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Application of Stochastic Processes to Almost Ideal Demand System (AIDS) Models

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I. INTRODUCTION

The world's worst recorded food disaster happened in 1943, in British-ruled India, and is known as the Bengal Famine. It is estimated that four million people died of hunger that year alone in eastern India (Ganguly, 2002). The reason for the severity of the famine was the hoarding of grains by Indian food traders who would sell goods at high prices in order to obtain large profits. When the British left India in 1947, the number one concern was to ensure that such a famine never happened again. It led to legislative measures that prohibit any person from monopolizing the food markets, and was a major contributor to the green revolution.

Although the green revolution did not start until some time later (1965-1978), many ideas and concepts were being generated in this post-famine period to alleviate the food problem that India was facing. In the early stages of planning, the general idea had been to expand cropland and simply plant more acres in order to produce more. The efforts to expand the farmland base did not ease the problem of hunger entirely; people were still dying from starvation (Ganguly, 2002). The Government of India (GOI) turned its attention to increasing production levels in order to become self-sufficient in food production.

One of the primary features that the green revolution brought about was cropping intensity, but the problem with this approach was the lack of water to produce multiple crops. The GOI solved the problem by implementing a plan that would allow for dams to be built that would capture the runoff water from the rains (Ganguly 2002). By building these dams, the GOI had a secondary water source that would allow for two or three growing seasons in the same year.

The second benefit of the green revolution was the introduction of high yielding varieties (HYV). In 1960, the total area under the HYV was only 1.9 million hectares; it increased to almost 15.4 million hectares by 1970, 43.1 million hectares by 1980, and had grown to 63.0 million hectares by 1990 (Worden and Heitzman, 1999). With the increase in area under HYV, the amount of food that was being produced multiplied greatly, with the greatest increases coming from wheat and rice production. By 1980, almost 75% of the total cropped area under wheat was sown with HYV, and almost 45% of total rice area was under HYV (Worden and Heitzman, 1999).

The adoption of new crop production technologies resulted in a significant increase in Indian food production, which has grown at an annual rate of 2.5 percent over the last four decades (Mohanty and Peterson, 2001). Currently, India is one of the leading producers of wheat, rice, and coarse grains and is largely self-sufficient in grain production with occasional imports and exports in years of shortages or surpluses respectively. Today, India ranks second in the world both in wheat and rice production with 68 and 89 million metric tonns (mmt) produced annually (USDA, 2001). Currently, the agricultural sector accounts for about 25 percent of the gross domestic product, and engages almost 67 percent of the labor force (CIA, 2002).

II. THE CONCEPT OF DEMAND FUNCTION

Measuring consumption is quite a difficult task. Consumption includes several components: all the individual expenditure on goods and services, a value for consumption that does not go through the market (home production, transfer in kind, etc.) and a value for durable goods possessed. For the latter some sort of consumption flow needs to be imputed. There is an important distinction to make between consumption and expenditure the former includes the value of service flows from durable items and assets (such as home, vehicles, washing machine, computers, etc.) whereas the latter includes current expenses on the purchase of these items. Theoretically, consumption is preferable to expenditure as it better reflects material resources, although in practice estimating the value of service flows involves crucial assumptions(such as definition of durable good, depreciation rate of different items, etc.). The methods adopted to construct consumption measures significantly vary among countries and over time. Most of the choices involved with the measurement of consumption are usually driven by data availability or by comparability over time within a country. There exist, however, good practice techniques and guidelines which one could look at when trying to construct an accurate measure of consumption. Total household consumption expenditure should comprise: food consumption, non food consumption, education expenditure and housing expenditure. In revising the method adopted by the Bolivian National Institute of Statistics (INE), it emerged, however, that computation of the total consumption expenditure was not clear and consistent.

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However, from 2005 onwards INE includes in the consumption aggregate only the current expenditure thus excluding the value of service flows of durables. Moreover, in computing current expenditure, durable goods and expenditures made in house repair and construction that are above a certain threshold are excluded because they are regarded as investment rather than consumption. As a result, a lack of consistency in the definition and construction of consumption aggregates provided by the INE is apparent and that simply prevents one from comparing those figures over time. Aware of that, the only feasible option for conducting sound research using consumption data is devoting intense effort to create original, consistent, accurate consumption estimates. This task, although very time consuming, represents a notable and original contribution of the present study to the literature. To estimate consumption figures the following components have been aggregated: Food consumption inside the household(food purchases, self-produced food, food from other sources-such as gifts, transfers in kind)

Food consumed outside the household (breakfast, drinks, lunch/dinner, snacks,..)

Non-food consumption (aggregate of about 40 categories related to current housing costs, domestic fuel and power, tobacco products, clothing and footwear, medical care and health expenses, transport, recreation, personal care, miscellaneous goods and services.

Education expenditure (tuition fees, transport, books and copies, uniform, etc.)

Housing expenditure (actual rent or rental equivalence value, expenses- gas, water, electricity, telephone-house repair-decoration)

The computation has been done at the (per capita) household level. When the expenditure was reported at the individual level, the household aggregate has been computed and the per capita mean has then been obtained dividing the household figure by the household size. As respondents are allowed for some modules to answer in Rupees, all the values in Rupees have been converted into real inside the house such as beds, TV, microwave oven, etc. Information on the method used to impute such values.

In the case of consumption, the willingness to buy a commodity, that is the demand for that, depends on the factors such as

1. The income of the consumer
2. The price of that commodity
3. Prices of other goods and services on which the consumer spends his income.
4. Tastes and preferences of the consumer, size of the family, social customs, expectations and advertisements. etc.

In the case of a farm or firm, the input demand for commodity depends on the factors like:

1. The total outlay or expenditure of the firm
2. The price of that commodity
3. Prices of other substitutes and complementary inputs
4. The nature of technology, etc.

Since the study is on consumer demand, we shall restrict to it only.

A consumer demand function for a commodity specifies the relationship between quantity of the commodity that the consumer is willing to buy and the demand factors. In mathematical form, the demand function for a commodity is express as.

$$Q_X = D_X[P_X, P_S, P_C, Y, T]$$

where Q_x is the quantity of the commodity X demanded, P_x is the price of the commodity, P_s denotes the price of other commodity which can be substituted for X, P_c is the price of the commodity which is the complement of commodity X,Y is the income of the consumer and T represents other demand factors such as tastes, preferences, social customs, etc, D_x indicates the functional form of the relationship. There may be more than one substitute and/or complementary goods for commodity X, In this situation the specification of the demand function for commodity X can be expanded by including their prices. The demand function may be linear or non-linear. It is a common observation that for most of the commodities, the willingness to buy decreases as price of the commodity increases. The law of demand states that other things being equal, the quantity demanded of the commodity varies inversely with its price. However in the case of consumer demand, an increase (decrease) in income of the consumer increases (decreases) the demand for the commodity. These commodities are called normal goods. For certain commodities, after a certain level of income of the consumer, the quantity demanded starts decreasing with increase in income. These are called inferior goods. In the case of food items, a rise in income causes an increase in quantity demanded in the initial stage but after a certain level of the consumer's income, the quantity demanded become invariable with respect to the income. There is a simple generalization of the relationship between consumer goods and income for a consumer, which is known as **Engel's law.** According to this law, as income increases the portion of income spent on food declines and the portion spent on comforts and luxuries increases.

III. ELASTICITY

Elasticity in demand theory is used to measure the responsiveness of the quantity demanded of a commodity to the change in price or income. It is defined as the unit percentage change in a demand factor. In symbols

$$\text{Elasticity} = \frac{(\Delta Q / Q)}{(\Delta X / X)}$$

Where Q is the quantity demanded of a commodity when X is the price and $\frac{dQ}{dX}.\frac{X}{Q} = \frac{\partial(\log Q)}{\partial(\log X)}$

is called the point elasticity since it gives the elasticity of demand at a point on the demand curve.

IV. CONSUMER BEHAVIOUR

A consumer at a time consumes one or more commodities. He gets satisfaction or derives utility by consuming the goods. The level of satisfaction or utility derived by him depends on quantities of the goods consumed. This is a basic consumer theory. According to this, we say that utility derived from consumption of a commodity depends exclusively on its quantity, other things being constant.

In symbols.

$$U_i = F_i(q_i), \quad i = 1,2........n$$

Where U_i is the level of utility.

q_i is the quantity of i^{th} commodity

and F_i denotes the shape of the relationship.

If there are n commodities,

$U= F(q_1 , q_2.......q_n)$

This is called utility function.

The first partial derivatives of U, otherwise known as marginal utilities are interpreted as the change in total utility by consuming one more extra unit of a commodity keeping the levels of other commodity constant. There is a law of diminishing marginal utility, according to this "As quantity consumed of a commodity increases, the marginal utility of that commodity tends to decline"

$$\text{i.e,} \quad \frac{\partial^2 U}{\partial q_i^2} < 0$$

A consumer at a time needs several goods and services for consumption. His income is limited. Now he wants to maximize his total utility by consuming all goods and services subject to the income constraint.

This can be formalized as

Max $U = F(q_1, q_2, ...q_n)$

Subject to

$y = P_1 q_1 + P_2 q_2 + ...+ P_n q_n$

Where y is the income, $P_1, P_2 \ldots P_n$ are fixed prices for $q_1, q_2 \ldots\ldots q_n$ respectively.

This can be solved by the use of Lagrange function

$L = F(q_1, q_2 \ldots q_n) + \lambda[y - P_1q_1 - P_2q_2 - \ldots\ldots P_nq_n]$

Equating the partial derivatives to zero,

$F_i = \lambda P_i$

Where

$$\left.\begin{array}{l} F_i = \dfrac{\partial F}{\partial q_i}, i = 1,2....n \\ \text{and} \qquad y - P_1q_1 - P_2q_2 ... - P_nq_n = 0 \\ \therefore \dfrac{F_1}{P_1} = \dfrac{F_2}{P_2} = ... = \dfrac{F_n}{P_n} = \lambda \end{array}\right\} \quad (1)$$

$$\text{i.e.,} \frac{MU_1}{P_1} = \frac{MU_2}{P_2} = ... = \frac{MU_n}{P_n} = MU \quad \text{of money} (= \lambda)$$

Where MU_i = marginal utility of U w.r.to q_i

$$\text{i.e.,} \frac{MU_i}{MU_j} = \frac{P_i}{P_j}$$

i.e., The ratio of marginal utilities of two commodities must be equal to the ratio of their prices. This ratio is defined as the rate of substitution between these goods.

V. DERIVATION OF DEMAND FUNCTION

The set of (n+1) equations in (1) for the (n+1) unknowns is solvable. The solution will be in terms of known variables. Let it be

$q_i = \phi_i(P_1, P_2, \ldots P_n, y), \qquad i = 1,2.....n$

$\lambda = \lambda^0(P_1, P_2, \ldots P_n, y)$

Where ϕ_I and λ^0 are functional relations. These are the demand functions for the different commodities. The above approach is called cardinal approach. The basic assumption for this was that utility could be measured through cardinal scale of measurement say in terms of 'utils'

VI. THE INDIFFERENCE CURVE ANALYSIS

This is an alternative approach in which the emphasis is given on comparing different utility levels instead of measuring them through

some cardinal scale. The indifference curve approach is based on the following assumptions.

1. There is complete consistency in ordering of preference by the consumer.
2. The consumer's preferences are not conflicting with each other.
3. An individual's preferences are such that he prefers more to less.
4. The goods consumed by the consumer are substitutable.
5. All commodities in the consumption basket of the consumer are divisible.
6. Individuals are rational in decision-making.

Based on the above assumptions, the indifference curve technique compares the different levels of satisfaction or utility rather than measuring them.

The **indifference curve** is the locus of different combinations of two or more goods, which yield the same level of satisfaction or utility to the consumer. It is also called as **iso-utility curve**.

In the consumption process we are substituting one commodity for the other but maintaining a constant level of utility derived from them. This is called **Rate of Commodity Substitution** (RCS). If this is computed for very small changes it is called Marginal Rate of Commodity Substitution (MRCS). Hence RCS or MRCS is that amount of a commodity say Q_2 to be given up per unit of another commodity say Q_1 for consumption if the consumer remains on the same indifference curve.

A consumer's MRCS between two goods (Q_2 and Q_1) for a constant level of utility will, in some way, depend on how many units of Q_2 and Q_1 he is currently consuming. That is the willingness to give up the commodity Q_2 in favour of the other commodity Q_1 directly relates to the quantity of Q_2. This is the reason why the magnitude of MRCS of Q_2 for Q_1 declines as we decrease quantity of Q_2 and increase quantity of Q_1.

The above phenomena can be well explained mathematically by the utility function.

Let U= F(q_1,q_2) be the utility function for two commodities.

$$dU = \frac{\partial U}{\partial q_1} dq_1 + \frac{\partial U}{\partial q_2} dq_2$$

By definition of the indifference curve, U does not change on it.

$$\therefore dU = 0$$

$$\therefore -\frac{dq_2}{dq_1} = \frac{\frac{\partial U}{\partial q_1}}{\frac{\partial U}{\partial q_2}} = \frac{F_1}{F_2}$$

The L.H.S. is the MRCS.

i.e, $$-\frac{dq_2}{dq_1} = \frac{MU_1}{MU_2} \qquad (2)$$

i.e., MRCS is equal to the ratio of marginal utilities of the two goods.

Since $\frac{\partial U}{\partial q_1} \geq 0$ and $\frac{\partial U}{\partial q_2} \geq 0$, we must have $\frac{dq_2}{dq_1}$ to be negative. This is possible only if the in difference curve is negatively sloped.

The curvature of the indifference curve will give the degree of substitution between the goods represented by it. A consumer attains equilibrium when he spends his limited income on various goods and services. To study this through indifference curve, we need the information on his budget constraint.

A budget constraint is the locus of all combinations of two goods(or more) which can be purchased with fixed income at fixed prices.

Let y be the fixed income, P_1 and P_2, the fixed prices of two commodities. Then the budget constraint is

$y = P_1q_1 + P_2q_2$

$$\therefore q_2 = \frac{y}{P_2} - \frac{P_1}{P_2} q_1$$

i.e, it is a downward sloping curve.

If we maximize

U=F(q_1,q_2)

Subject to the budget constraint, we get

$$\frac{MU_1}{MU_2} = \frac{P_1}{P_2}$$

From equation (2), we have

$$-\frac{dq_2}{dq_1} = \frac{MU_1}{MU_2}$$

Combining the above two.

$$MRCS = -\frac{dq_2}{dq_1} = \frac{MU_1}{MU_2} = \frac{P_1}{P_2}$$

Here $\frac{MU_1}{MU_2}$ is the slope of the indifference curve and $\frac{P_1}{P_2}$ is the slope of the budget line.

Thus at equilibrium position, the above two slopes must be equal.

i.e., the budget constraint is tangent to the indifference curve.

The combination of goods at the point of tangency would give the optimum level of satisfaction to consumers.

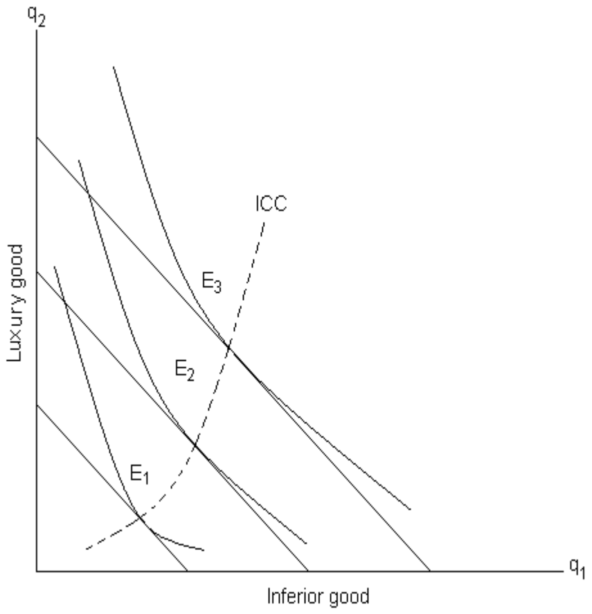

Fig. (i) Effects of income changes on consumer equilibrium (Price held constant)

Now we shift the budget line upward and for every shift get the equilibrium point on the indifference curve. The line formed by joining the equilibrium points on the indifference curve which we get by shifting the budget line up because of increase in income, other things being constant, is called the income-consumption curve (ICC). This is shown in Fig (i). This gives us how a consumer will consume the goods when his income rises.

The form of the ICC depends on consumer's behavior. If the ICC tilts towards a commodity axis then that commodity is treated as superior as compared to the other commodity. If it is negatively sloped, then both the commodities are normal goods. If it is negatively sloped, then one of the commodities is inferior. This is shown in Fig (ii).

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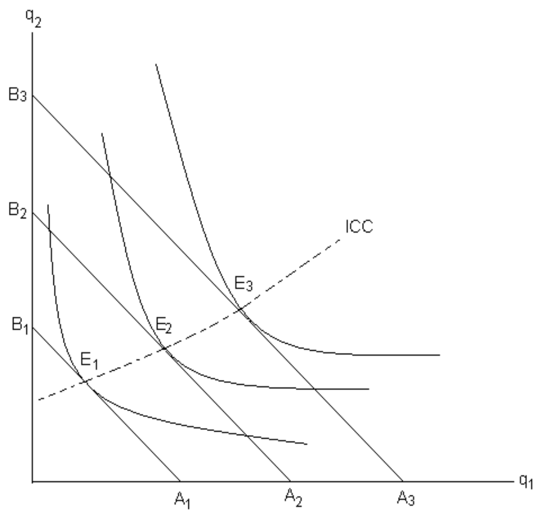

Fig. (ii): Downward sloping ICC

From the ICC we can draw the Engel curve. (An **Engel curve** is a relationship between quantity of a commodity say Q_1 purchased and income of the consumer, prices being constant).

For inferior goods the Engel curve will be negatively sloped, for luxury goods it will be positively sloped and for necessities very small positive slope as shown below in **Fig (iii).**

In the set of equations (1) if we allow all the variables to vary simultaneously, we get on taking total differentiation with two variables.

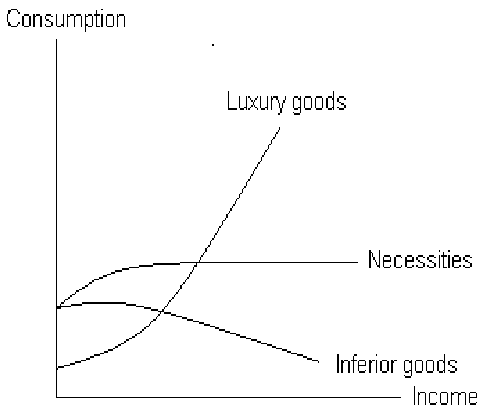

Fig. (iii) Engel Curve

$$\left.\begin{aligned} F_{11}\,dq_1 + F_{12}\,dq_2 - P_1 d\lambda &= \lambda\, dP_1 \\ F_{21}\,dq_1 + F_{22}\,dq_2 - P_2 d\lambda &= \lambda\, dP_2 \\ -P_1\,dq_1 - P_2\,dq_2 &= -\,dy + q_1 dP_1 + q_2 dP_2 \end{aligned}\right\} \quad (3)$$

These are three equations in the three unknowns dq_1, dq_2 and $d\lambda$ Solving (3) by using Crammer's Rule

$$dq_1 = \frac{\lambda dP_1(-P_2^2) - \lambda dP_2(-P_1P_2) + (-dy + q_1dP_1 + q_2dP_2)(-P_2F_{12} + P_1F_{22})}{D}$$

where

$$D = \begin{vmatrix} F_{11} & F_{12} & -P_1 \\ F_{21} & F_{22} & -P_2 \\ -P_1 & -P_2 & 0 \end{vmatrix}$$

$$dq_1 = \frac{-\lambda P_2^2 dP_1 + P_1P_2\lambda dP_2 + (-P_2F_{12} + P_1F_{22})(-dy + q_1dP_1 + q_2dP_2)}{D}$$

(4)

Assuming only price P_1 changes and other things are constant,
i.e., $dP_2=0$ and $dy=0$,
(4) becomes

$$\frac{\partial q_1}{\partial P_1} = \frac{-\lambda P_2^2 + (-P_2F_{12} + P_1F_{22})q_1}{D}$$

$$= -\frac{\lambda P_2^2}{D} + \frac{q_1(-P_2F_{12} + P_1F_{22})}{D}$$

i.e. the rate of change of q_1 w.r. to P_1 when all others are kept constant is given by

$$\frac{\partial q_1}{\partial P_1} = \frac{-\lambda P_2^2}{D} + \frac{q_1(-P_2F_{12} + P_1F_{22})}{D} \quad (5)$$

Similarly the rate of change of q_1 w.r. to income y when others are kept constant is given by

$$\frac{\partial q_1}{\partial y} = \frac{-(-P_2F_{12} + P_1F_{22})}{D} \quad (6)$$

Consider price changes compensated by income changes leaving utility unchanged.
i.e., $dU=0$
i.e., $F_1dq_1 + F_2dq_2 = 0$

i.e., $\frac{F_1}{F_2} = \frac{-dq_2}{dq_1}$ (7)

From equation (1)

$$\frac{F_1}{F_2} = \frac{P_1}{P_2} \quad (8)$$

Using (7) and (8)

$$\frac{P_1}{P_2} = -\frac{dq_2}{dq_1}$$

(ie) $P_1dq_1 + P_2dq_2 = 0$
Then the last equation in (3) becomes

$$-\,dy + q_1dP_1 + d_2dq_2 = 0 \quad (9)$$

Substituting (9) in (4),

$$\left[\frac{\partial q_1}{\partial P_1}\right]_{U=constant} = -\frac{\lambda P_2^2}{D} \quad (10)$$

From (5),(6) and (10) we get

$$\frac{\partial q_1}{\partial P_1} = \left(\frac{\partial q_1}{\partial P_1}\right)_{U=constant} - q_1\left(\frac{\partial q_1}{\partial y}\right)_{Prices=constant} \quad (11)$$

The above is called **Slutsky equation**

$\frac{\partial q_1}{\partial P_1}$ gives the total effect when P_1 alone changes.

$\left(\frac{\partial q_1}{\partial P_1}\right)_{U=constant}$ is called the substitution effect.

$q_1\left(\frac{\partial q_1}{\partial dy}\right)_{price=constant}$ is called the income effect.

Multiplying both sides of (11) by $\frac{P_1}{q_1}$ and multiplying the income effect on the R.H.S. by $\frac{y}{\bar{y}}$ we get

$E_{11} = e_{11} - \alpha_1\eta_1$ (12)

E_{11} is the price elasticity of the ordinary demand curve

e_{11} is the elasticity of compensated demand curve

α_1 is the proportion of the total expenditure spent on commodity Q_1 and η_1 is the income elasticity of demand for Q_1

Equations (11) and (12) can be extended to account for changes in the demand for one commodity resulting from changes in the price of the other commodity.

The expression for this is

$$\frac{\partial q_i}{\partial P_i} = \left(\frac{\partial q_i}{\partial P_j}\right)_{U=constant} - q_j\left(\frac{\partial q_i}{\partial y}\right)_{price=constant}$$

and $\varepsilon_{ij} = e_{ij} - \alpha_j\eta_i$ for i, j =1, 2

if i = j we get Slutsky's equations (11) and (12)

and if i≠j then

$$\frac{\partial q_i}{\partial P_j} \text{ or } \left(\frac{\partial q_i}{\partial P_j}\right)_{U=constant}$$

will show the cross effects indicating the change in quantity demanded of **i**[th] commodity when price of **j**[th] commodity changes.

VII. DERIVATION OF ROY'S IDENTITY

Consider Min U* = F* ($P_1, P_2, \ldots P_n$ y)

Subject to

The Lagrangian is Z = F* ($P_1, P_2, \ldots P_n$ y) + K(y - P_1q_1 - …. - P_nq_n)

Minimizing,

$$\frac{\partial F^*}{\partial P_i} - Kq_i = 0, \qquad i = 1,2\ldots\ldots\ldots\ldots\ldots n$$

$$\frac{\partial F^*}{\partial y} + K = 0$$

$$\frac{\partial F^*}{\partial K} = y - P_1q_1 - \ldots\ldots\ldots - P_n q_n = 0$$

From the first set of n equations

$$\frac{\frac{\partial F^*}{\partial P_1}}{q_1} = \frac{\frac{\partial F^*}{\partial P_2}}{q_2} = \ldots = \frac{\frac{\partial F^*}{\partial P_n}}{q_n} = K = \frac{-\partial F^*}{\partial y}$$

or

$$\frac{\frac{\partial U^*}{\partial P_1}}{q_1} = \frac{\frac{\partial U^*}{\partial P_2}}{q_2} = \ldots = \frac{\frac{\partial U^*}{\partial P_n}}{q_n} = K = \frac{-\partial U^*}{\partial y}$$

i.e. $$q_i^0 = -\frac{\frac{\partial U^*}{\partial P_i}}{\frac{\partial U^*}{\partial y}}, \qquad i = 1,2,\ldots\ldots\ldots n$$

This is 'Roy's Identity' which expresses the demand functions for the commodity in the consumption bundle of the consumer.

If $U^* = a_1\left(\frac{y}{P_1}\right)^{b_1} + a_2\left(\frac{y}{P_2}\right)^{b_2}$

Using Roy's identity

$$q_1 = \frac{-[-a_1b_1y^{b_1}P_1^{-b_1-1}]}{a_1b_1y^{b_1-1}P_1^{-b1} + a_2b_2y^{b_2-1}P_2^{-b_2}}$$

$$q_2 = \frac{-[-a_2b_2y^{b_2}P_2^{-b_2-1}]}{a_1b_1y^{b_1-1}P_1^{-b1} + a_2b_2y^{b_2-1}P_2^{-b_2}}$$

or

Generally

$$q_i = \frac{a_ib_iy_i^{b_i}P_i^{-b_i-1}}{\sum_j a_jb_jy^{b_j-1}P_j^{-b_j}}, \qquad i = 1,2 \text{ and } j = 1,2$$

VIII. LINEAR EXPENDITURE SYSTEM (LES)

In this a special type functional form is assumed for the utility function. This function satisfies the additive property.

i.e., Sum of different types of expenditure equals total expenditure and is homogeneous of degree one in income and prices.

i.e., U is assumed to be

$$U = \beta_1\log(q_1 - \overline{q_1}) + \beta_2\log((q_2 - \overline{q_2}) + \ldots + \beta_n\log(q_n - \overline{q_n})$$

where $\overline{q_i}$ is the quantity of $Q_i$commondity purchased and q_i is a

constant, i = 1,2,….n

Maximizing this utility function subject to the budget constraint, we have

$$\beta_i = \lambda P_i(q_i - \overline{q_i}) \qquad i = 1,2,\ldots n$$

$$\sum_{i=1}^{n}\beta_i = \lambda\sum_{i=1}^{n}P_i(q_i - \overline{q_i})$$

$$\therefore \lambda = \frac{1}{\sum_{i=1}^{n}P_i(q_i - \overline{q}_i)} = \frac{1}{\sum_{i=1}^{n}P_iq_i - \sum_{i=1}^{n}P_i\overline{q}_i)}$$

Here

$$y - \sum_{i=1}^{n}P_i\overline{q_i}$$

is defined as super-numerary income or uncommitted income and

β_i are the marginal propensities to consume.

(A) is interpreted as the expenditure (P_i q_i) on a commodity at the optimum which is a sum of committed expenditures on that commodity $(P_i \overline{q_i})$ and a portion (β_i) of the super-numerary income. The demand function derived in (B) can be written as

$$x_{ih} = b_{ih} + \frac{a_i}{P_i}(\mu_h - \sum_{k=1}^{n} P_k b_{kh}) \ldots\ldots (C)$$

$i = 1,2,\ldots n$

Here x_{ih} is the quantity and P_i is the price of good i and μ_h is the total expenditure of house hold *h*. The b_{ih} here can be made as an explicit function of the characteristics of household such as age, education, occupation, number of members etc.

Let

$$b_{ih} = \sum_{g=1}^{m} C_{ig} Z_{gh}$$

Where C_{ig} is the effect of the g^{th} characteristic and Z_{gh} is either a dummy variable or a quantitive variable.

The marginal budget shares (a_i) can also be expressed as a function of household characteristics say

$$a_{ih} = \sum_{g=1}^{m} a_{ig} Z_{gh} \text{ with } \sum a_i = 1$$

IX. ALMOST IDEAL DEMAND SYSTEM (AIDS) & RESULTS

To derive the equation for AIDS, the structure of the cost function is essential. Consider a production process with two inputs say X_1 and X_2. Let C^0 be the amount of money, which a producer has for the expenditure on the two inputs. Let the input price be P_1 and P_2 for X_1 and X_2 respectively. If x_1 and x_2 are the quantities of the two inputs X_1 and X_2 then the cost line is given by

$C^0 = P_1x_1 + P_2x_2$.

This gives the locus of all combination of the two inputs which the producer can buy using his fixed outlay (C^0) at fixed input prices.

Let the production function is of the form

$$y = A x_1^{a} x_2^{\beta}$$

and the iso-cost line is

$C = P_1x_1 + P_2x_2 + b$ (general form)

Then

$$\frac{\partial y}{\partial x_1} = \frac{\alpha A}{x_1}, \quad \frac{\partial y}{\partial x_2} = \frac{\beta A}{x_2}$$

and the equilibrium condition is

$$\frac{\frac{\partial y}{\partial x_1}}{\frac{\partial y}{\partial x_2}} = \frac{\alpha x_2}{\beta x_1} = \frac{P_1}{P_2}$$

From the iso-cost equation

$\mathbf{P_1 x_1 = C - P_2x_2 - b}$

Substituting in the expansion path

$$\alpha x_2 P_2 - \beta(C - P_2 x_2 - b) = 0$$

$$\therefore x_2 = \frac{\beta}{\alpha+\beta}\left(\frac{C-b}{P_2}\right)$$

$$x_1 = \frac{\alpha}{\alpha+\beta}\left(\frac{C-b}{P_1}\right)$$

$$\therefore y = A\left[\frac{\alpha}{\alpha+\beta}\left(\frac{C-b}{P_1}\right)\right]^{\alpha}\left[\frac{\beta}{\alpha+\beta}\left(\frac{C-b}{P_2}\right)\right]^{\beta}$$

$$\therefore y = A\left[\frac{\alpha}{\alpha+\beta}\left(\frac{C-b}{P_1}\right)\right]^{\alpha}\left[\frac{\beta}{\alpha+\beta}\left(\frac{C-b}{P_2}\right)\right]^{\beta}$$

$$\therefore y^{\frac{1}{\alpha+\beta}} = \left[\frac{A\alpha^{\alpha}\beta^{\beta}}{P_1^{\alpha}P_2^{\beta}}\right]^{\frac{1}{\alpha+\beta}} \cdot \frac{C-b}{\alpha+\beta}$$

$$\therefore C = (\alpha+\beta)(A\alpha^{\alpha}\beta^{\beta})^{-\frac{1}{\alpha+\beta}} y^{\frac{1}{\alpha+\beta}} P_1^{\frac{\alpha}{\alpha+\beta}} P_2^{\frac{\beta}{\alpha+\beta}} + b$$

$\alpha+\beta$ is called the returns to scale, be denoted by R. Then

$$C = R[A\alpha^{\alpha}\beta^{\beta}]^{-\frac{1}{R}} y^{\frac{1}{R}} P_1^{\frac{\alpha}{R}} P_2^{\frac{\beta}{R}} + b$$

i.e., C is of the form

$$C = K y^{\frac{1}{R}} P_1^{\frac{\alpha}{R}} P_2^{\frac{\beta}{R}} + b$$

Where

$$K = R[A\alpha^{\alpha}\beta^{\beta}]^{-\frac{1}{R}}$$

In the case of n inputs, it takes the form

$$C = K y^{\frac{1}{R}} P_1^{\frac{\alpha_1}{R}} P_2^{\frac{\alpha_2}{R}} \ldots\ldots P_n^{\frac{\alpha_n}{R}} + b$$

where $K = R[A\alpha_1^{\alpha_1}\alpha_2^{\alpha_2}\ldots\ldots\alpha_n^{\alpha_n}]^{\frac{-1}{R}}$

and $R = \alpha_1 + \alpha_2 + \ldots + \alpha_n$

This cost structure is the basic for the assumption of AIDS

To derive the AIDS model, let the logarithm of the cost function for a given utility level U and price P be of the form

$$\text{LogC}(U,P) = a_0 + \sum_k a_k \log P_k + \frac{1}{2}\sum_k\sum_j g_{kj} \log P_k \log P_j + UB_0 \Pi P_k^{B_k} \ldots\ldots (16)$$

By definition $\frac{\partial C}{\partial Pi} = q_i(U,P) = q_i(\text{say})$

Where $C = C(U,P)$

i.e.. $\frac{\partial C}{\partial P_i} = q_i$

$\therefore \frac{P_i}{C} \cdot \frac{\partial C}{\partial P_i} = \frac{P_i q_i}{C}$

i.e., $\frac{\partial \log C}{\partial \log P_i} = \frac{P_i q_i}{C} = w_i (say)$

w_i is the expenditure share of the **i**[th] good

From (16)

$$\frac{\partial \log C}{\partial \log P_i} = a_i + \sum_j g_{ij} \log P_j + UB_0 B_i' \prod_k P_k^{B_k}$$

i.e.. $w_i = a_i + \sum_j g_{ij} \log P_j + UB_0 B_i \prod_k P_k^{B_k}$ ……………(17)

Denote C(U.P) in (16) by Y. Then

$$\log Y = a_0 + \sum a_k \log P_k + \frac{1}{2} \sum_k \sum_j g_{kj} \log P_k \log P_j + UB_0 \prod_k P_k^{B_k}$$

$$\therefore UB_0 \prod_k P_k^{B_k} = \log Y - [a_0 + \sum_k a_k \log P_k + \frac{1}{2} \sum_k \sum_j g_{kj} \log P_k \log P_j]$$

Substituting this in (17)

$$w_i = a_i + \sum_j g_{ij} \log P_j + B_i [\log Y - (a_0 + \sum a_k \log P_k + \frac{1}{2} \sum_k \sum_j g_{kj} \log P_k \log P_j)]$$

Then the AIDS in the budget share form is

$$w_i = a_i + \sum_j g_{ij} \log P_j + B_i \log\left(\frac{Y}{P}\right), \quad i = 1,2,\ldots\ldots n$$

Where P is the price index defined in terms of individual prices given by

$$\mathrm{Log}P = a_0 + \sum_k a_k \log P_k + \frac{1}{2} \sum_k \sum_j g_{kj} \log P_k \log P_j$$

$g_{ij} = g_{ji}$: Symmetry condition

These conditions will ensure that the system satisfies the additivity, homogeneity in prices and income and the Slutsky symmetry conditions.

Here a_i, B_i, g_{ij} ; i,j = 1,2……n are parameters.

The non-linearity of these sets of equations requires the use of maximum likelihood methods of estimation.

For estimation purpose P is approximated to the price index given by $\log P = \sum_k w_k P_k$

where w_k is the weight of P_k

In addition to the above, if the household size also is to be taken in the function, it is modified as

$$w_i = a_i + \sum_j g_{ij} \log p_j + B_1 \log\left(\frac{Y}{P}\right) + \theta_i \log S, \quad i = 1,2\ldots\ldots n$$

The demand elasticities corresponding to the above AIDS are

(i) Own price elasticity $e_{ii} = \frac{g_{ii} - B_i w_i}{w_i}$

(ii) Cross price elasticity $e_{ij} = \frac{g_{ij} - B_i w_j}{w_i}$

(iii) Real expenditure elasticity $e_{iy} = \frac{B_i}{w_i} + \frac{g_{ii} - B_i w_i}{w_i}$

(iv) Household size elasticity $e_{is} = \frac{\theta_i - B_i}{w_i}$

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