Effect of Sweep Angle and a Half Sine Wave on Roll Damping Derivative of a Delta Wing

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ABSTRACT--- This paper presents the effect of sweep angle on a roll of damping derivative of a delta wing with half sine wave for an attached shock case in supersonic/hypersonic flow has been studied analytically. The Ghosh Strip theory is replicated. By combining this with the similitude at high-speed flows lead to giving a piston theory. The initial conditions for the applicability of the theory are that the attached wave must be attached with the leading edge of the wing. The results of the present study reveals that with the increments in the sweep angles; it results in continuous decrease in the roll damping derivative, it is also seen that the magnitude of the decrement for lower sweep angle is considerable as compared to the higher values of the sweep angles due to the drastic change in the surface area of the wing. Roll damping derivative progressively increases with the angle of attack; however, with the increase in the inertia level of the flow, it results in the decrement in the damping derivative and later conforms to the Mach number independent principle. Effect of the leading edge bluntness and viscous effects are neglected. Results have been obtained for the supersonic/hypersonic flow of perfect gases over a wide range of angle of attack, planform area for different Mach numbers. In the present study, attention is on the effect of sweep angle of the wing on roll damping derivative at a different angle of attack and inertia level has been studied. In the contemporary theory, Leeward surface is taken along with shock waves attached with the leading edge.

Keywords — Delta wing, Hypersonic, Leeward surface, Sweep angle.

I. INTRODUCTION

Ghosh [1] has developed a 2D large deflection hypersonic similitude. The resulting piston theory is not restricted to slender shapes as in the cases of Lighthill’s [2] and Miles [3] piston theories. Ghosh’s piston theory [4] has been applied to oscillating plane ogives to predict \( C_{m\alpha} \). The similitude was extended to non-slender cones/quasi cones, and a new kind of piston motion, called conico-annular piston motion was given by Ghosh [5]. Oscillating delta wings at significant incidence was treated by Ghosh [6], Etkin [7] and Levin [8] have shown the separate effects of the pitch rate and incidence rate on the pitching moment. The plane piston theory of Ghosh [9] was applied with the inclusion of wave reflection effect to obtain a converged solution for \( C_{m\alpha} \) non-slender wedges/plane ogives with the rate of \( \alpha \) in hypersonic flow. Ghosh [10] has given a unified hypersonic similitude, and a consequent piston theory which is valid for wedges/quasi-wedges for any Mach number greater than 1 and \( E \leq 0.3 \) provided bow shock is attached. Hui et al. [11] have studied the problem of stability of an oscillating flat plate wing of arbitrary plan form placed at a specified mean angle of attack in supersonic/ hypersonic flow by applying strip theory. During the derivations of the theory, it is assumed that at each spanwise station the flow is independent of the location of the strips, and the flow remains two-dimensional. with the shock being attached. To assess the overall stability the moment derivatives due to the pitch rate as well as incident rate should be evaluated. In the present work, the unified similitude of Ghosh [12] along with the extended theory of Crasta & Khan [15]-[20] is combined with strip theory to obtain the unsteady moment derivative for a wing whose front edge is straight.

In this paper, the authors have attempted to study the Stiffness derivative with different pivot positions (h), which gives accurate results in comparison with the theory developed, by Liu as well as Crasta & Khan.

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independent of the location of the strips, and the flow remains two-dimensional, with the shock being attached. To assess the overall stability the moment derivatives due to the pitch rate as well as incident rate should be evaluated. In the present work, the unified similitude of Ghosh [12] along with the extended theory of Crasta & Khan [15]-[20] is combined with strip theory to obtain the unsteady moment derivative for a wing whose front edge is straight.

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II. ANALYSIS

Let the roll be \( \rho \) and the rolling moment is \( L \), defined according to the right-hand system of reference

\[
L = 2 \int \left( \int_0^c \left( z = f(x) \right) p z dz \right) dx
\]

The local piston Mach number normal to the wing surface is given by

\[
M_p = (M_{\infty}) \sin \alpha_0 \frac{z}{a_{\infty}} \bar{p}
\]

The non-dimensionalized was achieved by dividing with the dynamic pressure, area, a span of the wing, and a characteristic time factor

\[
\therefore -C_p = \frac{1}{U_{\infty} c^3 b (\cot \varepsilon - 4A_H)}
\]

\[
\therefore -C_{p_p} = \frac{\sin \alpha_0 f(S_1)}{(\cot \varepsilon - 4A_H \cot \varepsilon) / \pi}
\]

\[
S_1 = M_{\infty} \sin \alpha_0
\]

\[
f(S_1) = \frac{(\gamma + 1)}{2S_1} \left[ 2S_1 + \frac{(B + 2S_1^2)}{(B + S_1^2)^{1/2}} \right]
\]

On the Leeward surface, the piston Mach number

\[
M_p = \frac{Z_p}{a_\theta \cos \mu_\theta}
\]

For the rotation in the right-handed direction the acoustic pressure ratio

\[
P_p / P_\theta = 1 + \gamma M_p
\]

The rolling Moment

\[
\bar{L} = -\int P_p (L - x) zdz
\]

\[
= \int \frac{P_\theta \gamma z}{a_\theta \cos \mu_\theta} (L - x) zdz
\]

\[
= \frac{P_\theta \gamma}{a_\theta \cos \mu_\theta} \int z^2 (L - x) dz
\]

The evaluation of the above integral is precisely the same as Khan (1984) only the coefficient

\[
p_\infty AF(S_1) / a_\infty \cos \phi
\]

has been replaced here by

\[
\frac{P_\theta \gamma}{a_\theta \cos \mu_\theta}
\]

\[
\therefore -C_{p_p} = \text{R.H.S of (1) } \left( \frac{P_\theta \gamma}{a_\theta \cos \mu_\theta} \right)
\]

\[
= \frac{2P_\theta}{a_\infty} \frac{a_\theta}{M_{\infty}} M_\rho \frac{1}{\sqrt{M_{\rho} - 1}} \frac{1}{(\cot \varepsilon - 4A_H \cot \varepsilon) / \pi}
\]

\[
\cot^2 \varepsilon \left( \frac{A_p}{2\pi} - \frac{A_H}{\pi^2} (\varepsilon^2 - 4) \right) +
\]

\[
\frac{4}{9} \cot \varepsilon(A_p^2 + A_H^2) - \frac{4}{9} A_H^3 - 16A_p A_H^2 - \frac{16A_p^2 A_H}{15\pi}
\]

\[
\therefore -C_{p_p} = \text{R.H.S of (1) + (2)}
\]

When \( A_\iota \to 0 \)
III. RESULTS & DISCUSSIONS

Figure 1 depicts the rolling derivative vs. sweep angle for $\delta = 5^\circ$ and Mach number five. It is seen that the rolling derivative decreases with a sweep angle for a half sine wave. Up to 20 degrees the decrement is linear then the nonlinearity creeps in. The reason for this trend is due to the decrease in the wing surface area for the nose section, and shift of larger area towards the trailing edge. The magnitude of the decrease in the damping derivative due to roll is large for lower sweep angle and for sweep angles more than twenty degrees there is a sudden drop in the magnitude.

Figure 2 depicts the rolling derivative vs. sweep angle for $\delta = 10^\circ$ and Mach number five. The same trend is seen as above with an increase in the magnitude due to the increase in the angle $\delta$.

Figure 3 depicts the rolling derivative vs. sweep angle for $\delta = 15^\circ$ and Mach number five. The graph is on similar lines as above with an increase in the magnitude due to the increase in the angle $\delta$.

Figure 4 depicts the rolling derivative vs. sweep angle for $\delta = 20^\circ$ and Mach number five. It is observed that as semi vertex angle increases the magnitude of roll damping derivative increases and decreases with increase in sweep angle. The decrease in roll damping derivative with sweep angle is abrupt from 10 to 20 degrees and then seems to decrease steadily. The reason may be due to the variations in the surface area of the wing and also the flow deflection angle $\delta$.
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Figure 5 depicts the rolling derivative vs. sweep angle for $\delta = 5^\circ$, $M = 9$.

Figure 6 depicts the rolling derivative vs. sweep angle for $\delta = 10^\circ$, $M = 9$.

Figure 7 depicts the rolling derivative vs. sweep angle for $\delta = 15^\circ$, $M = 9$.

Figure 8 depicts the rolling derivative vs. sweep angle for $\delta = 20^\circ$, $M = 9$.

Figure 9 depicts the rolling derivative vs. sweep angle for $\delta = 5^\circ$, $M = 15$.

Figure 7 depicts the rolling derivative vs. sweep angle for $\delta = 15^\circ$, and Mach number 9 as semi vertex angle increases the magnitude of rolling derivative increases due to an increase in the surface area of the wing. For this case also the same trend continues.
Figure 10 depicts the rolling derivative vs. sweep angle for $\delta = 10^0$, $M = 15$. The similar trend is seen. Figure 11 depicts the rolling derivative vs. sweep angle for $\delta = 20^0$, $M = 15$. The same trend as above is seen. Figure 12 depicts the rolling derivative vs. sweep angle for $\delta = 5^0$, $M = 20$. Figure 13 depicts the rolling derivative vs. sweep angle for $\delta = 10^0$, $M = 20$. Figure 14 depicts the rolling derivative vs. sweep angle for $\delta = 15^0$, $M = 20$. There is no much difference in the magnitude of rolling derivative with respect to increase in Mach number from 15 to 20 degrees thus validating the Mach Number Independence principle.
Figure 14 depicts the rolling derivative vs. sweep angle for $\delta = 15^\circ$ and Mach number 15. The similar trend with variation only in magnitude is seen.

![Figure 15 Rolling derivative vs. sweep angle for $\delta = 20^\circ$, M = 20.](image)

Figure 15 depicts the rolling derivative vs. sweep angle for $\delta = 20^\circ$ and Mach Number 20 as semi vertex angle increases the magnitude of rolling derivative increases due to an increase in the surface area of the wing.

**CONCLUSIONS:**

Based on the above discussions, we may draw the following conclusions:

- The effect of sweep angle on a roll of damping derivative of a delta wing with half sine wave for an attached shock case in supersonic/hypersonic flow has been studied analytically.
- The results of the present study reveal that with the increase in the sweep angle, there is a continuous decrease in the roll damping derivative.
- It is also seen that the magnitude of the decrement for lower sweep angle is considerable as compared to the higher values of the sweep angles due to the drastic change in the surface area of the wing.
- Roll damping derivative progressively increases with the angle of attack; however, with the increase in the Mach number, it results in the decrement in the damping derivative and later conforms to the Mach number independence principle.
- Effects of wave reflection, leading-edge bluntness, and viscosity have not been taken into account.
- Results have been obtained for the supersonic/hypersonic flow of perfect gases over a wide range of angle of attack, the surface area of the wing for different Mach numbers.
- In the contemporary theory, Leeeward surface is taken along with shock waves attached with the leading edge.

**REFERENCES**