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Circular Index for the Book, Fan and Tridegreed Graphs

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Abstract— The universe of diagram thought is well off in its hypothetical improvement comparatively to in finding programming zones. one of the all around examined thoughts in diagram statute is the hole between vertices in a charts, explicitly while a diagram is used in demonstrating genuine overall inconveniences. in this paper a round separation among any vertices of a graphx is portrayed. so also for charts like digital book, Fan and Tridegreed diagrams the round record has been watched.

Keywords: Wiener Polynomial, Wiener Index, Detour Polynomial, Detour Index, Wiener – Detour Matrix, Circular Polynomial and Circular Index.

1. INTRODUCTION

The spherical separation d(u, v) between vertices u, v of a diagram is characterized as d0(u, v) = D(u, v) + d(u, v), wherein d(u, v) is the separation a few of the vertices u and v and D(u, v) is the skip separation between the vertices u and v. The roundabout separation assumes a full-size assignment in calculated management.

The number one thing of a transporter is to restriction the fuel and automobile fees to circulate the merchandise. offer us a threat to just accept that a distribution center is located at a city X and Y be a aim city. A merchandise bearer takes a long day out to suitable the products from the city X and Y covering each one of the towns amongst X and Y. On the arrival trip the most brief route might be selected to touch base to the distribution middle X.

the goods transporter day experience can be spoken to with the aid of a chart G, wherein the vertices examine to the cities and vertices are adjoining in G if and simply if there can be a proper away avenue associating the referring to towns which does no longer go through a few extraordinary city. on this paper roundabout listing of a chart is characterized utilizing Wiener Index and Detour Index. in addition the round list of unique charts have been found using roundabout Polyonomial.

2. CIRCULAR INDEX

Definition 2.1

Let G(p, q) be a simple graph with p vertices and q edges.

Let d(u, v) denote the shortest distance between two vertices u, v ∈ V(G). The Wiener polynomial of a graph G is denoted by WP(G, x) and is defined as WP(G : x) = $\sum_{u,v\in V(G)} x^{d(u,v)}$. The Wiener Index of G is

WI = $\sum WP'(G:1)$, where ′ denotes the derivative of WP(G:x) with respect to x.

Definition 2.2

Let D(u, v) denote the longest distance between two vertices u, v∈V(G). The detour distance polynomial of a graph G is denoted by DP(G:y) = $\sum_{u,v\in V(G)} y^{D(u,v)}$. The Detour Index is given by

DI = $\sum WDP'(G:1)$, where ′ denotes the derivative of DP(G:y) with respect to y.

Definition 2.3

A Wiener Detour Matrix of G is a rectangular matrix of order ' p' with entries zero along the primary diagonal, above the main diagonal the entries are D(u, v) and underneath the principle diagonal the entries are d(u, v), where i ≠ j.

Definition 2.4

Let $d^o(u, v)$ denote the circular distance between two vertices u, v ∈ V(G). The circular polynomial of a graph G is denoted by CP(G:z) = $\sum_{u,v\in V(G)} z^{d^0(u,v)}$. The circular index of G is CI = CP′(G:1) where′denotes the derivative of CP(G:z) with respect to 'z'.

3. CIRCULAR INDEX OF BOOK, FAN AND TRIDEGREED GRAPHS & RESULTS

Theorem 3.1

The circular index of Book graph is CI(G) = $3n^2 + 5n + 3$, n ≥ 3.

Proof :

Let G = $K_{1,1,n}$ be a Book graph. The vertex set of G is V(G) = n + 2

and the edge set of G is E(G) = {v_1v_2, v_1v_i, v_2v_i ; i = 3, 4, . . . , n}.

Consider K1,1,2 (fig.1) the corresponding Wiener – Detour Matrix (fig.2) are as follows :

WDM(K1,1,2)

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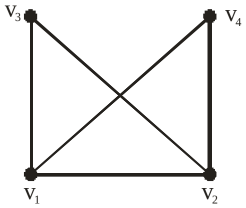

Fig.1

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array}\begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 2 & 2 & 2 \\ 1 & 0 & 2 & 2 \\ 1 & 1 & 0 & 3 \\ 1 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

Fig.2

From the above WDM(K1,1,2), the Wiener Polynomial, Detour polynomial and Circular polynomial are WP = $x^2 + 5x$; DP = $y^3 + 5x^2$;CP = $z^5 + 5z^3$ and the corresponding indices are 7, 13 and 20 respectively.

Now consider K1,1,3, (fig.3) and the corresponding Wiener – Detour Matrix of K1,1,3 (fig.4) are as follows :

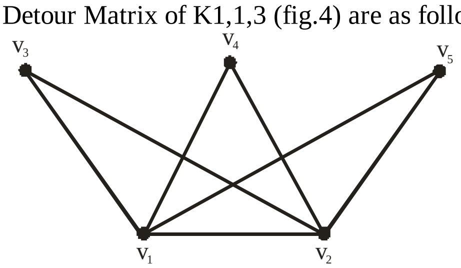

Fig.3

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}\begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 2 & 3 & 3 & 3 \\ 1 & 0 & 3 & 3 & 3 \\ 1 & 1 & 0 & 4 & 4 \\ 1 & 1 & 2 & 0 & 4 \\ 1 & 1 & 2 & 2 & 0 \end{bmatrix} \end{array}$$

Fig.4

From the above WDM(K1,1,3), the Wiener Polynomial, Detour Polynomial and Circular Polynomial are

WP = $3x^2 + 7x$; DP = $3y^4 + 6y^3 + y^2$; CP = $z^3 + 6z^4 + 3z^6$, and the corresponding indices are 13, 32 and 45 respectively.

If 'n' increased by one, the graph $K_{1,1,4}$ is obtained and the coefficient of x^2 in the corresponding Wiener Polynomial is also increased by three and the coefficient of x is increased by 2. In the same way coefficient of x^3, coefficient of x^4 in Detour Polynomial respectively increased by 2 and 3. Similarly the coefficient of circular polynomial are also increased.

- in general for any ' n' , n ≥ 3

WP = $\frac{n(n-1)}{2}x^2 + (2n+1)x$; DP = $y^2 + 2ny^3 + \frac{n(n-1)}{2}y^4$ and

CP = $z^3 + 2nz^4 + \frac{n(n-1)}{2}z^6$ and the corresponding indices are

WI = $n^2 + n + 1$; DI = $2(n^2 + 2n + 1)$ and CI = $3n^2 + 5n + 3$ respectively.

Theorem 3.2

The Circular Index of Tridegreed graph is CI(G) = $\frac{1}{2}\left[n^3 - n^2 + 2\right]$, ∀ n > 7.

Proof :

Let G = J(n) = $C_{n-3} + P_3$, n > 7 be a tridegreed graph. The vertex set of G is,

V(G) = {$v_1, v_2, v_3, \ldots, v_n$} and the edge set of G is

E(G) = {$v_1v_2, v_2v_3, v_iv_j, v_jv_{j+1}, v_4v_n$; ∀ i = 1, 2, 3 & j = 4, 5, . . . n}

Now consider J(8) = C5P3 (fig.5) and the corresponding Wiener – Detour Matrix (fig.6) are as follows.

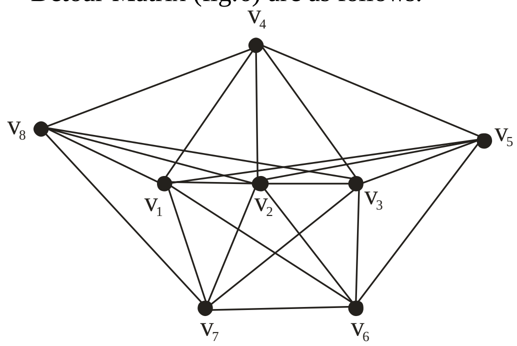

Fig.5

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array}\begin{array}{c} \begin{array}{cccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\ \begin{bmatrix} 0 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \\ 1 & 0 & 7 & 7 & 7 & 7 & 7 & 7 \\ 2 & 1 & 0 & 7 & 7 & 7 & 7 & 7 \\ 1 & 1 & 1 & 0 & 7 & 7 & 7 & 7 \\ 1 & 1 & 1 & 1 & 0 & 7 & 7 & 7 \\ 1 & 1 & 1 & 1 & 1 & 0 & 7 & 7 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 7 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Fig.6

From the above WDM(J(8)), the Wiener Polynomial, Detour Polynomial and the Circular Polynomial are as follows

WP = $27x + x^2$; DP = $28y^7$; CP = $27z^8 + z^9$ and the corresponding indices are 29, 196 and 225 respectively. If n is increased by 1, the graph J(9) is obtained and there is no change in the coefficient of x2 in Wiener Polynomial and the coefficient of x is increased by 8 and the Detour Polynomial is 36y8. Similarly the coefficients are increased in circular polynomial.

WP = $35x + x^2$

DP = $36y^8$

CP = $35z^9 + z^{10}$ and the corresponding indices are 37, 288 and 325 respectively.

In general, for every n > 7. The Wiener Polynomial, the Detour Polynomial and the Circular Polynomial are as follows :

WP = $\frac{n(n-1)-2}{2}x + x^2$; DP =

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$\frac{n(n-1)}{2}y^{n-1}$ and

CP $= \frac{n(n-1)-2}{2}z^n + z^{n+1}$; $n > 7$ respectively.

Also the Wiener Index, the Detour Index and the Circular Index are

$WI[J(n)] = \frac{1}{2}\left[n^2 - n + 2\right]$; $DI[J(n)] = \frac{1}{2}\left[n^3 - 2n^2 + n\right]$ and

$CI[J(n)] = \frac{1}{2}\left[n^3 - 2n^2 + n\right]$, $n \geq 6$ respectively.

C-program to calculate the circular index for Tridegreed graph, $\forall n > 7$.

```
#include<iostream.h>
#include<conio.h>
void main(void)
  {
    int i,j,n,a[20][20],wd=0,wi=0;
    cout<<"\nEnter the No. of Vertices:\t";
    cin>>n;
    for(i=1;i<n;i++)
  {
    for(j=1;j<=i;j++)
      {
        wd=wd+n-1;
        wi=wi+1;
      }
  }
    cout<<"\nWiener Index is      =\t"<<wi+1;
    cout<<"\nWiener Detour Index is=\t"<<wd;
    cout<<"\nWiener Circular Index =\t"<<wi+1+wd<<"\n";
    for(i=1;i<=n;i++)
 {
    for(j=1;j<=n;j++)
      {
        if(i==j)
      {
        a[i][j]=0;
      }
    else
      {
        if(i<j)
          {
          a[i][j]=n-1;
          }
        if(i>j)
          {
            a[i][j]=1;
            if(j==1 && i==3)
            a[i][j]=2;
          }
        }
      }
    }
  for(i=1;i<=n;i++)
    {
     for(j=1;j<=n;j++)
    {
     cout<<a[i][j]<<"     ";
    }
     cout<<"\n";
    }
  getch();
 }
```

Theorem 3.3

The circular index of Fan Graph (F_n) is,

$$CI = -\frac{1}{720}\left[5n^6 - 213n^5 + 3725n^4 - 34515n^3 + 173390n^2 - 459192n + 498240\right]$$

, $n \geq 4$.

Proof :

Let G = Fn be the Fan Graph for n ≥4. Also Let $V(G) = \{v_1, v_2, v_3, \ldots, v_n\}$and

$E(G) = \{v_1v_i, v_iv_j ; \forall i < j, i = 2, 3, \ldots n\}$be the vertex set and edge set respectively.

Now consider n = 4, the Fan Graph F4 (fig.7) and the corresponding Wiener – Detour Matrix fig.(8) are as follows :

WDM

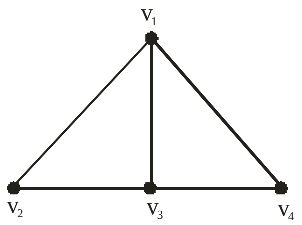

Fig.7

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 3 & 2 & 3 \\ 1 & 0 & 3 & 3 \\ 1 & 1 & 0 & 3 \\ 1 & 2 & 1 & 0 \end{bmatrix} \end{array}$$

Fig.8

From the above WDM, the Wiener Polynomial, Detour Polynomial and Circular Polynomial are as follows.

$WP = 5x + x^2$; $DP = y^2 + 5y^3$ and $CP = z^3 + 4z^4 + z^5$ respectively

Similarly we increased the dimension of n by '1', then there is no change in their Wiener distance, but in the Detour distance, is increased by one to every pair of vertices. Therefore the corresponding Wiener Polynomial, Detour

Polynomial and Circular Polynomial of F5 are as follows.

$WP = 7x + 3x^2$; $DP = 2y^3 + 8y^4$ and $CP = 2z^4 + 5z^5 + 3z^6$ respectively.

Similarly, for all n≥4, the Wiener Polynomial is $WP = (2n - 1)x + \frac{(n-1)(n-2)}{2}x^2$ and the Wiener Index is $WI = n^2 - n + 1$.

But in the Detour polynomial we analysed. For some cases,For n is even, then

Case (i) If n = 10, 16, 22, . . .

$$DP = y^{\frac{n}{2}} + \sum_{K=1}^{\frac{n-4}{6}} 2y^{\frac{n}{2}+K} + \sum_{K=1}^{\frac{n-1}{3}} [5+(K-1)9]\, y^{\left(\frac{2n-2}{3}+K\right)},$$

where n = 4, $DP = y^2 + 5y^3$.

Case (ii) If n = 12, 18, 24, ...

$$DP = y^{\frac{n}{2}} + \sum_{K=1}^{\frac{n-2}{6}} 2y^{\frac{n}{2}+K} + \sum_{K=1}^{\frac{n-6}{3}} [8+(K-1)9]\, y^{\left(\frac{2n-1}{3}\right)+K}$$

where n = 6, DP = y3 + 3y5+11y5.

Case (iii) If n = 8, 14, 20, ...

DP =

For n is odd, then

Case (i) If n = 5, 11, 17, . . .

$$DP = \sum_{K=1}^{\frac{n+1}{6}} 2y^{\left(\frac{n-1}{2}+K\right)} + \sum_{K=1}^{\frac{n-2}{3}} [8+(K-1)9]\, y^{\left(\frac{5n-13}{6}+K\right)}$$

Case (ii) If n = 7, 13, 19, . . .

$$DP = \sum_{K=1}^{\frac{n-1}{6}} 2y^{\left(\frac{n-1}{2}+K\right)} + \sum_{K=1}^{\frac{n-1}{3}} [5+(K-1)9]\, y^{\left(\frac{2n-2}{3}+K\right)}$$

Case (iii) If n = 9, 15, 21, . . .

$$DP = \sum_{K=1}^{\frac{n-3}{6}} 2y^{\left(\frac{n-1}{2}+K\right)} + 3y^{\frac{2n}{3}} + \sum_{K=1}^{\frac{n-3}{3}} [11+(K-1)9]\, y^{\left(\frac{2n}{3}+K\right)}$$

From the above Detour polynomial and Wiener polynomial the circular polynomial is also found. Therefore, the Circular index of the Fan Graph is,

$$CI = -\frac{1}{720}\left[5n^6 - 213n^5 + 3725n^4 - 34515n^3 + 173390n^2 - 459192n + 498240\right],$$

$\forall\, n \ge 4$

CONCLUSION

on this way, we've determined the roundabout listing for sure diagrams like e-book, fan and tridegreed charts. This type of roundabout document assumes a excellent assignment within the improvement and capacity of devices starting with one spot then onto the subsequent, which restrict the time and price of spherical excursion of suitable's bearer

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