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Quotient Square Sum Cordial Labeling

T.M. Selvarajan, Swapna Raveendran

***Abstract---** Let G = (V, E) be a simple graph and $\varphi : V \rightarrow \{1, 2, ... | V |\}$ be a bijection, for each edge uv assigned the label 1 if $\lfloor \frac{[\varphi(u)]^2 + [\varphi(v)]^2}{(\varphi(u)+ \varphi(v))} \rfloor$ is odd and 0 if $\lfloor \frac{[\varphi(u)]^2 + [\varphi(v)]^2}{(\varphi(u)+ \varphi(v))} \rfloor$ is even. φ is called quotient square sum cordial labeling if $| e_{\varphi}(0) - e_{\varphi}(1)| \leq 1$, where $e_{\varphi}(0)$ and $e_{\varphi}(1)$ denote the number of edges labeled with 0 and labeled with 1 respectively. A graph which admits a quotient square sum cordial labeling is called quotient square sum cordial graph. In this paper path P_m, cycle C_m, star $K_{1,n}$, friendship graph F_n, bistar $B_{n,n}$, $C_4 \cup P_n$, $K_{m,2}$ and $K_{m,2} \cup P_n$ are shown to be quotient square sum cordial labeling .*

***Keywords---** Quotient Square Sum Cordial Labeling, Friendship Graph, Wheel Graph and Double Fan Graph.*

AMS Mathematical Subject Classification (2010)--- 05C78

In this paper, the authors defined quotient square sum cordial labeling and prove the existence or nonexistence of quotient square sum cordial labeling for some families of graphs.

I. INTRODUCTION

The field of Graph Theory plays an important role in various areas of pure and applied sciences. A labeling of a graph G is a mapping that carries a set of graph elements, usually the vertices and edges into a set of numbers, usually integers. Many kinds of labeling have been studied and an excellent survey of graph labeling can be found in [3].

Graph Labeling of a graph G is an assignment of integers either to the vertices or edges or both subject to certain conditions. Graph labeling is a very powerful tool that eventually makes things in different fields very ease to be handled in mathematical way. Nowadays graph labeling has much attention from different brilliant researches in graph theory which has rigorous applications in many disciplines, e.g., communication networks, coding theory, optimal circuits layouts, astronomy, radar and graph decomposition problems. Recently the concept of prime graceful labeling was introduced by T.M. Selvarajan and R. Subramoniam in the year 2018.They studied the prime graceful labeling of various graphs in [5]. In [6] M. Sudha and A. Chandra Babu studied the relation between Different types of Graceful Graphs The symbol $\lfloor x \rfloor$ stands for largest integer less than or equal to x.

DISCUSSION & RESULTS

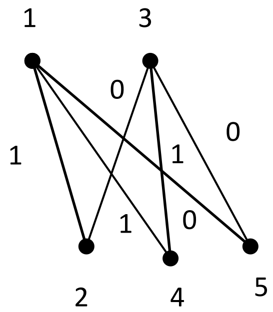
Definition 1.1

Let G be a (p,q) graph. Let φ : V(G)→ {1, 2, 3, ...p} be a one- one map. For each edge uv assign the label $[\frac{\varphi(u)}{\varphi(v)}]$ or $[\frac{\varphi(v)}{\varphi(u)}]$ according as φ (u) ≥ φ(v) or φ(v) ≤ φ(u). f is called a quotient cordial labeling of G, if $| e_{\varphi}(0) - e_{\varphi}(1)| \leq 1$, where $e_{\varphi}(0)$ and $e_{\varphi}(1)$ respectively denote the number of edges labeled with even integers and number of edges labeled with odd integers. A graph with a quotient cordial labeling is called quotient cordial graph.

Definition 1.2

Let G = (V, E) be a simple graph and $\varphi : V \rightarrow \{1, 2, ... | V |\}$ be a bijection, for each edge uv assigned the label 1 if $\lfloor \frac{[\varphi(u)]^2 + [\varphi(v)]^2}{(\varphi(u)+ \varphi(v))} \rfloor$ is odd and 0 if $\lfloor \frac{[\varphi(u)]^2 + [\varphi(v)]^2}{(\varphi(u)+ \varphi(v))} \rfloor$ is even. φ is called quotient square sum cordial labeling if $| e_{\varphi}(0) - e_{\varphi}(1)| \leq 1$, where $e_{\varphi}(0)$ and $e_{\varphi}(1)$ denote the number of edges labeled with 0 and labeled with 1 respectively. A graph which admits a quotient square sum cordial labeling is called quotient square sum cordial graph.

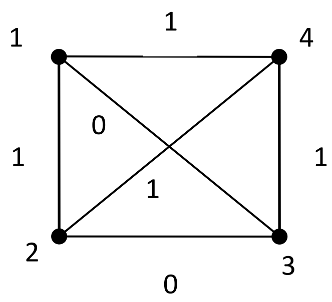
Example 1.3



For e(23) (edge 23) , $\lfloor \frac{(4+9)}{(2+3)} \rfloor$ =⌊2.6⌋ =2.

$e_{\varphi}(0) = 3$ and $e_{\varphi}(1) = 3 \Rightarrow | e_{\varphi}(0) - e_{\varphi}(1)| \leq 1$, therefore $K_{2,3}$ admits quotient square sum cordial labeling .

Example 1.4



$e_{\varphi}(0) = 2$ and $e_{\varphi}(1) = 4 \Rightarrow | e_{\varphi}(0) - e_{\varphi}(1)| > 1$

The complete graph K_4 does not admit quotient square sum cordial labeling.

Theorem 1.5

For n ≥ 3, the path P_n admits quotient square sum cordial labeling.

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Proof

Let $P_n = \{v_1, v_2, \ldots, v_n\}$ be a path with $n \geq 3$,where v_k and v_{k+1} are adjacent.

Define : $\{v_1, v_2, \ldots, v_n\} \rightarrow \{1, 2, \ldots, n\}$ by $\varphi(v_k)= k$.

$\frac{[\varphi(v_k)]^2+[\varphi(v_{k+1})]^2}{[\varphi(v_k)+\varphi(v_{k+1})]} = \frac{k^2+(k+1)^2}{\{(k+(k+1)\}} = k + \frac{1}{2} + \frac{1}{4k+2}$

$\frac{1}{2} + \frac{1}{4k+2} < 1$ for all $k > 0$, $\lfloor \frac{[\varphi(v_k)]^2+[\varphi(v_{k+1})]^2}{[\varphi(v_k)+\varphi(v_{k+1})]} \rfloor = k$

$$e(v_k v_{k+1}) = \begin{cases} 1 \text{ if } k \text{ is odd} \\ 0 \text{ if } k \text{ is even} \end{cases}$$

If n is even, $e(v_1v_2) =1$, $e(v_2v_3)=0$, $e(v_3v_4) = 1, \ldots, e(v_{n-1}v_n) = 1$,

then $e_\varphi(0) = \frac{n}{2}$ and $e_\varphi(1) = \frac{n}{2} - 1 \Rightarrow | e_\varphi(0) - e_\varphi(1)| = 1$

If n is odd, $e(v_1v_2) =1$, $e(v_2v_3)=0$, $e(v_3v_4) = 1, \ldots,$ and $e(v_{n-1}v_n) = 0$,

Then $e_\varphi(0) = \frac{(n-1)}{2}$ and $e_\varphi(1) = \frac{(n-1)}{2} \Rightarrow | e_\varphi(0) - e_\varphi(1)| = 0$.

Theorem 1.6

The star graph $K_{1,n}$ admits quotient square sum cordial labeling.

Proof

Let v_1 be the central vertex of degree n.

Let $v_2, \ldots, v_n, v_{n+1}$ be the n pendant vertices attached to it.

Define φ: $\{v_1, v_2, \ldots, v_n, v_{n+1}\} \rightarrow \{1, 2, \ldots, n,n+1\}$ by $\varphi(v_k) = k$.

$\frac{[\varphi(v_1)]^2+[\varphi(v_k)]^2}{[\varphi(v_1)+\varphi(v_k)]} = \frac{1+k^2}{[1+k]} = (k - 1) + \frac{2}{(k+1)}$

$\frac{2}{(k+1)} < 1$ for $k \geq 2$, $\lfloor \frac{[\varphi(v_1)]^2+[\varphi(v_k)]^2}{[\varphi(v_1)+\varphi(v_k)]} \rfloor = k - 1$ for $k \geq 2$

$$e(v_1 v_k) = \begin{cases} 1 \text{ if } k \text{ is even} \\ 0 \text{ if } k \text{ is odd} \end{cases}$$

If n is even, $e(v_1 v_k) =1$, for $k = 2,4,6, \ldots, n$ and $e(v_1 v_k) =0$, for $k = 3,5, \ldots, (n + 1)$

$e_\varphi(0) = \frac{n}{2}$ and $e_\varphi(1) = \frac{n}{2} \Rightarrow | e_\varphi(0) - e_\varphi(1)| = 0$.

If n is odd, $e(v_1 v_k) =1$, for $k = 2,4,6, \ldots, (n -1)$ and $e(v_1 v_k) = 0$, for $k = 3,5, \ldots, n$.

$e_\varphi(0) = \frac{(n-1)}{2}$ and $e_\varphi(1) = \frac{(n+1)}{2} \Rightarrow | e_\varphi(0) - e_\varphi(1)| = 1$

Theorem 1.7

The graph $K_{n,2}$ is an quotient square sum cordial graph for $n \neq 3$.

Proof

The graph $K_{n,2}$ contains $(n + 2)$ vertices and 2n edges.

Label the vertices having degree n of $K_{n,2}$ with 1 and 2 remaining vertices with integers 3,4, 5, ..., (n+2) .

Define φ: $\{v_1, v_2, \ldots, v_n, v_{n+1}, v_{n+2}\} \rightarrow \{1, 2, \ldots, n,n+1,n+2\}$ by

$\varphi(v_1) = 1$, $\varphi(v_2) = 2$ and $\varphi(v_k)= k$ for $3 \leq k \leq n+2$.

By Theorem 1.6, $\lfloor \frac{[\varphi(v_1)]^2+[\varphi(v_k)]^2}{[\varphi(v_1)+\varphi(v_k)]} \rfloor = k - 1$ for $k \geq 2$.

$$e(v_1 v_k) = \begin{cases} 1 \text{ if } k \text{ is even} \\ 0 \text{ if } k \text{ is odd} \end{cases}$$

$\frac{[\varphi(v_2)]^2+[\varphi(v_k)]^2}{[\varphi(v_2)+\varphi(v_k)]} = \frac{4+k^2}{[2+k]} = (k - 2) + \frac{8}{(k+2)}$

$\frac{8}{(k+2)} < 1$ for $k > 6$, $\lfloor \frac{[\varphi(v_2)]^2+[\varphi(v_k)]^2}{[\varphi(v_2)+\varphi(v_k)]} \rfloor = k - 2$ for $k > 6$.

For $k >6$, $e(v_2 v_k) = \begin{cases} 0 \text{ if } k \text{ is even} \\ 1 \text{ if } k \text{ is odd} \end{cases}$

Case 1 If n is even $e(v_1 v_k) =1$, for $k = 4,6, \ldots, n + 2$, $e(v_1 v_k) = 0$, for $k = 3,5, \ldots, (n - 1)$ and $e(v_2 v_k) = 0$, for $k = 8,10 \ldots, n + 2$,

$e(v_2 v_k) = 1$, for $k = 7,9, \ldots, (n + 1)$.

$e(v_2 v_3) = 0$, $e(v_2 v_5) = 0$, $e(v_2 v_4) =1$ and $e(v_2 v_6) =1$.

$e_\varphi(0) = n$ and $e_\varphi(1) = n \Rightarrow | e_\varphi(0) - e_\varphi(1)| = 0$.

Case 2: If n is odd and $n \neq 3$. $e(v_1 v_k) =1$, for $k = 4,6, \ldots,$ $e(v_1 v_k) = 0$, for $k = 3,5, \ldots, n$, $e(v_2 v_k) =0$, for $k = 8,10 \ldots, n + 2$,

$e(v_2 v_k) =1$, for $k = 7,9, \ldots, (n + 2)$.

$e(v_2 v_3) = 0$, $e(v_2 v_5) = 0$, $e(v_2 v_4) =1$ and $e(v_2 v_6) =1$.

$e_\varphi(0) = n$ and $e_\varphi(1) = n \Rightarrow | e_\varphi(0) - e_\varphi(1)| = 0$

Theorem 1.8

The fan graph $F_n = Pn + K_1$ is a quotient square sum cordial graph.

Proof

The fan graph $F_n = K_1 + P_n$ contains $n + 1$ vertices and $(2n - 1)$ edges. Label the vertex having degree n of F_n with 1 and the remaining vertices by 2,3,4,…,n, $(n + 1)$.

Case 1: If n is even, using Theorem 1.5 and Theorem 1.6, $e_\varphi(0) = n$ and $e_\varphi(1) = (n - 1) \Rightarrow | e_\varphi(0) - e_\varphi(1)| = 1$.

Case 2 If n is odd, using Theorem 1.5 and Theorem 1.6, $e_\varphi(0) = (n - 1)$

and $e_\varphi(1) = n \Rightarrow | e_\varphi(0) - e_\varphi(1)| = 1$. **Theorem 1.9.** The Double fan graph $F_{2,n}$ is a quotient square sum cordial graph for $n \neq 3$.

Proof : The Double fan graph $F_{2,n}$ contains n+2 vertices and (3n− 1) edges. Label the vertices having degree n of double fan graph $F_{2,n}$ with 1 and 2, the remaining vertices by 3,4,…,n, $(n + 1)$, $(n + 2)$

Case 1: If n is even, using Theorem 1.5 and Theorem 1.7, $e_f(0) = \frac{3n}{2}$ and $e_\varphi(1) = \frac{3n}{2} - 1 \Rightarrow | e_\varphi(0) - e_\varphi(1)| = 1$.

Case 2: If n is odd, using Theorem 1.5 and Theorem 1.7, $e_\varphi(0) = \frac{(3n-1)}{2}$ and $e_\varphi(1) = \frac{(3n-1)}{2} \Rightarrow | e_\varphi(0) - e_\varphi(1)| = 0$.

Theorem 1.10

For $n \geq 3$, the cycle Cn is a quotient square sum cordial graph.

Proof

Let $C_n = \{v_1, v_2, \ldots, v_n, v_1\}$ be the cycle, where v_k and v_{k+1} are adjacent,

v_n and v_1 are adjacent for $k = 1, 2, 3, \ldots, n$.

case(i): n is odd

Define φ: $\{v_1, v_2, \ldots, v_n\} \rightarrow \{1, 2, \ldots, n\}$ by $\varphi(v_k)= k$ for $k = 1, 2, 3, \ldots, n$.

$\frac{[\varphi(v_k)]^2+[\varphi(v_{k+1})]^2}{[\varphi(v_k)+\varphi(v_{k+1})]} = \frac{k^2+(k+1)^2}{\{(k+(k+1)\}} = k + \frac{1}{2} + \frac{1}{4k+2}$

$\frac{1}{2} + \frac{1}{4k+2} < 1$ for all $k >0$

$\lfloor \frac{[\varphi(v_k)]^2+[\varphi(v_{k+1})]^2}{[\varphi(v_k)+\varphi(v_{k+1})]} \rfloor = k$

$e(v_k v_{k+1}) = \begin{cases} 1 \; if \; k \; is \; odd \\ 0 \; if \; k \; is \; even \end{cases}$

$\frac{[\varphi(v_n)]^2+[\varphi(v_1)]^2}{[\varphi(v_n)+\varphi(v_1)]} = \frac{n^2+1}{(n+1)} = (n-1) + \frac{2}{n+1}$

$\lfloor \frac{[\varphi(v_n)]^2+[\varphi(v_1)]^2}{[\varphi(v_n)+\varphi(v_1)]} \rfloor = (n-1)$

$e(v_1v_2) = 1$, $e(v_2v_3) = 0$, $e(v_3v_4) = 1, \ldots, e(v_{n-1}v_n) = 0$ and $e(v_nv_1) = 0$.

$e_\varphi(0) = \frac{(n-1)}{2}$ and $e_\varphi(1) = \frac{(n+1)}{2} \Rightarrow | e_\varphi(0) - e_\varphi(1)| = 1$

Case (ii) : n is even

Define $\varphi : \{v_1, v_2, \ldots, v_n\} \rightarrow \{1, 2, \ldots, n\}$ by

$\varphi(v_k) = \begin{cases} k \; if \; 1 \le k \le n-4 \\ n \; if \; k = n-3 \\ k-1 \; if \; n-2 \le k \le n \end{cases}$

$\lfloor \frac{[\varphi(v_k)]^2+[\varphi(v_{k+1})]^2}{[\varphi(v_k)+\varphi(v_{k+1})]} \rfloor = k$ for $1 \le k \le n-4$

$\frac{[\varphi(v_{n-3})]^2+[\varphi(v_{n-4})]^2}{[\varphi(v_{n-3})+\varphi(v_{n-4})]} = (n-2) + \frac{8}{(2n-4)}$

$\frac{8}{(2n-4)} < 1$ for $n > 6$

$\lfloor \frac{[\varphi(v_{n-3})]^2+[\varphi(v_{n-4})]^2}{[\varphi(v_{n-3})+\varphi(v_{n-4})]} \rfloor = (n-2)$ for $n > 6$

$\frac{[\varphi(v_{n-3})]^2+[\varphi(v_{n-2})]^2}{[\varphi(v_{n-3})+\varphi(v_{n-2})]} = (n-1) - \frac{1}{2} + \frac{9}{(4n-3)}$

$-\frac{1}{2} + \frac{9}{(4n-3)} < 1$ for $n > 3$

$\lfloor \frac{[\varphi(v_{n-3})]^2+[\varphi(v_{n-2})]^2}{[\varphi(v_{n-3})+\varphi(v_{n-2})]} \rfloor = (n-1)$ for $n > 3$

$\frac{[\varphi(v_1)]^2+[\varphi(v_n)]^2}{[\varphi(v_1)+\varphi(v_n)]} = (n-2) + \frac{2}{n}$

$\lfloor \frac{[\varphi(v_1)]^2+[\varphi(v_n)]^2}{[\varphi(v_1)+\varphi(v_n)]} \rfloor = (n-2)$

$e(v_1v_2) = 1$, $e(v_2v_3) = 0$, $e(v_3v_4) = 1, \ldots, e(v_{n-4}v_{n-3}) = 0$, $e(v_{n-3}v_{n-2}) = 1$,

$e(v_{n-1}v_n) = 0$ and $e(v_nv_1) = 0$.

$e_\varphi(0) = \frac{n}{2}$ and $e_\varphi(1) = \frac{n}{2} \Rightarrow | e_\varphi(0) - e_\varphi(1)| = 0$.

Theorem 1.11

For $n > 3$, the wheel graph $W_n = K_1 + C_n$ admits quotient square sum cordial labeling.

Proof

The wheel graph $W_n = K_1 + C_n$ contains $n + 1$ vertices and 2n edges. Label the vertex having degree n of W_n with 1 and the remaining vertices by 2,3,4,…,n, $(n + 1)$.

Case 1

If n is even, using Theorem 1.6 and Theorem 1.11, $e_\varphi(0) = n$

and $e_\varphi(1) = n \Rightarrow | e_\varphi(0) - e_\varphi(1)| = 0$.

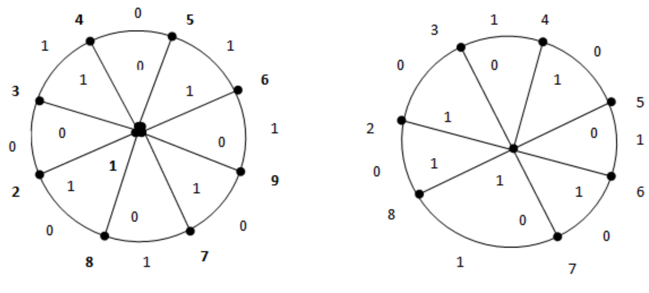
Case 2

If n is odd, using Theorem 1.6 and Theorem 1.11, $e_\varphi(0) = n$

and $e_\varphi(1) = n$

$\Rightarrow | e_\varphi(0) - e_\varphi(1)| = 0$.

Example 1.12



$W_8 = K_1 + C_8$ $W_7 = K_1 + C_7$

Theorem 1.13

The bistar graph $B_{n,n}$ admits quotient square sum cordial labeling.

Proof

Bistar graph $B_{n,n}$ contains (2n+2) vertices and (2n+1) edges.

Let $|V(G)| = 2n + 2$; $|E(G)| = 2n + 1$. Bistar graph $B_{n,n}$ has exactly two vertices of degree n, let them be u_0 and v_0.

Define φ: $\{v_0, v_1, v_2, \ldots, v_n, u_0, u_1, u_2, \ldots, u_n\} \rightarrow \{1, 2, \ldots, 2n, 2n+1, 2n+2\}$ by $\varphi(v_0) = 1$, $\varphi(u_0) = 2$, $\varphi(v_i) = k + 2$; $1 \le k \le n$ and

$\varphi(u_k) = n + k$; $3 \le k \le (n+2)$

$\frac{[\varphi(v_1)]^2+[\varphi(v_k)]^2}{[\varphi(v_1)+\varphi(v_k)]} = \frac{1+k^2}{[1+k]} = (k-1) + \frac{2}{(k+1)}$

$\frac{2}{(k+1)} < 1$ for $k \ge 2$

$\lfloor \frac{[\varphi(v_1)]^2+[\varphi(v_k)]^2}{[\varphi(v_1)+\varphi(v_k)]} \rfloor = k - 1$ for $k > 2$

$\frac{[\varphi(u_0)]^2+[\varphi(u_k)]^2}{[\varphi(u_0)+\varphi(u_k)]} = \frac{4+(n+k)^2}{[2+n+k]} = (n + k - 2) + \frac{8}{(n+k+2)}$

$\frac{8}{(n+k+2)} < 1$ for all $k > 3$

$\lfloor \frac{[\varphi(u_0)]^2+[\varphi(u_k)]^2}{[\varphi(u_0)+\varphi(u_k)]} \rfloor = (n + k - 2)$

If n is even,

$e(v_0v_k) = \begin{cases} 1 \; if \; k \; is \; even \\ 0 \; if \; k \; is \; odd \end{cases}$ and $e(u_0u_k) = \begin{cases} 0 \; if \; k \; is \; even \\ 1 \; if \; k \; is \; odd \end{cases}$

$e(u_0 v_0) = 1$, $e(v_0 v_k) = 1$, for k = 2,4,6, ..., n and

$e(v_1 v_k) = 0$, for k = 1,3,5, ..., $(n-1)$

$e_\varphi(0) = \frac{n-2}{2}$ and $e_\varphi(1) = \frac{n}{2} \Rightarrow | e_\varphi(0) - e_\varphi(1)| = 1$.

If n is odd

$e(v_0v_k) = \begin{cases} 1 \; if \; k \; is \; even \\ 0 \; if \; k \; is \; odd \end{cases}$ and $e(u_0u_k) = \begin{cases} 1 \; if \; k \; is \; even \\ 0 \; if \; k \; is \; odd \end{cases}$

$e(v_1 v_k) = 1$, for k = 2,4,6, ..., $(n-1)$ and $e(v_1 v_k) = 0$, for k = 3,5, ..., n .

$e_\varphi(0) = \frac{(n-1)}{2}$ and $e_\varphi(1) = \frac{(n-1)}{2} \Rightarrow | e_\varphi(0) - e_\varphi(1)| = 0$

Theorem 1.14

The friendship graph $F_n^{(3)}$ admits quotient square sum cordial labeling.

Proof

The friendship graph $F_n^{(3)}$, is a planar undirected graph with 2n + 1 vertices and 3n edges constructed by joining n copies of the triangle graph T with a common vertex.

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Let v_1 be the central vertex of degree 2n.

Friendship graph contains n triangles $T_1, T_2, \ldots, T_n$.

Define f: V(G)→ {1, 2, ...,2n,2n+1} by $\varphi(v_1) = 1$, label the outer

vertices of the triangles $T_1, T_3, T_5, \ldots$ by $\varphi(v_k) = 2k, 1 \le k \le n$ and label the outer vertices of the triangles $T_2, T_4, T_6, \ldots$ by $\varphi(v_k) = 2k+1, 1 \le k \le n$.

$\frac{[\varphi(v_1)]^2+[\varphi(v_k)]^2}{[\varphi(v_1)+\varphi(v_k)]} = \frac{1+k^2}{[1+k]} = (k - 1) + \frac{2}{(\square+l)}$

$\frac{2}{(k+1)} < 1$ for $k \ge 2$

$\lfloor\frac{[\varphi(v_1)]^2+[\varphi(v_k)]^2}{[\varphi(v_1)+\varphi(v_k)]}\rfloor = (k - 1)$ for $k \ge 2$

$\frac{[\varphi(v_k)]^2+[\varphi(v_{k+1})]^2}{[\varphi(v_k)+\varphi(v_{k+1})]} = \frac{k^2+(k+2)^2}{\{(k+(k+2)]} = (k + 1) + \frac{1}{k+1}$

$\frac{1}{k+1} < 1$ for all $k > 1$

$\lfloor\frac{[\varphi(v_k)]^2+[\varphi(v_{k+1})]^2}{[\varphi(v_k)+\varphi(v_{k+1})]}\rfloor = (k + 1)$

Case 1

If n is even, $e_\varphi(0) = \frac{3n}{2}$ and $e_\varphi(1) = \frac{3n}{2} \Rightarrow |e_\varphi(0) - e_\varphi(1)| = 0$.

Case 2

If n is odd, $e_\varphi(0) = \frac{3n-1}{2}$ and $e_\varphi(1) = \frac{3n+1}{2}$

$\Rightarrow |e_\varphi(0) - e_\varphi(1)| = 1$.

Theorem 1.15

The friendship graph $Fr_n^{(3)}$ admits quotient square sum cordial

labeling .

Proof

The friendship graph $Fr_n^{(3)}$ is an undirected graph with $3n + 1$ vertices and 5n edges constructed by joining n copies of the fan graph F_3 with a common vertex.

Let v_1 be the central vertex of degree 3n.

Define φ: V(G)→ {1, 2, ...,2n,... , 3n, $3n + 1$} by $\varphi(v_1) = 1$,

label the outer vertices $v_2, v_3, \ldots, v_{3n}, v_{3n+1}$ of the friendship graph $Fr_n^{(3)}$

by $\varphi(v_k) = k, 2 \le k \le 3n + 1$

$\lfloor\frac{[\varphi(v_1)]^2+[\varphi(v_k)]^2}{\varphi(v_1)+\varphi(v_k)]}\rfloor = (k - 1)$ for $k \ge 2$

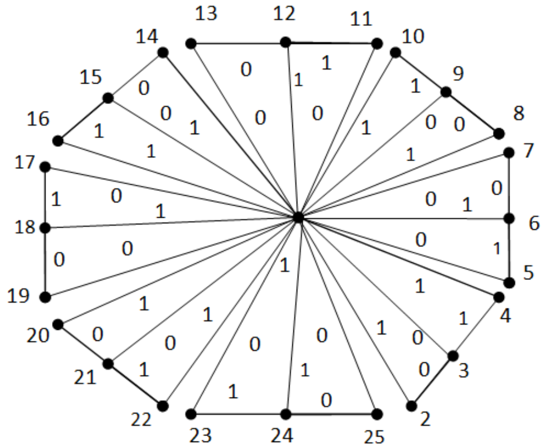
$\frac{[\varphi(v_k)]^2+[\varphi(v_{k+1})]^2}{[\varphi(v_k)+\varphi(v_{k+1})]} = k + \frac{1}{2} + \frac{1}{4k+2}, \lfloor\frac{[\varphi(v_k)]^2+[\varphi(v_{k+1})]^2}{[\varphi(v_k)+\varphi(v_{k+1})]}\rfloor = k$

Case 1: If n is even , $e_\varphi(0) = \frac{5n}{2}$ and $e_\varphi(1) = \frac{5n}{2} \Rightarrow |e_\varphi(0) - e_\varphi(1)| = 0$

Case 2: If n is odd, $e_\varphi(0) = \frac{5n-1}{2}$ and $e_\varphi(1) = \frac{5n+1}{2}$

$\Rightarrow |e_\varphi(0) - e_\varphi(1)| = 1$.

Example 1.16



The friendship graph $Fr_8^{(3)}$

$e_\varphi(0) = 20$ and $e_\varphi(1) = 20 \Rightarrow |e_\varphi(0) - e_\varphi(1)| = 0$.

Theorem 1.17

The graph $P_n \cup K_{m,2}$ ($m \ne 3$) is a quotient square sum cordial graph.

Proof

The graph $P_n \cup K_{m,2}$ contains $(n + m + 2)$ vertices and $(2m + n - 1)$ edges .Using Theorem 1.5 and Theorem 1.7, $P_n \cup K_{m,2}$ admits quotient square sum cordial labeling.

Theorem 1.18

The graph $P_n \cup C_4$ is a quotient square sum cordial graph.

Proof

The graph $P_n \cup C_4$ contains $(n + 4)$ vertices and $(n + 3)$ edges .

Label the vertices of C_4 by 1,3,2,4 and the remaining vertices

by 5,6,…, $(n + 4)$.

For C_4 , $e_\varphi(0) = e_\varphi(1) = 2$

Using Theorem 1.5,

$P_n \cup C_4$ admits quotient square sum cordial labeling.

II. CONCLUSION

The investigation of labeled graph is very important due to its various applications in many fields. It is very interesting to study the various types of graphs which admits quotient square sum cordial labeling. It is an open area of research to discuss some more similar results for various graphs.

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