Consistent Network Emerge for N-Hop Network

Girija M.S, Thiagarajan K, Tapas Bapu B.R

Abstract:
Communication is all about how you get your message across the network. Network theory is a part of grid theory in which a network can be depicted as a grid with nodes and links. Connected grid and completely connected grid representation of the network is converted to agenda reliance chart which illustrates the data dependencies and control dependencies. Fixation of the source node and sink node is made from the agenda reliance chart and the possible numbers of n-hop networks are determined through agenda reliance chart.

Index Terms: Agenda Reliance Chart, Grid, N-Hop Network, Shortest Path

I. INTRODUCTION

In a communication set-up, a node can send, store or receive data. In other words, any node can play the role source or sink node or intermediate node to enhance the process of communication. A hop is just a path portion between resource node and sink node [1]. The hop count refers to the number of nodes which are intermediate between the source node and sink node. There are many n-hop networks. In general, n-hop network means that there are n intermediate nodes between the source node and sink node. Grid theory is a study of grids which is used to model the network with nodes as vertices and paths as edges. Grid theory approach has become a critical component in the computing field including network and security [2]. Network theory [6], a study of grids has wide number of applications in many disciplines.

Preliminary and Definitions

1. Grid:
A grid [4] is an ordered pair G = (V, E) where, V is the vertices set and E is the set of connections between vertices.

2. Undirected Grid:
An undirected grid [3] is a grid in which links have no direction i.e., the edge (a, b) is identical to (b, a).

3. Directed Grid:
A directed grid [3] is a grid in which edges have orientation. Here, edge is a set of pair of ordered vertices.

4. Connected Grid:
A grid is said to be connected if there is a path between every pair of vertices.

5. Degree:
The degree [5] of a vertex of the grid is the number of links incident to the vertex.

6. Regular Grid:
A grid [7] is said to be a regular grid if every node has the same degree.

7. Connecting Way:
Connecting way [8] is a finite or infinite sequence of links which connects a sequence of vertices.

8. Level:
Level is a division of the nodes into subsets that have the same distance from a given root vertex.

9. Agenda Reliance Chart (ARC):
Agenda Reliance Chart in computer science is a representation of grid that makes data dependencies and control dependencies explicit.

The organization of the paper as follows, Section II illustrates the block diagram, Section III presents the examples of 2-Regular grid, Section IV deals with 3-Regular grid. Section V presents complete grid, Section VI describes the propositions and Section VII ends up with the conclusion and future work.

II. BLOCK DIAGRAM OF PROPOSED SYSTEM

Fig. 1 illustrates the representation of the regular grid approach for N-hop networks.

![Figure 1. Block Diagram](image)

Here,
GR – graphical representation,
ELG- equivalent level graph,
So & S1 are source node and sink node.
III. STUDY ON POSSIBLE N–HOP NETWORK IN 2-REGULAR GRID (N>=1)

Example 3.1

Consider the following grid G₀ with 4 nodes and 4 links as shown below in Fig. 2.

![Figure 2. Grid G₀](image)

The possible 1-hop and 2-hop networks are shown in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Hop Networks of G₀</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1-Hop</strong></td>
</tr>
<tr>
<td>1-3-4</td>
</tr>
<tr>
<td>1-2-4</td>
</tr>
<tr>
<td>2-1-3</td>
</tr>
<tr>
<td>2-4-3</td>
</tr>
<tr>
<td>3-1-2</td>
</tr>
<tr>
<td>3-4-2</td>
</tr>
<tr>
<td>4-2-1</td>
</tr>
<tr>
<td>4-3-1</td>
</tr>
</tbody>
</table>

Corresponding Agenda Reliance Chart [5] for G₀ is shown in Fig.3 (a).

![Figure 3(a). ARC of G₀](image)

Now, fixing 1 as the source node (since it is at the first level) and 4 as the sink node (because it is at the last level), the possible 1-hop networks (required level) are listed as below in Table 2(a).

<table>
<thead>
<tr>
<th>Table 2(a). Required Hop Networks of G₀</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Levels</strong></td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
</tbody>
</table>

Observation:
Here, we observed from Fig. 3(a) & 3(b) that in 1-hop network every node of the given network can act as the source node as well as the sink node according to the fixation of the user (as user friendly).

IV. STUDY ON N–HOP NETWORK IN 3–REGULAR GRID (N >= 1)

Example 4.1:
Consider the following grid G₁ with 6 nodes and 9 links as shown in Fig. 4

![Figure 4. Grid G₁](image)

Fig. 5.shows the Agenda Reliance Chart (ARC) or level based structure of G₁.
Now, fixing 1 as the source node (since it is at the first level) and 3 & 4 as the sink node (since it is at the last level) the possible 1-hop and 2-hop networks are as follows in Table 3.

### Table 3. Required Hop Networks of G₁

<table>
<thead>
<tr>
<th>Levels</th>
<th>Nodes</th>
<th>1-Hop</th>
<th>2-Hop</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>1-5-4</td>
<td>1-2-4-3</td>
</tr>
<tr>
<td>B</td>
<td>5,2,6</td>
<td>1-2-4</td>
<td>1-2-3-4</td>
</tr>
<tr>
<td>C</td>
<td>4,3</td>
<td>1-2-3</td>
<td>1-5-4-3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1-6-3</td>
<td>1-6-3-4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1-6-5-4</td>
</tr>
</tbody>
</table>

**Observation:**
If any node is fixed as the source node (since every node is having equal degree), all the other nodes can be acted as the sink node. Sink node can be reached through only 1-hop network as shown in Fig. 5, because the distance between source node and sink node is maximum 2.

**Example 4.2**
Consider the following grid G₂ with 8 vertices and 12 edges as shown in Fig.6.

![Grid G₂](image)

**Figure 6. Grid G₂**

Fig. 7. Shows ARC for G₂:

![ARC of G₂](image)

**Figure 7. ARC of G₂**

Now, fixing 1 & 2 as the source node (since it is at the first level) and 3 & 4 as the sink node (since it is at the last level) the possible 1-hop, 2-hop and 3-hop networks are shown in Table 4.

### Table 4. Required Hop Networks of G₂

<table>
<thead>
<tr>
<th>Levels</th>
<th>Nodes</th>
<th>1-Hop</th>
<th>2-Hop</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1,2</td>
<td>1-2-3</td>
<td>1-6-5-4</td>
</tr>
<tr>
<td>B</td>
<td>6,7</td>
<td>2-1-4</td>
<td>2-7-8-3</td>
</tr>
<tr>
<td>C</td>
<td>5,8</td>
<td>1-6-5-8-3</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>3,4</td>
<td>2-7-8-5-4</td>
<td></td>
</tr>
</tbody>
</table>

**Observation:**
Here, at most 3-hop network can be constructed as shown in Table 4 after fixing up the source node and the sink node.

**Example 4.3**
Consider the following asymmetrical grid G₃ with 10 vertices and 15 edges as shown in Fig. 8.

![Grid G₃](image)

**Figure 8. Grid G₃**

ARC of G₃ is shown in Fig. 9.

![ARC of G₃](image)

**Figure 9. ARC of G₃**

Now, fixing 1 as the source node (since it is at the first level) and 9 as the sink node (since it is at the last level), the only possible 3-hop networks are shown in Table 5.

### Table 5. Required Hop Networks of G₃

<table>
<thead>
<tr>
<th>Levels</th>
<th>Nodes</th>
<th>3-Hop</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>1-2-8-7-9</td>
</tr>
<tr>
<td>B</td>
<td>2,3,4</td>
<td>1-2-8-0-9</td>
</tr>
<tr>
<td>C</td>
<td>8,5</td>
<td>1-4-5-0-9</td>
</tr>
<tr>
<td>D</td>
<td>7,0,6</td>
<td>1-4-5-6-9</td>
</tr>
<tr>
<td>E</td>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>
networks are not possible. But 3-hop network can be constructed as shown in Table 5.

V. COMPLETE GRID

Consider any regular complete grid as shown in Fig. 10. with n nodes.

![Grid G4](image)

Figure 10. Grid G4

(1) ARC of G₃ as shown in Fig 11 shows that (n-1) number of 1-Hop will be available even though it is n perfect colorable as shown in Fig. 12.

![ARC of G4](image)

Figure 11. ARC of G₄

![Grid Coloring of ARC of G4](image)

Figure 12. Grid Coloring of ARC of G₄

(2) If grid is complete regular then, ARC will have only 2 levels. The possible 1-Hop networks are shown in Table 6.

<table>
<thead>
<tr>
<th>Levels</th>
<th>Nodes</th>
<th>1-Hop</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>A</td>
<td>B-A-D</td>
</tr>
<tr>
<td>1</td>
<td>B,C,D</td>
<td>B-A-C</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B-C-D</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C-A-D</td>
</tr>
</tbody>
</table>

VI. PROPOSITIONS

1. In a given r-Regular network if there are n levels in Level based program dependence grid then, at most (n-2) hop network can be constructed.

2. In a given r-Regular network if there are p points, q edges, and n levels with as the source node and v as the set of vertices, then,

\[ p \leq n \cdot \deg(s) \leq \sum \deg(v - s) \]

VII. CONCLUSION AND FUTURE WORK

Regular network is taken into consideration and is converted to agenda reliance chart to identify the maximum levels of network. Source node and sink node is fixed based on the Agenda Reliance Chart and N-hop networks are determined to obtain the optimized path. The work can be extended in future by considering the ancestor networks.

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REFERENCES

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