Effect of Sweep Angle and a Half Sine Wave on Roll Damping Derivative of a Delta Wing

Renita Sharon Monis, Aysha Shabana, Asha Crasta, S. A. Khan

ABSTRACT--- This paper presents the effect of sweep angle on a roll of damping derivative of a delta wing with half sine wave for an attached shock case in supersonic/hypersonic flow has been studied analytically. The Ghosh Strip theory is replicated. By combining this with the similitude at high-speed flows lead to giving a piston theory. The initial conditions for the applicability of the theory are that the attached wave must be attached with the leading edge of the wing. The results of the present study reveals that with the increments in the sweep angles; it results in continuous decrease in the roll damping derivative, it is also seen that the magnitude of the decrement for lower sweep angle is considerable as compared to the higher values of the sweep angles due to the drastic change in the surface area of the wing. Roll damping derivative progressively increases with the angle of attack; however, with the increase in the inertia level of the flow, it results in the decrement in the damping derivative and later conforms to the Mach number independence principle. Effect of the leading edge bluntness and viscous effects are neglected. Results have been obtained for the supersonic/hypersonic flow of perfect gases over a wide range of angle of attack, planform area for different Mach numbers. In the present study, attention is on the effect of sweep angle of the wing on roll damping derivative at a different angle of attack and inertia level has been studied. In the contemporary theory, Leeward surface is taken along with shock waves attached with the leading edge.

Keywords — Delta wing, Hypersonic, Leeward surface, Sweep angle.

I. INTRODUCTION

Ghosh [1] has developed a 2D large deflection hypersonic similitude. The resulting piston theory is not restricted to slender shapes as in the cases of Lighthill's [2] and Miles [3] piston theories. Ghosh's piston theory [4] has been applied to oscillating plane ogives to predict C_{m_q} . The similitude was extended to non-slender cones/quasi cones, and a new kind of piston motion, called conico-annular piston motion was given by Ghosh [5]. Oscillating delta wings at significant incidence was treated by Ghosh [6]. Etkin [7] and Levin [8] have shown the separate effects of the pitch rate and incidence rate on the pitching moment. The plane piston theory of Ghosh [9] was applied with the inclusion of wave reflection effect to obtain a converged solution for $C_{m_{\dot\alpha}}$ non-slender wedges/plane ogives with the rate of α in hypersonic flow. Ghosh [10] has given a unified hypersonic similitude, and a consequent piston theory which is valid for wedges/quasi-wedges for any Mach number greater than 1 and $E \le 0.3$ provided bow shock is attached. Hui et al. [11] have studied the problem of stability of an oscillating flat plate wing of arbitrary plan form placed at a specified mean angle of attack in supersonic/ hypersonic flow by applying strip theory. During the derivations of the theory, it is assumed that at each spanwise station the flow is independent of the location of the strips, and the flow remains two-dimensional. with the shock being attached. To assess the overall stability the moment derivatives due to the pitch rate as well as incident rate should be evaluated. In the present work, the unified similitude of Ghosh [12] along with the extended theory of Crasta & Khan [15]-[20] is combined with strip theory to obtain the unsteady moment derivative for a wing whose front edge is straight.

In this paper, the authors have attempted to study the Stiffness derivative with different pivot positions (h), which gives accurate results in comparison with the theory developed, by Liu as well as Crasta & Khan.

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II. ANALYSIS

Let the roll be p and the rolling moment is L, defined according to the right-hand system of reference

$$L = 2\int_0^c \left(\int_0^{z=f(x)} pzdz \right) dx$$

The local piston Mach number normal to the wing surface is given by

$$M_P = (M_\infty) Sin\alpha_0 \frac{z}{a_\infty} \bar{p}$$

The non-dimensionalized was achieved by dividing with the dynamic pressure, area, a span of the wing, and a characteristic time factor

$$\therefore -C_{l_p} = \frac{1}{\int_\infty U_\infty C^3 b(\cot\varepsilon - \frac{4A_H}{\pi})} C^3$$

$$\therefore -C_{l_p} = \frac{\sin\alpha_0 f(S_1)}{(Cot^2\varepsilon - \frac{4A_H}{\pi}\cot\varepsilon)} \left[\begin{array}{l} \frac{\cot^2\varepsilon}{12} + \cot^2\varepsilon \frac{A_F}{2\pi} - \frac{A_H}{\pi^3}(\pi^2 - 4) + \\ \frac{1}{4}\cot\varepsilon(A_F{}^2 + A_H{}^2) - \frac{4}{9\pi} A_H{}^3 \\ -\frac{16}{9}\frac{A_F A_H}{\pi^2} - \frac{16}{15\pi} A_F{}^2 A_H \end{array} \right] \quad (1)$$

Where $S_1 = M_\infty Sin\alpha_0$

$$f(S_1) = \frac{(\gamma+1)}{2S_1}\left[2S_1 + \frac{(B+2S_1{}^2)}{(B+S_1{}^2)^{1/2}}\right]$$

On the Leeward surface ,the piston Mach number

$$M_p = \frac{Z_{\bar{p}}}{a_\theta \cos\mu_\theta}$$

For the rotation in the right-handed direction the acoustic pressure ratio

$$\frac{p_p}{P_\theta} = 1 + \gamma M_p$$

The rolling Moment

$$\bar{L} = -\int P_p (L - x) z dz$$

$$= \int \frac{p_\theta \gamma z}{a_\theta \cos\mu_\theta} (L - x) z dz$$

$$= \frac{p_\theta \gamma}{a_\theta \cos\mu_\theta} \int z^2 (L - x) dz$$

The evaluation of the above integral is precisely the same as Khan (1984) only the coefficient

$$\frac{p_\infty AF(S_1)}{a_\infty \cos\phi}$$

has been replaced here by

$$\frac{p_\theta \gamma}{a_\theta \cos\mu_\theta}$$

$$\therefore -C_{l_p} \, Contribution \; from \; Leeward \; surface =$$

$$R.H.S. of (1) * \frac{p_\theta \gamma}{a_\theta \cos\mu_\theta} / \frac{p_\infty AF(S_1)}{a_\infty \cos\phi}$$

$$= \frac{2p_\theta}{p_\infty}\frac{a_\infty}{a_\theta}\frac{M_\theta}{M_\infty}\frac{1}{\sqrt{M_\theta{}^2} - 1}\frac{1}{(\cot^2\varepsilon - \frac{4A_H Cot\varepsilon}{\pi})}[\frac{\cot^3\varepsilon}{12} +$$

$$\cot^2\varepsilon(\frac{A_F}{2\pi} - \frac{A_H}{\pi^3}(\pi^2 - 4)) +$$

$$\frac{1}{4}\cot\varepsilon(A_F{}^2 + A_H{}^2) - \frac{4}{9\pi}A_H{}^3 - \frac{16}{9}\frac{A_F A_H}{\pi^2} - \frac{16 A_F{}^2 A_H}{15\pi}] \quad (2)$$

$$\therefore -C_{l_p} = R.H.S\left[(1) + (2)\right]$$

When $A_F = 0$

$$\therefore -C_{l_p} = \frac{\sin\alpha_0 f(S_1)}{(Cot^2\varepsilon - \frac{4A_H}{\pi}\cot\varepsilon)}\begin{bmatrix} \frac{\cot^2\varepsilon}{12} + \cot^2\varepsilon(-\frac{A_H}{\pi^3}(\pi^2-4) + \\ \frac{1}{4}\cot\varepsilon(A_H{}^2) - \frac{4}{9\pi}A_H{}^3 \end{bmatrix} +$$

$$\frac{2p_\theta}{p_\infty}\frac{a_\infty}{a_\theta}\frac{M_\theta}{M_\infty}\frac{1}{\sqrt{M_\theta{}^2}-1}\frac{1}{(\cot^2\varepsilon - \frac{4A_H Cot\varepsilon}{\pi})}[\frac{\cot^3\varepsilon}{12} + \cot^2\varepsilon(-\frac{A_H}{\pi^3}(\pi^2-4)) + \frac{1}{4}\cot\varepsilon(A_H{}^2) - \frac{4}{9\pi}A_H{}^3]$$

III. RESULTS & DISCUSSIONS

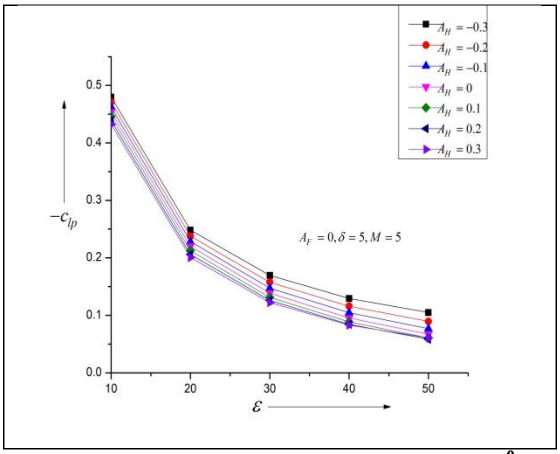

Fig. 1Rolling derivative vs. sweep angle for δ = 5^0, M = 5.

Figure1 depicts the rolling derivative vs. sweep angle for δ = 5^0 and Mach number five. It is seen that the rolling derivative decreases with a sweep angle for a half sine wave. Up to 20 degrees the decrement is linear then the nonlinearity creeps in. The reason for this trend is due to the decrease in the wing surface area for the nose section, and shift of larger area towards the trailing edge. The magnitude of the decrease in the damping derivative due to roll is large for lower sweep angle and for sweep angles more than twenty degrees there is a sudden drop in the magnitude.

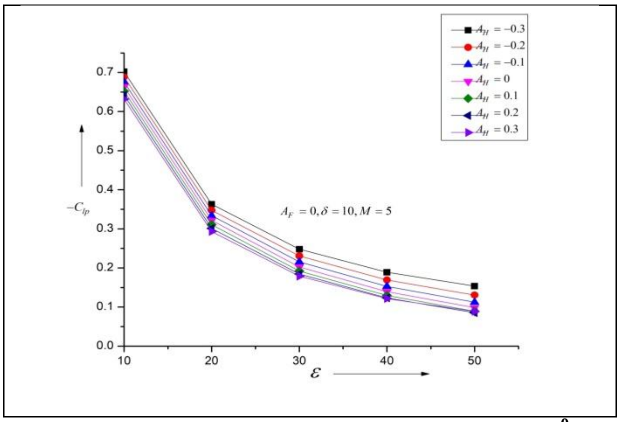

Fig. 2 Rolling derivative vs. sweep angle for δ = 10^0, M = 5.

Figure 2 depicts the rolling derivative vs. sweep angle for δ = 10^0 and Mach number five. It is seen that the rolling derivative decreases with a sweep angle for a half sine wave. The same trend is seen as was for flow deflection angle of five degrees, but the magnitude is increased from 0.45 to 0.7

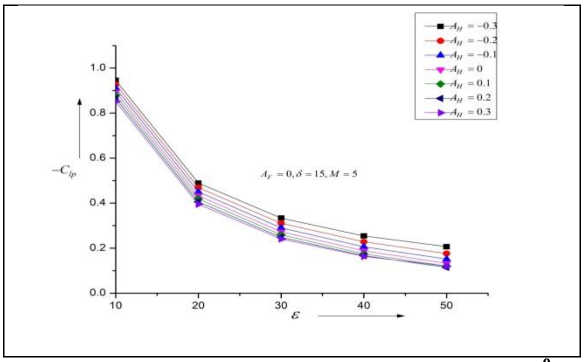

Fig. 3 Rolling derivative vs. sweep angle for δ = 15^0, M = 5.

Figure3 depicts the rolling derivative vs. sweep angle for δ = 15^0 and Mach number five. The graph is on similar lines as above with an increase in the magnitude due to the increase in the angle δ.

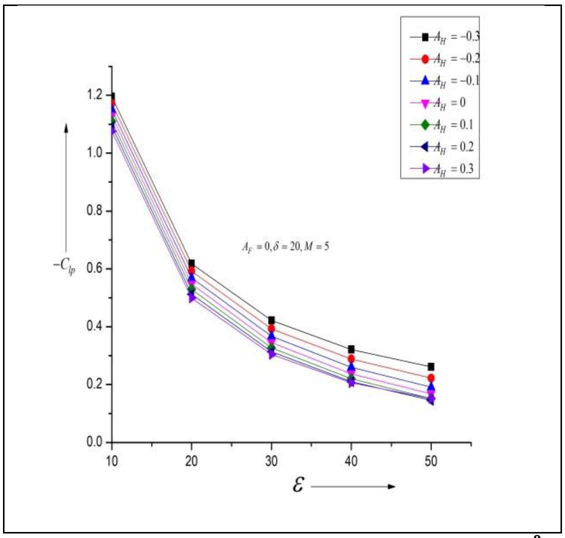

Fig. 4 Rolling derivative vs. sweep angle for δ = 20^0, M = 5.

Figure 4 depicts the rolling derivative vs. sweep angle for δ = 20^0, and Mach number five. It is observed that as semi vertex angle increases the magnitude of roll damping derivative increases and decreases with increase in sweep angle. The decrease in roll damping derivative with sweep angle is abrupt from 10 to 20 degrees and then seems to decrease steadily .The reason may be due to the variations in the surface area of the wing and also the flow deflection angle δ.

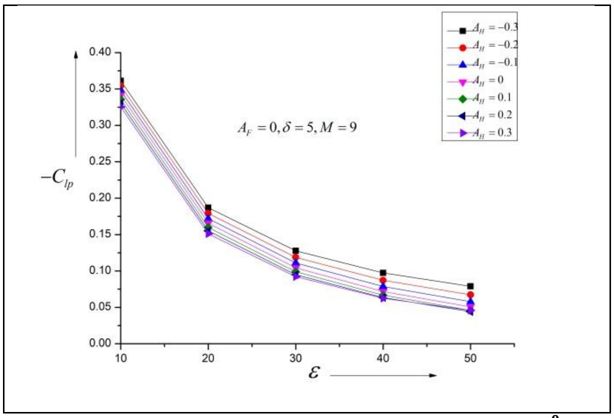

Fig. 5 Rolling derivative vs. sweep angle for δ = 5^0, M = 9.

Figure5 depicts the rolling derivative vs. sweep angle for δ = 5^{0}and Mach Number Nine. It is seen that the rolling derivative decreases with a sweep angle for a half sine wave. The magnitude of rolling derivative decreases with increase in Mach number otherwise the trend is on similar lines.

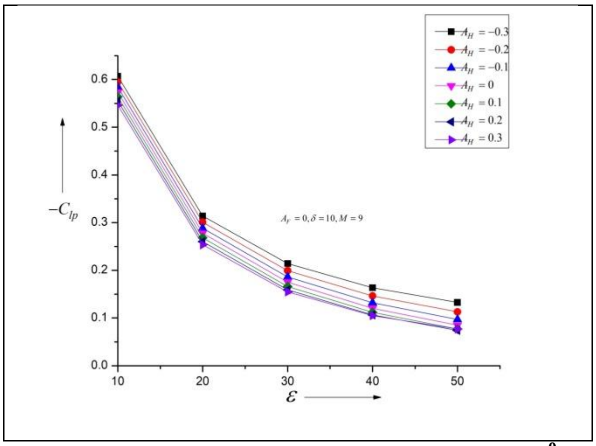

Fig. 6 Rolling derivative vs. sweep angle for δ = 10^0, M = 9.

Figure 6 depicts the rolling derivative vs. sweep angle for the flow deflection angle δ = 10^0, and Mach Number 9 as semi vertex angle increases the magnitude of rolling derivative increases due to increase in the surface area of the wing.

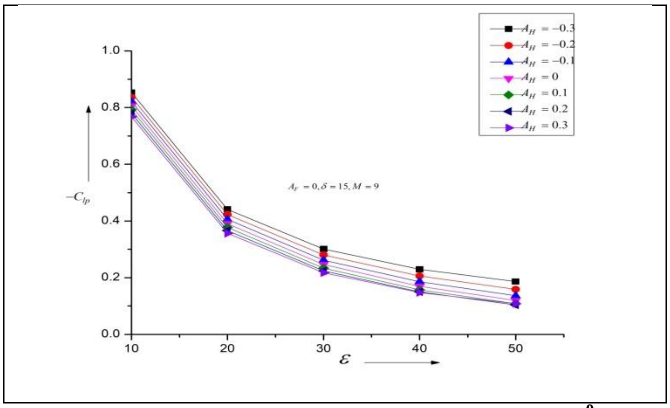

Fig. 7 Rolling derivative vs. sweep angle for δ = 15^0, M = 9.

Figure 7 depicts the rolling derivative vs. sweep angle for δ = 15^{0,} and Mach number 9 as semi vertex angle increases the magnitude of rolling derivative increases due to an increase in the surface area of the wing. For this case also the same trend continues.

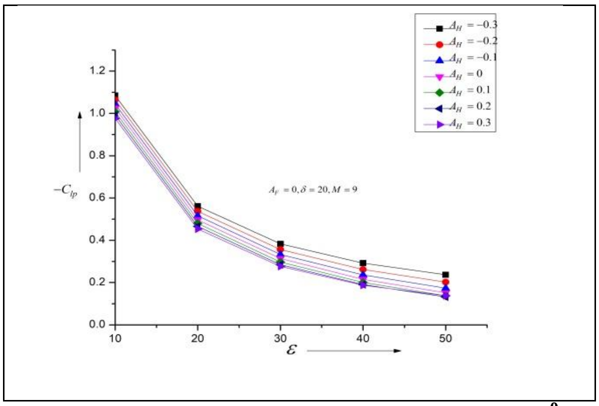

Fig. 8 Rolling derivative vs. sweep angle for δ = 20^0, M = 9.

Figure 8 depicts the rolling derivative vs. sweep angle for δ = 20^0 and Mach Number 9. There is a considerable increase in rolling derivative with an increase in the semi vertex angle. Further, there is an abrupt decrease in rolling with sweep angle between 10 to 20 degrees then a steady decrease in the magnitude is seen, and the reasons for this trend may be due to the decrease in the plan form area at the nose and then increase in the surface area towards the trailing edge.

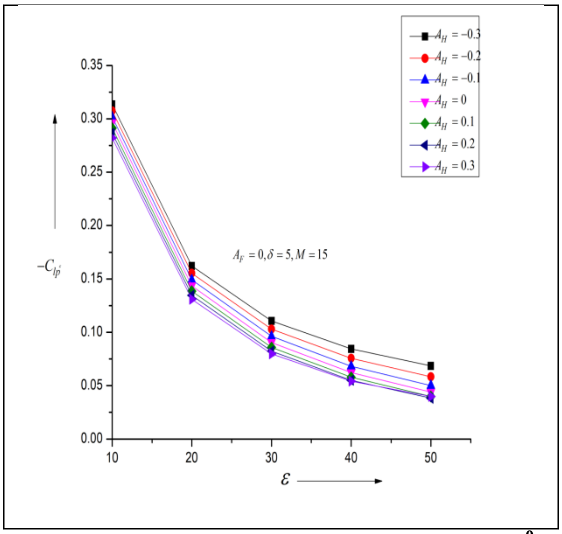

Fig. 9 Rolling derivative vs. sweep angle for δ = 5^0, M = 15.

Figure 9 depicts the rolling derivative vs. sweep angle for δ = 5^0 and Mach Number 15. The rolling moment derivative decreases with an increase in the Mach number from 9 to 15. Otherwise, the same trend as earlier is seen.

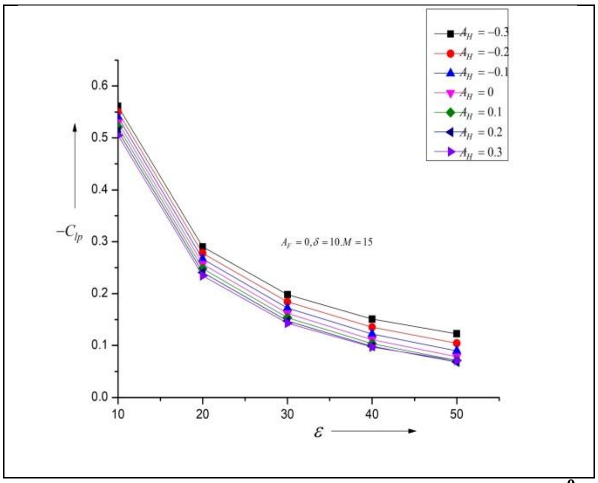

Fig. 10 Rolling derivative vs. sweep angle for δ = 10^0, M = 15.

Figure 10 depicts the rolling derivative vs. sweep angle for δ = 10^0 and Mach Number 15 as semi vertex angle increases the magnitude of rolling derivative increases due to an increase in plan form area.

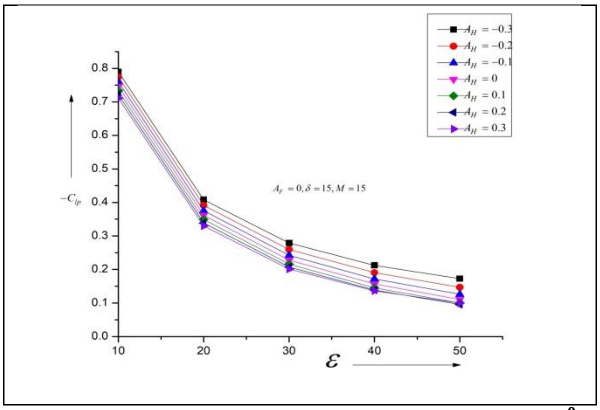

Fig. 10 Rolling derivative vs. sweep angle for δ = 15^0, M = 15.

Figure10 depicts the rolling derivative vs. sweep angle for δ = 15^{0}and Mach Number 15. The similar trend is seen.

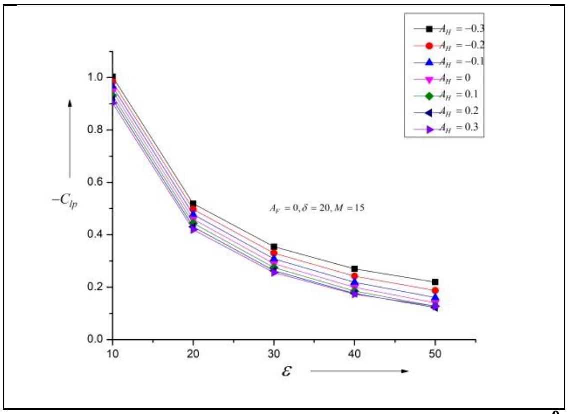

Fig. 11 Rolling derivative vs. sweep angle for δ = 20^0, M = 15.

Figure 11 depicts the rolling derivative vs. sweep angle for δ = 20^0 and Mach number 15. The similar trend with variation only in magnitude is seen.

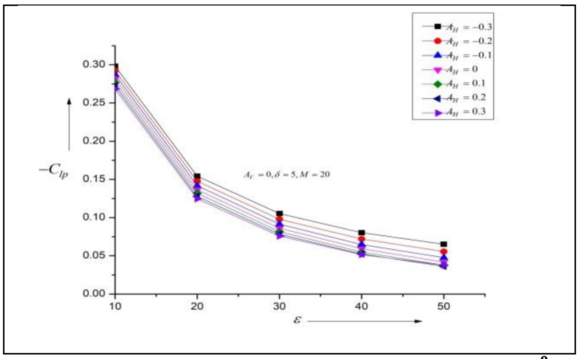

Fig. 12 Rolling derivative vs. sweep angle for δ = 5^0, M = 20.

Figure12 depicts the rolling derivative vs. sweep angle for δ = 5^0 and Mach number 20. There is no much difference in the magnitude of rolling derivative with respect to increase in Mach number from 15 to 20 degrees thus validating the Mach Number Independence principle.

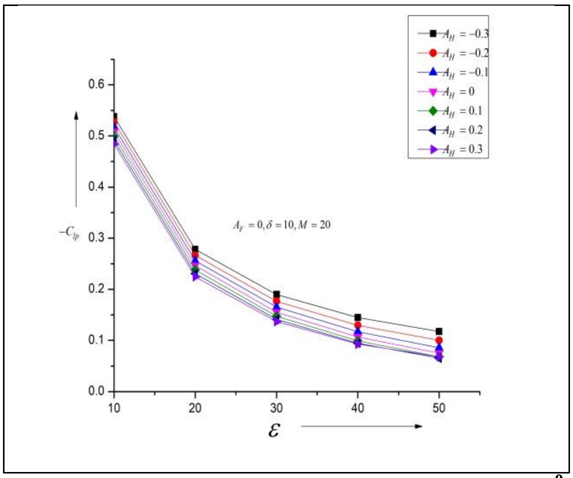

Fig. 13 Rolling derivative vs. sweep angle for δ = 10^0, M = 20.

Figure13 depicts the rolling derivative vs. sweep angle for δ = 10^0 and Mach Number 20. The same trend as above is seen

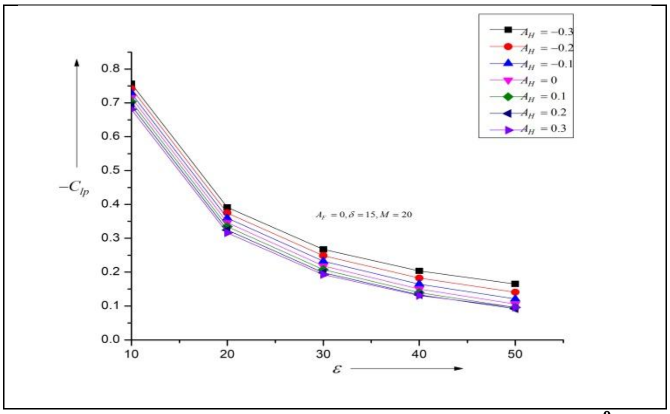

Fig. 14 Rolling derivative vs. sweep angle for δ = 15^0, M = 20.

Figure14 depicts the rolling derivative vs. sweep angle for $\delta = 15^0$ and Mach number 15. The similar trend with variation only in magnitude is seen.

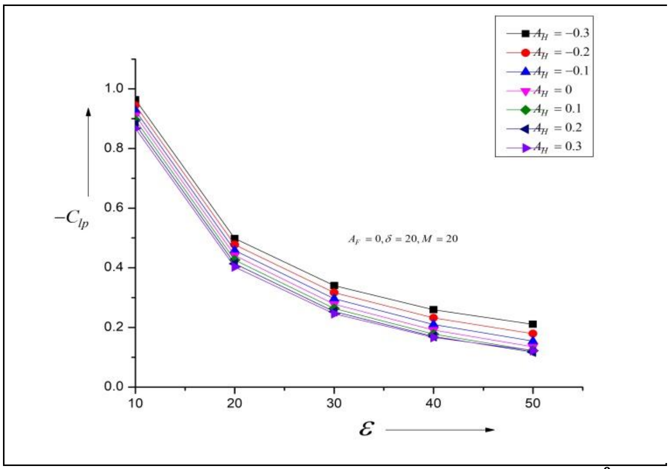

Fig. 15 Rolling derivative vs. sweep angle for $\delta = 20^0$, M = 20.

Figure15 depicts the rolling derivative vs. sweep angle for $\delta = 20^0$ and Mach Number 20 as semi vertex angle increases the magnitude of rolling derivative increases due to an increase in the surface area of the wing.

CONCLUSIONS:

Based on the above discussions we may draw the following concussions:

- The effect of sweep angle on a roll of damping derivative of a delta wing with half sine wave for an attached shock case in supersonic/hypersonic flow has been studied analytically.
- The results of the present study reveal that with the increase in the sweep angle; there is a continuous decrease in the roll damping derivative.
- It is also seen that the magnitude of the decrement for lower sweep angle is considerable as compared to the higher values of the sweep angles due to the drastic change in the surface area of the wing.
- Roll damping derivative progressively increases with the angle of attack; however, with the increase in the Mach number, it results in the decrement in the damping derivative and later conforms to the Mach number independence principle.
- Effects of wave reflection, leading-edge bluntness, and viscosity have not been taken into account.
- Results have been obtained for the supersonic/hypersonic flow of perfect gases over a wide range of angle of attack, the surface area of the wing for different Mach numbers.
- In the contemporary theory, Leeward surface is taken along with shock waves attached with the leading edge.

REFERENCES

1. Ghosh, K.,1977, "*A New Similitude for Airfoils in Hypersonic Flow,"* Proceedings of the 6th Canadian Congress of Applied Mechanics, Vancouver, Canada, May 29-June 3, pp. 685-686.
2. Light Hill, M.J., 1953, *"Oscillating Aerofoil at High Mach Numbers,"* Journal of Aeronautical Sciences, Vol. 20, June, pp. 804-812.
3. Miles, J. W., 1960, "*Unsteady flow at hypersonic speeds, Hypersonic flow*," Butterworth's Scientific Publications, London, pp. 185-197.
4. Ghosh K, Mistry, B.K, 1980, "*Large incidence hypersonic similitude and Oscillating non-planar wedges*," AIAA Journal, Vol., 18, No. 18, August, pp.1004 -1006.
5. Ghosh K., 1984, "*Hypersonic large deflection similitude for quasi-wedges and quasi cones*," The Aeronautical Journal, March,pp.70-76.
6. Ghosh, K. 1984, "*Hypersonic Large Deflection Similitude for oscillating delta wings,"* Aeronautical Journal, Oct., pp. 357-361.
7. Etkin, E, 1972, "*Dynamics of atmospheric flight,"* Wiley, New York.
8. Levin, D. A. 1984, "*Vortex lattice method for calculating longitudinal dynamic stability derivatives of oscillating delta wings*," AIAA Journal, January, pp. 6-12.
9. Ghosh K. and Vempathi, M. and Das, D. 1985, "*Hypersonic flow past non-slender wedges, cones and ogives in oscillation*," The Aeronautical Journal, August, pp. 247-256.
10. Ghosh K., 1986, "*Unified supersonic/hypersonic similitude for oscillating wedges and plane ogives*," AIAA Journal, Vol.24, No.7, July, pp.1205-1207.
11. Hui W. H. et al., 1982, *"Oscillating Supersonic/Hypersonic wings at High Incidence*," AIAA Journal, Vol. 20. Issue 3, March, pp. 299-304.
12. Ghosh K, 1983, "*Unified similitude for wedge and cone with attached shock*, " extended abstract published in Canadian Congress of Appl. Mech., University of Saskatchewan, Saskatoon, Canada, May 31- June 3, pp.533-544.
13. Hui, W.H., 1969, "*Stability of oscillating wedges and caret wings in hypersonic and supersonic flows,"* AIAA Jr., Vol. 7, No. 8, August, pp. 1524-1530.
14. Lui, D. D. and Hui, W. H., 1977, "*Oscillating delta wings with attached shock waves*," AIAA Jr., June, pp. 804-812.
15. Khan S.A. and Asha Crasta, 2010, "*Oscillating Supersonic delta wings with Curved Leading Edges,"* Advanced Studies in Contemporary Mathematics, Vol. 20, No.3, pp. 359-372.
16. Asha Crasta and Khan S. A., 2012, "*Oscillating Supersonic delta wing with Straight Leading Edges,"* International Journal of Computational Engineering Research, Vol. 2, Issue 5, September, pp.1226-1233.
17. Asha Crasta and S.A., Khan, 2013, "*Stability Derivatives in the Newtonian Limit*," International Journal of Advanced Research in Engineering and Technology, Vol.4, Issue 7, Nov-Dec., pp. 276-289.
18. Asha Crasta, S. A. Khan,2014, *"Effect of angle of incidence on roll damping derivative of a delta wing"* International Journal of Emerging Trends in Engineering and developments, Vol. 2, Issue 4, March 2014, pp. 343-356.
19. Asha Crasta, S. Pavitra and Khan S. A., 2016,*"Estimation of Surface Pressure Distribution On a Delta Wing with Curved Leading Edges in Hypersonic/Supersonic Flow,"* International Journal of Energy, Environment and Economics, Vol. 24, Issue No.1, pp. 67-73, e-ISSN: 2349-7688.
20. Aysha Shabana ,Renita S Monis, Asha Crasta and Khan S. A.,2018, *"Effect of semi vertex angle on stability derivatives for an oscillating cone for the constant value of specific heat ratio,"* International Journal of Engineering & Technology, Vol. 7, Issue No., pp. 386-390.