E_{3,2}$ . That is,

$$E_{2,2} = \{uv \in E(G); d_G(u) = d_G(v) = 2\}; |E_{2,2}| = m - 2.$$

$$E_{3,2} = \{uv \in E(G); d_G(u) = 3, d_G(v) = 2\}; |E_{3,2}| = 2.$$

$$E_{3,1} = \{uv \in E(G); d_G(u) = 3, d_G(v) = 1\}; |E_{3,1}| = 1.$$

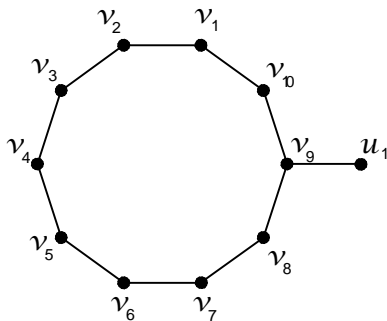


Fig 1:  $T_{10,1}$  Graph

**Case 2:**

Vertex and edge partition of tadpole graphs  $T_{m,n}$  where  $m \geq 3$  and  $n > 1$  is considered below.

In this case we split  $V(G)$  into three subsets  $V_1, V_2$  &  $V_3$  as the following three partitions.

$$V_1 = \{v \in V(G); d_1(v) = d_G(v) = 1\}; |V_1| = 1$$

$$V_2 = \{v \in V(G); d_2(v) = d_G(v) = 2\}; |V_2| = m + n - 2,$$

$$\text{and } V_3 = \{v \in V(G); d_3(v) = d_G(v) = 3\}; |V_3| = 1$$

Similarly, we consider the partition of edge set  $E(G)$  of  $G$  as  $E_{2,1}, E_{2,2}$  &  $E_{2,3}$ . That is,

$$E_{2,2} = \{uv \in E(G); d_G(u) = d_G(v) = 2\}; |E_{2,2}| = m + n - 4.$$

$$E_{2,3} = \{uv \in E(G); d_G(u) = 3, d_G(v) = 2\}; |E_{2,3}| = 3.$$

$$E_{2,1} = \{uv \in E(G); d_G(u) = 2, d_G(v) = 1\}; |E_{2,1}| = 1.$$

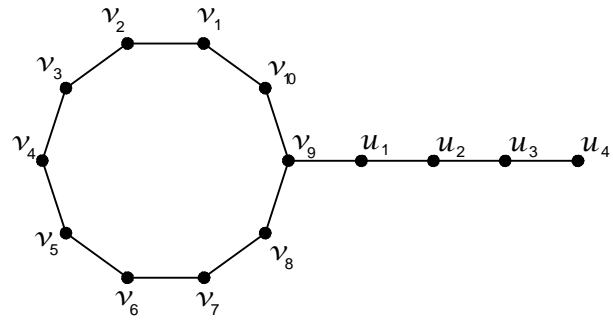


Fig2:  $T_{10,4}$  Graph

**Theorem 2.1:** The first HZBI and their polynomial of a Tadpole graph are

$$HM_1(T_{(m,n)}) = 16m + 34 \text{ where } m \geq 3 \text{ and } n = 1$$

$$HM_1(T_{m,n}, x) = 2x^{25} + (m - 1)x^{16} \text{ where } m \geq 3 \text{ and } n = 1$$

**Proof:** Let  $G = T_{m,n}$  be a tadpole graph. Let the vertex set  $V(G) = \{v_1, v_2, \dots, v_n\} \cup \{u_1, u_2, \dots, u_n\}$ . Where  $v_i \in V(C_m)$  and  $u_i \in P_n$ . We have,

$$\begin{aligned} HM_1(T_{m,n}) &= \sum_{v_1 v_2 \in E(G)} [d_G(v_1) + d_G(v_2)]^2 \\ &= \sum_{E_1} [d_2(v_1) + d_2(v_2)]^2 + \sum_{E_2} [d_3(v_1) + d_2(v_2)]^2 \\ &\quad + \sum_{E_3} [d_3(v_1) + d_1(v_2)]^2 \\ &= (m - 2)(2 + 2)^2 + 2(3 + 2)^2 + 1(3 + 1)^2 \\ &= 16(m - 2) + 50 + 16 \end{aligned}$$

$$HM_1(T_{m,n}) = 16m + 34$$

Now for the first hyper-Zagreb polynomial of  $T_{m,n}$ , we have

$$\begin{aligned} HM_1(T_{m,n}, x) &= \sum_{v_1 v_2 \in E} x^{[d_G(v_1) + d_G(v_2)]^2} \\ &= \sum_{E_1} x^{[d_2(v_1) + d_2(v_2)]^2} + \sum_{E_2} x^{[d_3(v_1) + d_2(v_2)]^2} \\ &\quad + \sum_{E_3} x^{[d_3(v_1) + d_1(v_2)]^2} \\ &= (m - 2)x^{[2+2]^2} + 2x^{[3+2]^2} + x^{[3+1]^2} \\ &= 2x^{25} + (m - 2)x^{16} + x^{16} \\ HM_1(T_{m,n}, x) &= 2x^{25} + (m - 1)x^{16} \end{aligned}$$

**Theorem 2.2:** The first HZBI and their polynomial of a Tadpole graph are

$$HM_1(T_{(m,n)}) = 16m + 16n + 20 \text{ where } m \geq 3 \text{ and } n > 1$$

$$HM_1(T_{m,n}, x) = 3x^{25} + (m + n - 4)x^{16} + x^9 \text{ where } m \geq 3 \text{ and } n > 1$$

**Proof:** Let  $G = T_{m,n}$  be a tadpole graph. As defined in theorem 2.1, we consider the vertex set  $V(G)$ .

Consider

$$\begin{aligned} HM_1(T_{m,n}) &= \sum_{v_1 v_2 \in E(G)} [d_G(v_1) + d_G(v_2)]^2 \\ &= \sum_{E_1} [d_2(v_1) + d_2(v_2)]^2 + \sum_{E_2} [d_3(v_1) + d_2(v_2)]^2 \\ &\quad + \sum_{E_3} [d_3(v_1) + d_1(v_2)]^2 \\ &= (m + n - 4)(2 + 2)^2 + 3(3 + 2)^2 + 1(2 + 1)^2 \\ &= 16(m + n - 4) + 75 + 9 \\ &= 16m + 16n + 20 \end{aligned}$$



Now for the first HZb polynomial of  $T_{m,n}$ , we have

$$\begin{aligned} HM_1(T_{m,n}, x) &= \sum_{uv \in E} x^{[d_G(v_1)+d_G(v_2)]^2} \\ &= \sum_{E_1} x^{[d_2(v_1)+d_2(v_2)]^2} + \sum_{E_2} x^{[d_3(v_1)+d_2(v_2)]^2} + \\ &\sum_{E_3} x^{[d_2(v_1)+d_1(v_2)]^2} \\ &= (m+n-4)x^{(2+2)^2} + 3x^{(3+2)^2} + x^{(2+1)^2} \\ &= (m+n-4)x^{16} + 3x^{25} + x^9 \\ HM_1(T_{m,n}, x) &= 3x^{25} + (m+n-4)x^{16} + x^9 \end{aligned}$$

**Theorem 2.3:**The second HZbI and their polynomial of a Tadpole graph are

$$\begin{aligned} HM_2(T_{m,n}) &= 16m + 49 \text{ where } m > 3 \text{ and } n = 1 \\ HM_2(T_{m,n}, x) &= 3x^{36} + (m-2)x^{16} + x^4 \text{ where } m \\ &> 3 \text{ and } n = 1 \end{aligned}$$

**Proof:** Let  $G = T_{m,n}$  be a tadpole graph and define the vertex set  $V(G)$  as in theorem 2.1. Consider

$$\begin{aligned} HM_2(T_{m,n}) &= \sum_{v_1 v_2 \in E} [d_G(v_1)d_G(v_2)]^2 \\ &= \sum_{E_1} [d_2(v_1)d_2(v_2)]^2 + \sum_{E_2} [d_3(v_1)d_2(v_2)]^2 \\ &\quad + \sum_{E_3} [d_3(v_1)d_1(v_2)]^2 \\ &= (m-2)(2 \times 2)^2 + 2(3 \times 2)^2 + (3 \times 1)^2 \\ &= 16(m-2) + 72 + 9 \end{aligned}$$

$$HM_2(T_{m,n}) = 16m + 49$$

Now, for the second hyper-Zagreb polynomial of a  $T_{m,n}$ , we have

$$\begin{aligned} HM_2(T_{m,n}, x) &= \sum_{uv \in E} x^{[d_G(v_1)d_G(v_2)]^2} \\ &= \sum_{E_1} x^{[d_2(v_1)d_2(v_2)]^2} + \sum_{E_2} x^{[d_3(v_1)d_2(v_2)]^2} \\ &\quad + \sum_{E_3} x^{[d_3(v_1)d_1(v_2)]^2} \\ &= (m-2)x^{[2 \times 2]^2} + 2x^{[3 \times 2]^2} + x^{[3 \times 1]^2} \\ &= (m-2)x^{16} + 3x^{36} + x^4 \\ HM_2(T_{m,n}, x) &= 2x^{36} + (m-2)x^{16} + x^4 \end{aligned}$$

**Theorem 2.4:**The second HZbI and their polynomial of a Tadpole graph are

$$\begin{aligned} HM_2(T_{m,n}) &= 16m + 16n + 48 \text{ where } m > 3 \text{ and } n \\ &> 1 \\ HM_2(T_{m,n}, x) &= 3x^{36} + (m+n-4)x^{16} + x^4 \text{ where } m \\ &> 3 \text{ and } n > 1 \end{aligned}$$

**Proof:** Let  $G = T_{m,n}$  be a tadpole graph. As defined in theorem 2.1, we consider the vertex set  $V(G)$ .

Consider

$$\begin{aligned} HM_2(T_{m,n}) &= \sum_{v_1 v_2 \in E} [d_G(v_1)d_G(v_2)]^2 \\ &= \sum_{E_1} [d_2(v_1)d_2(v_2)]^2 + \sum_{E_2} [d_3(v_3)d_2(v_2)]^2 \\ &\quad + \sum_{E_3} [d_2(v_1)d_1(v_2)]^2 \\ &= (m+n-4)(2 \times 2)^2 + 3(3 \times 2)^2 + (2 \times 1)^2 \\ &= 16(m+n-4) + 108 + 4 \end{aligned}$$

$$HM_2(T_{m,n}) = 16m + 16n + 48$$

Now, for the second hyper-Zagreb polynomial of a  $T_{m,n}$ , we have

$$HM_2(T_{m,n}, x) = \sum_{v_1 v_2 \in E} x^{[d_G(v_1)d_G(v_2)]^2}$$

$$\begin{aligned} &= \sum_{E_1} x^{[d_2(v_1)d_2(v_2)]^2} \\ &\quad + \sum_{E_2} x^{[d_3(v_1)d_2(v_2)]^2} + \sum_{E_3} x^{[d_2(v_1)d_1(v_2)]^2} \\ &= (m+n-4)x^{[2 \times 2]^2} + 3x^{[3 \times 2]^2} + x^{[2 \times 1]^2} \\ &= (m+n-4)x^{16} + 3x^{36} + x^4 \\ HM_2(T_{m,n}, x) &= 3x^{36} + (m+n-4)x^{16} + x^4 \end{aligned}$$

**Corollary 2.5:** The difference between first and second HZbI of a Tadpole graph  $T_{m,n}$  where ( $m > 3$  and  $n > 1$ ) is

$$HM_2(T_{m,n}) - HM_1(T_{m,n}) = 28$$

**Corollary 2.6:** The difference between first and second HZbI of a Tadpole graph  $T_{m,n}$ , where ( $m > 3$  and  $n = 1$ ) is

$$HM_2(T_{m,n}) - HM_1(T_{m,n}) = 15$$

**Corollary 2.7:**The first HZbI of Tadpole graph is equal to sum of the first HZbI of cycle  $C_m$  and path  $P_n$ . Where ( $m > 3$  and  $n > 1$ )

$$HM_1(T_{m,n}) = HM_1(C_m) + HM_1(P_n)$$

**Corollary 2.8:**The first HZbI of Tadpole graph is equal to sum of first HZbI of cycle  $C_m$  and sixteen times  $n$ . Where ( $m > 3$  and  $n = 1$ )

$$HM_1(T_{m,n}) = HM_1(C_m) + 16n$$

**Theorem 2.9:** The Randic index of Tadpole graph  $G$  are

$$\kappa(G) = \frac{(m-2)}{2} + 1.394 \text{ where } m \geq 3 \text{ and } n = 1$$

**Proof:** Consider the graph  $G = T_{m,n}$  is a tadpole graph. We consider the vertex set  $V(G)$  as defined in Theorem 2.1. We have,

$$\begin{aligned} \kappa(G) &= \sum_{v_1 v_2 \in E(G)} \frac{1}{\sqrt{d_G(v_1)d_G(v_2)}} \\ &= \sum_{v_1 v_2 \in E(G)} \frac{1}{\sqrt{d_2(v_1)d_2(v_2)}} + \sum_{v_1 v_2 \in E(G)} \frac{1}{\sqrt{d_3(v_1)d_2(v_2)}} + \\ &\quad \sum_{v_1 v_2 \in E(G)} \frac{1}{\sqrt{d_3(v_1)d_1(v_2)}} \\ &= (m-2) \frac{1}{\sqrt{2 \times 2}} + 2 \frac{1}{\sqrt{3 \times 2}} + \frac{1}{\sqrt{3 \times 1}} \\ \kappa(G) &= \frac{(m-2)}{2} + 1.394 \end{aligned}$$

**Theorem 2.10:** The Randic index Tadpole graphs are

$$\kappa(T_{m,n}) = \frac{(m+n-4)}{2} + 1.932 \text{ where } m \geq 3 \text{ and } n > 1$$

**Proof:** Let  $G = T_{m,n}$  be a tadpole graph. Define  $V(G)$  as in Theorem 2.1. Consider

$$\begin{aligned} \kappa(G) &= \sum_{v_1 v_2 \in E(G)} \frac{1}{\sqrt{d_G(v_1)d_G(v_2)}} \\ &= \sum_{v_1 v_2 \in E(G)} \frac{1}{\sqrt{d_2(v_1)d_2(v_2)}} + \sum_{v_1 v_2 \in E(G)} \frac{1}{\sqrt{d_3(v_1)d_2(v_2)}} + \\ &\quad \sum_{v_1 v_2 \in E(G)} \frac{1}{\sqrt{d_2(v_1)d_1(v_2)}} \\ &= (m+n-4) \frac{1}{\sqrt{2 \times 2}} + 3 \frac{1}{\sqrt{3 \times 2}} + \frac{1}{\sqrt{2 \times 1}} \\ &= \frac{(m+n-4)}{2} + 1.932 \\ \kappa(G) &= \frac{(m+n-4)}{2} + 1.932 \end{aligned}$$



**Theorem 2.11:** The sum connectivity indexes of tadpole graph are

$$X(G) = \frac{(m-2)}{2} + 1.394 \text{ where } m \geq 3 \text{ and } n = 1$$

**Proof:** As defined in Theorem 2.1, we consider the set of vertices  $V(G)$ . We have

$$\begin{aligned} \chi(G) &= \sum_{v_1 v_2 \in E(G)} \frac{1}{\sqrt{d_G(v_1) + d_G(v_2)}} \\ &= \sum_{v_1 v_2 \in E(G)} \frac{1}{\sqrt{d_2(v_1) + d_2(v_2)}} + \sum_{v_1 v_2 \in E(G)} \frac{1}{\sqrt{d_3(v_1) + d_2(v_2)}} \\ &\quad + \sum_{v_1 v_2 \in E(G)} \frac{1}{\sqrt{d_3(v_1) + d_1(v_2)}} \\ &= (m-2) \frac{1}{\sqrt{2+2}} + 2 \frac{1}{\sqrt{3+2}} + \frac{1}{\sqrt{3+1}} \\ \chi(G) &= \frac{(m-2)}{2} + 1.394 \end{aligned}$$

**Theorem 2.12:** The sum connectivity indexes of tadpole graph are

$$X(G) = \frac{(m+n-4)}{2} + 1.919 \text{ where } m \geq 3 \text{ and } n > 1$$

**Proof:** We denote sum connectivity  $X(G)$  and let  $G = T_{m,n}$  be a tadpole graph. Consider

$$\begin{aligned} X(G) &= \sum_{v_1 v_2 \in E(G)} \frac{1}{\sqrt{d_G(v_1) + d_G(v_2)}} \\ &= \sum_{v_1 v_2 \in E(G)} \frac{1}{\sqrt{d_2(v_1) + d_2(v_2)}} + \sum_{v_1 v_2 \in E(G)} \frac{1}{\sqrt{d_3(v_1) + d_2(v_2)}} \\ &\quad + \sum_{v_1 v_2 \in E(G)} \frac{1}{\sqrt{d_2(v_1) + d_1(v_2)}} \\ &= (m+n-4) \frac{1}{\sqrt{2+2}} + 3 \frac{1}{\sqrt{3+2}} + \frac{1}{\sqrt{2+1}} \\ &= \frac{(m+n-4)}{2} + 1.919 \end{aligned}$$

**Theorem 2.13:** The atom-bond connectivity index of tadpole graph is

$$ABC(G) = \frac{m+n}{\sqrt{2}} + 2.231 \text{ where } m \geq 3 \text{ and } n = 1$$

**Proof:** Let  $G = T_{m,n}$  be a tadpole graph. Consider

$$\begin{aligned} ABC(G) &= \sum_{v_1 v_2 \in E(G)} \sqrt{\frac{d_G(v_1) + d_G(v_2) - 2}{d_G(v_1) d_G(v_2)}} \\ &= \sum_{v_1 v_2 \in E(G)} \sqrt{\frac{d_2(v_1) + d_2(v_2) - 2}{d_2(v_1) d_2(v_2)}} + \sum_{v_1 v_2 \in E(G)} \sqrt{\frac{d_3(v_1) + d_2(v_2) - 2}{d_3(v_1) d_2(v_2)}} \\ &\quad + \sum_{v_1 v_2 \in E(G)} \sqrt{\frac{d_2(v_1) + d_1(v_2) - 2}{d_G(v_1) d_G(v_2)}} \\ &= (m-2) \sqrt{\frac{2+2-2}{2 \times 2}} + 2 \sqrt{\frac{3+2-2}{3 \times 2}} + \sqrt{\frac{3+1-2}{3 \times 1}} \\ ABC(G) &= \frac{m-2}{\sqrt{2}} + 2.231 \end{aligned}$$

**Theorem 2.14:** The atom-bond connectivity index of tadpole graph is

$$ABC(G) = \frac{m+n}{\sqrt{2}} \text{ where } m \geq 3 \text{ and } n > 1$$

**Proof:** Let  $G = T_{m,n}$  be a tadpole graph. Consider

$$\begin{aligned} ABC(G) &= \sum_{v_1 v_2 \in E(G)} \sqrt{\frac{d_G(v_1) + d_G(v_2) - 2}{d_G(v_1) d_G(v_2)}} \\ &= \sum_{v_1 v_2 \in E(G)} \sqrt{\frac{d_2(v_1) + d_2(v_2) - 2}{d_2(v_1) d_2(v_2)}} + \sum_{v_1 v_2 \in E(G)} \sqrt{\frac{d_3(v_1) + d_2(v_2) - 2}{d_3(v_1) d_2(v_2)}} \\ &\quad + \sum_{v_1 v_2 \in E(G)} \sqrt{\frac{d_2(v_1) + d_1(v_2) - 2}{d_2(v_1) d_1(v_2)}} \\ &= (m+n-4) \sqrt{\frac{2+2-2}{2 \times 2}} + 3 \sqrt{\frac{3+2-2}{3 \times 2}} + \sqrt{\frac{2+1-2}{2 \times 1}} \\ ABC(G) &= \frac{m+n}{\sqrt{2}} \end{aligned}$$

### III. RESULTS

#### 3.1 First hyper index and polynomial

Sl. No	Notation and Graph	First hyper index	Polynomial
1	$HM_1(T_{(m,n)})$ where $m \geq 3$ and $n > 1$	$16m + 16n + 20$	$3x^{25} + (m+n-4)x^{16} + x^9$
2	$HM_1(T_{(m,n)})$ $m \geq 3$ and $n = 1$	$16m + 34$	$2x^{25} + (m-1)x^{16}$

#### 3.2 Second hyper index and polynomial

Sl. No	Notation and Graph	Second hyper index	Polynomial
1	$HM_2(T_{(m,n)})$ where $m \geq 3$ and $n > 1$	$16m + 16n + 48$	$3x^{36} + (m+n-4)x^{16} + x^4$
2	$HM_2(T_{(m,n)})$ $m \geq 3$ and $n = 1$	$16m + 49$	$3x^{36} + (m+n-4)x^{16} + x^4$

#### 3.3 The difference between first and second hyper-Zagreb index

Sl. No	Graph	Difference
1	$HM_1(T_{(m,n)})$ where $m \geq 3$ and $n > 1$	28
2	$HM_1(T_{(m,n)})$ $m \geq 3$ and $n = 1$	15

#### 3.4 Connectivity indices





Sl.no	Notation and Graph	$m \geq 3$ and $n > 1$	$m \geq 3$ and $n = 1$
1	$\aleph(T_{(m,n)})$	$\frac{(m+n-4)}{2} + 1.932$	$\frac{(m-2)}{2} + 1.394$
2	$X(T_{(m,n)})$	$\frac{(m+n-4)}{2} + 1.919$	$\frac{(m-2)}{2} + 1.394$
3	$ABC(T_{(m,n)})$	$\frac{m+n}{\sqrt{2}}$	$\frac{m-2}{\sqrt{2}} + 2.231$

#### IV. CONCLUSION

- For the Tadpole graph first HZbI is equal to sum of the first HZbI of cycle and path. Here ( $G = T_{m,n}$  where  $m > 3$  and  $n > 1$ ).
- For the Tadpole graph first HZbI is equal to sum of the first HZbI of cycle and sixteen times  $n$ . Here ( $G = T_{m,n}$  where  $m > 3$  and  $n = 1$ ).
- The Randic index of Tadpole graph is equal to the sum connectivity indexes of tadpole graph. where  $m \geq 3$  and  $n = 1$ .

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