International Journal of Recent Technology and Engineering (IJRTE)
ISSN: 2277-3878, Volume-8, Issue-2S3, July 2019

Robot Forward and Inverse Kinematics Research using Matlab

D. Sivasamy, M. Dev Anand, K. Anitha Sheela

Abstract--- *The joint arrangement of every robot can be described by the Denavit Hardenberg parameters. These parameters are enough to obtain a working of the robot described and Presented is a Matlab program which modelled Sorbet era 5u pluse given a set of corresponding DH parameters. The prim aim of this paper is to develop forward and inverse kinematic models of Sorbet era 5u plus using Matlab GUI in order to optimize the manipulative task execution. Forward kinematics analysis is done for the flexible twist angle, link lengths, and link offsets of each joints by varying joint angles to specify the position and orientation of the end effectors. Forward analysis can be used to provide the position of some point on the end effectors together with the orientation of the end effectors measured relative to a coordinate system fixed to ground for a specified set of joint variables. This simulation allows the user to get forward kinematics and inverse kinematics of Scorbot era 5u Plus of the modelled robot in various link length parameters and joint angles and corresponding end effectors position and orientation is going to validate with Rob cell software and compared with Lab view measured values.*

Keywords--- *Robot, Scorbot era 5u Plus, DH Representation, Matlab GUI. Robocell, Labview.*

I. INTRODUCTION

The Denavit-Hartenberg illustration of forward kinematic equations of robots has grown to be the standard procedure for model robots and their motions. The technique summarizes the relationship between two joints in concise set of four parameters. Any robot can be modelled using the D-H representation.

A computer code has been formed in Matlab to implement the modelling of any robot with only the DH parameters as input. The purpose of the simulator is to create an accurate Forward kinematic and inverse kinematics representation of any type of robot and its motions. The simulator also allows for the independent manipulation of each joint of the modelled robot.

Presented in this study are the details of this Scorbot era 5u Plus as well as background on the DH representation and some analysis on how effectively the Scorbot era 5u Plus model compared with Labview and Matlab.

II. BACKGROUND

In 1955, Denavit and Hartenberg in print a paper [1] explaining a kinematic notation that was eventually adapted as a way to represent robots. The method defines robots as a sequence of joints, each with a degree of freedom. Each joint has its own reference frame complete with a z and x axis, the intersection of which defines the joint's origin. Each joint is defined as either *prismatic*, when the motion is a linear translation along the joint's z axis, or revolute, when the motion is a rotation about the joint's z axis. Each joint is defined iteratively in terms of the transformation necessary from the previous joint. The first joint is defined as a transformation from a reference origin and axis. This technique is detailed in Figure 1 and Figure 2.

The transformation from joint *n* and joint *n+1* has four steps as follows.

1. First, rotate an angle of θ_{n+1} about the z_n axis. This aligns x_n with x_{n+1}.
2. Second, translate along the z_n axis a distance d_{n+1} to make the x_n and x_{n+1} axis collinear.
3. Third, translate along the xn axis a distance αn+1. This makes the two origins in the same location.

Finally, rotate the z_n axis about the x^{n+1} axis an angle of αn+1. This process aligns both the origins and reference frames of joint *n* and joint *n+1* and is depicted in Figure 2.

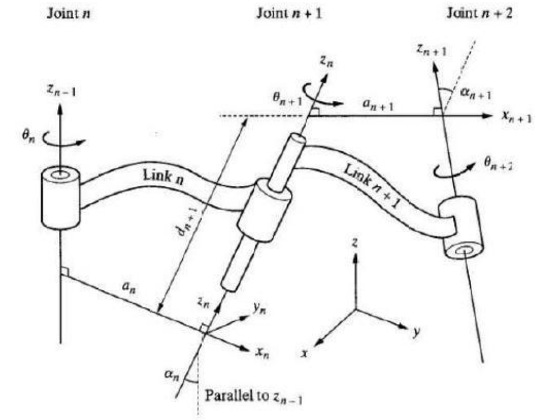

Figure 1: Denavit Hartenberg Representation

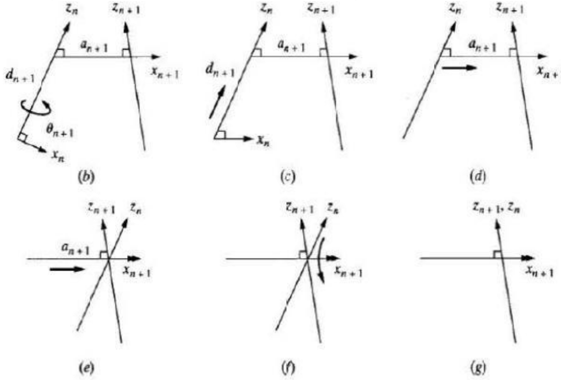

Figure 2: Transformation from Joint n and joint n+1 in DH Representation

---

Revised Manuscript Received on July 10, 2019.

D. Sivasamy*, External Research Scholar, ECE, Jawaharlal Nehru Technological University, Hyderabad-85, Andhra Pradesh, India. (e-mail: sivasamy.d@gmail.com)

M. Dev Anand, Professor and Director Research, Department of Mechanical Engineering, Noorul Islam Centre for Higher Education, Kumaracoil, Thuckalay, Kanyakumari District, Tamil Nadu, India.

K. Anitha Sheela, Professor & Head, ECE, Jawaharlal Nehru Technological University, Hyderabad-85, Telangana, India.

The transformation between reference frame *i - 1* and reference frame *i* can be easily calculated by following these above steps and is shown in Equation.

$$T_{i}^{i-1} = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Following this technique iteratively, each joint can be described by the previous until all of the joints of the robot have been described. The resultant parameters θ, d, a, α are the critical parameters to define one joint in terms of the previous. The values of these parameters for each joint are often represented in a table known as a DH table. Every robot can be described by its DH table. For a more exhaustive review of the Denavit-Hartenberg Representation see references [1] and [2].

This simulator uses the DH parameters in the table to model the robot and its motions. This is described in detail in the following sections.

Given the set of joint-link parameters, the problem of finding the position and orientation of the end-effectors with respect to known reference frame for an n-DOF manipulator is referred as Forward Kinematic Model. This model gives the position and orientation of the end-effectors as a function of the joint variables and other joint-link constant parameters. For a given position and orientation of the end-effectors, with respect to an immobile or inertial reference frame, it is required to find a set of joint variables that would bring the end-effectors in the specified position and orientation. This is referred as Inverse Kinematic Model.

III. SCORBOT era 5u PLUS

For this research the SCORBOT ER 5u Plus is used. It is a vertical articulated robot, with five revolute joints. With gripper attached, the robot has six DOF (Degrees of Freedom). Figure 4 identifies the Scorbot arm links and Figure 5 identifies the Scorbot arm joints. This design permits the end-effectors to be positioned and oriented arbitrarily within a large work space. The ranges of all the axes are as follows: Axis 1: Base rotation (±155^o), Axis 2: Shoulder rotation (-35^o to +130^o), Axis 3: Elbow rotation (±130^o), Axis 4: Wrist pitch (±130^o), Axis 5: Wrist roll (±570° - electrically, Unlimited mechanically).

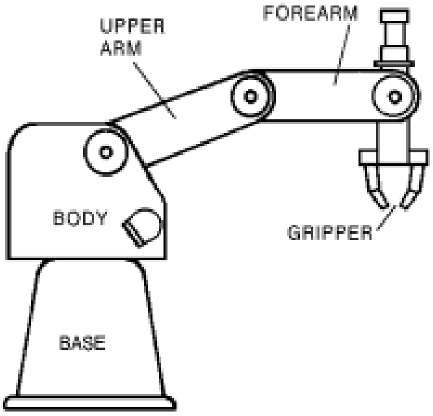

Figure 3: Scorbot Arm Links

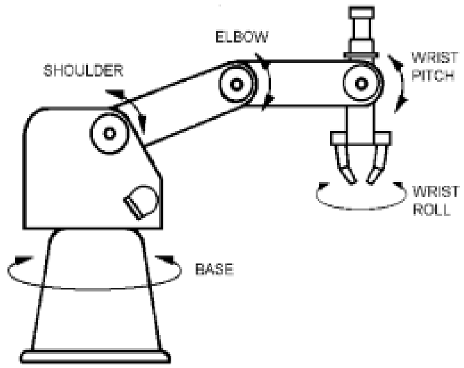

Figure 4: SCORBOT Arm Joints

Table 1: D-H Parameter for Scorbot era 5u Plus

Joint	α_i	a_i	d_i	θ_i
1	$-\pi/2$	16	349	θ_1
2	0	221	0	θ_2
3	0	221	0	θ_3
4	$\pi/2$	0	0	$\pi/2+\theta_4$
5	0	0	145	θ_5

Table 2: Movements of the Joints

Axis	Joint	Motion
1	Base	Rotates the Body
2	Shoulder	Raises and Lowers the Upper
3	Elbow	Raises and Lowers the Forearm
4	Wrist	Raises and Lowers the End
5	Wrist Roll	Rotates the End Effector

Table 3: Range of Joints for SCORBOT era 5u Plus

Joint	Joint Name	Range
1	Base	±155
2	Shoulder	-35to +130
3	Elbow	±130
4	Wrist Pitch	±130
5	Wrist Roll	±570

[6] explained an alternative formulation for the development of homogeneous matrix method on the basis of motion at the joints of serial mechanism through the numerical study of a 5 DOF serial robot and verified the end-results of the formulation [8] for serial manipulator robot.

Presented some formulation for velocity analysis and acceleration analysis for serial robots.[12] established the coordinates of robot kinematics mathematical model and the target matrix using D-H coordinate transformation method. PRO/E is used to model the robot and MATLAB functions are used for the analysis. [15] controlled the robot manipulator arm of Lynx-6 robot by traditional PID controller, Neuro Fuzzy logic controller and compared the results obtained by those methods. These are the limitations of the above works.

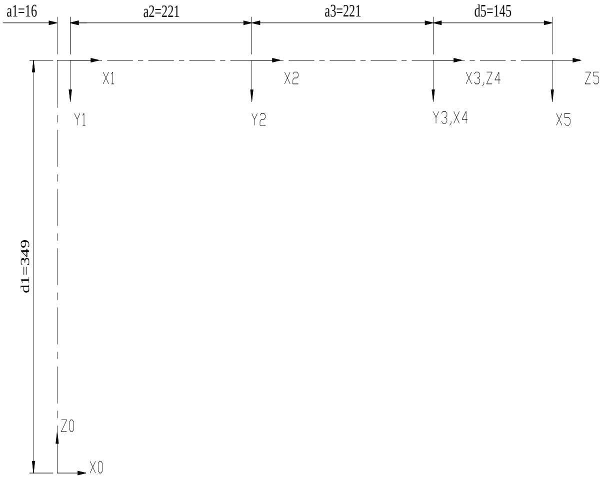

Figure 5: Forward Kinematics Analysis

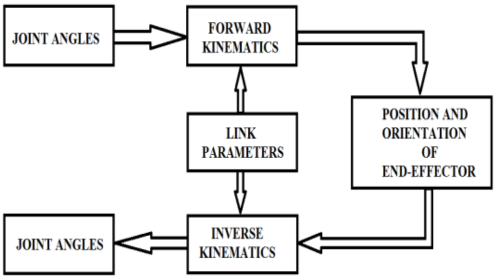

Figure 6: Forward and Inverse Kinematics

1. It is evident from the available literature that the concepts are not well developed for the Reach ability analysis, path and workspace analysis of general serial mechanism. Reason being, the mathematical formulations for these matrices are not much helpful for the numerical computation for kinematics analysis of multi-body mechanism.
2. The end results are not validated using geometric methods.
3. Above methods not user friendly we can't alter the link length parameters and joint angles the proposed method can use unknown users can operate easy way.

[9] Presented the analysis of a simplified model of radial lines stretching motion, the condition for radial linear motion of the end actuator of wafer-handling robot. The dynamic simulation of polar coordinates was realized based on Microsoft Visual Studio 2008 and Pro/toolkit.[14] analysed the direct kinematics of position of non contact legs of a Hexapod Spider-like Mobile Robot[19] established the kinematics model of practical series mechanical arm to act the manipulations with parallel executive mechanism, and solved the problem using D-H transformation.

The 3D model of the arm was created by Pro/E. Movement analysis upon series mechanical arm confirmed its correctness and feasibility, and also offered a theoretical basis to follow-up structural design. [1] investigated the forward kinematics analysis of Scorbot ER V Plus robot as found in literature. But this work has the following major limitations:

1. The experimental and theoretical results are not accurate.
2. For the home position and all the other set of parameters, it gave only approximate results.

[18] Simulated the workspace based on Matlab program using analytical method. [16] used Monte Carlo method to analyze the workspace of an industrial robot and modeled the robot with PRO/E. The relationship between the robot position and joint variables was analyzed. But to verify the correctness of kinematics equations, simulations were not performed. [13] deduced a formulation of modular robot based on D-H and presented the kinematics simulation based on MATLAB. But the workspace was not simulated in this work.

Two user defined algorithms were developed using MATLAB for Joint space and Cartesian space tracking [2]. The limitations of the above works are:

Here straight line movement of the end effectors was only considered. In this only 5 parameters in Cartesian space are considered (x, y, z, roll and pitch) without considering position and orientation vectors while analyzing forward and inverse kinematics. The 3D representation of the path and workspace are not shown in the presentable form transformation matrix.

After establishing D-H coordinate system for each link, a homogeneous transformation matrix can easily be developed considering frame {i-1} and frame {i} transformation consisting of four basic transformations. The overall complex homogeneous matrix of transformation can be formed by consecutive applications of simple transformations. This transformation consists of four basic transformations. **T_1:** A rotation about z_{i-1} axis by an angle θ_i. **T_2:** Translation along z_{i-1} axis by distance d_i. **T_3**.

Translation by distance a_i along x_i axis and **T4:** Rotation by angle α_i about x_i axis. From the transformation matrix, the position and orientation of the end effector is extracted with respect to base. It is given as shown below.

$$
{}^{o}T_1 = \begin{bmatrix} c_1 & 0 & -s_1 & a_1c_1 \\ s_1 & 0 & c_1 & a_1s_1 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}T_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}T_3 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_3 & c_3 & 0 & a_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{3}T_4 = \begin{bmatrix} -s_4 & 0 & c_4 & 0 \\ c_4 & 0 & s_4 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{4}T_5 = \begin{bmatrix} c_5 & -s_5 & 0 & d_5 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

The overall complex homogeneous matrix of transformation is as given below.

$$T_e = {}^{o}T_5 = {}^{o}T_1\,{}^{1}T_2\,{}^{2}T_3\,{}^{3}T_4\,{}^{4}T_5$$

$$
{}^{0}T_{5}=\begin{bmatrix} -s_1s_5-c_1s_{234}c_5 & -s_1c_5+c_1s_{234}s_5 & c_1c_{234} & c_1(a_1+a_2c_2+a_3c_{23}+c_{234}d_5) \\ c_1s_5-s_1s_{234}c_5 & c_1c_5+s_1s_{234}s_5 & s1c_{234} & s_1(a_1+a_2c_2+a_3c_{23}+c_{234}d_5) \\ -c_{234}c_5 & c_{234}s_5 & -s_{234} & d_1-a_2s_2-a_3s_{23}-s_{234}d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

IV. RESULT KINAMATICS

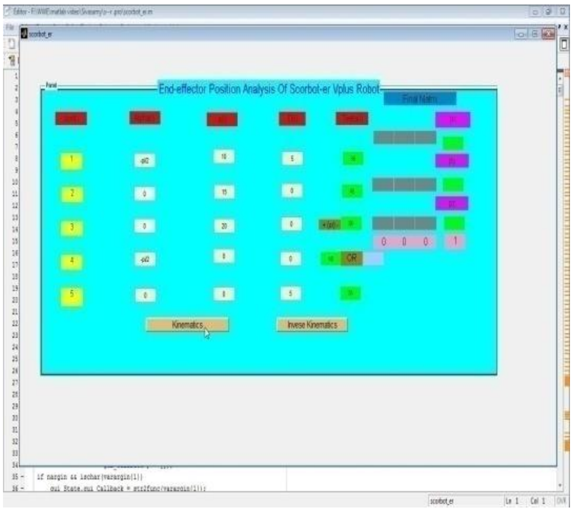

Figure 7: Kinematics Analysis of Scotbot ER 5u Plus

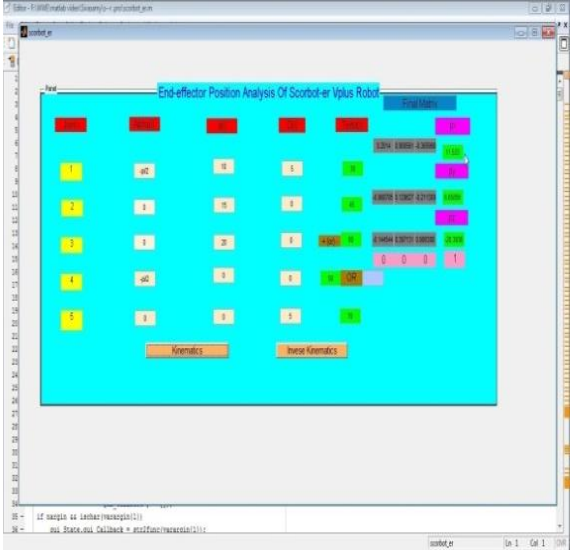

Figure 8: End - Effectors Position of Matrix Values

It is compared value is validated with Robocell software and compared result with Mat LAB Gui based on the [17] forward and inverse kinematics analysis of Scorbot Er 5u Plus using Labview. We have taken reading of 10 arbitrary Theta values of each joints have been taken and end-effectors position and orientation have been recorded that was compared with standard Robocell software and it was compared with Labview result.

V. INVERSE KINEMATICS ANALYSIS

The problem of inverse kinematics analysis had been investigated by researchers as reported in many literatures.[7][11][14][13].The design of controller had been investigated by researchers as reported in some literature. [4][5][15][16][19][11]. Various approaches had been used for inverse kinematics analysis. The analytical procedure for solving an inverse kinematics problem and to determine the joint coordinates of each joint using Matlab program, was already available in literature [1] and [8], [13] using Matlab & Labview. Some researchers have addressed the inverse kinematics problem of serial robot manipulators using conventional techniques like geometric, algebraic and iterative methods. [5][9][16] and some others have solved using widely published methodologies like ANN (Artificial Neural Network), ANFIS (Adaptive Neuro-Fuzzy Inference System), RNN (Random Neural Network), Bio-mimetic approach. [14], [16], [15],[15].A controller based on Inverse Kinematics is designed. This serves the purpose by applying Artificial Neural Network (ANN) and an Adaptive Neuro-Fuzzy Inference System (ANFIS) method. [18]illustrated, a 5 DOF serial robot manipulator with prismatic arm joint and offset wrist with the pre-given uncertain orientation vectors is solved by virtual joint method. Inverse kinematics calculation of arc welding robot is achieved by RBF (Radial Basis Function) of six-input and single output. The forward and inverse kinematics is seen as a nonlinear mapping between the joint space and the operation space of the robot. [18] obtained results of the final end-effectors trajectory errors of the proposed neural network model of the inverse kinematics problem and they are verified by applying proper direct kinematics virtual Labview instrumentation by Olaru. [16]. But inverse kinematics in Labview is not done in their work. Song J. (2012) adopted the D-H method to do the kinematics analysis of 4 DOF articulated egg plant-picking robot and inverse kinematics is solved by using the simplified inverse transformation method. It is shown by tests that the error of the forward kinematics solution is ±1.5mm, while the error of the inverse kinematics solution is ±1.31°. The limitations of the above works are: 1.Time consumption and expensive in implementation. The problems solved in the above works are only suitable for fixed parameters. Labview is not used to solve the problems. Where i =Joint Number, α_i=Twist angle, a_i=link length, d_i=link offset, θ_i=joint angle.

$$
[{}^{o}T_1]^{-1}=\begin{bmatrix} c_1 & s_1 & 0 & -a_1 \\ 0 & 0 & -1 & d_1 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
[{}^{1}T_2]^{-1}=\begin{bmatrix} c_2 & s_2 & 0 & -a_2 \\ -s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
[{}^{2}T_3]^{-1}=\begin{bmatrix} c_3 & s_3 & 0 & -a_3 \\ -s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
[{}^{3}T_4]^{-1}=\begin{bmatrix} -s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ c_4 & s_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
[{}^{4}T_5]^{-1}=\begin{bmatrix} c_5 & s_5 & 0 & 0 \\ -s_5 & c_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Using matrix equality, the inverse kinematics equations are obtained as explained in the following steps.

Base Joint Angle

The overall complex homogeneous matrix of transformation is taken from the kinematics final equation as said below.

$$T_e = {}^oT_5 = {}^oT_1\,{}^1T_2\,{}^2T_3\,{}^3T_4\,{}^4T_5$$

$${}^oT_1\,{}^1T_2\,{}^2T_3\,{}^3T_4\,{}^4T_5 = {}^oT_5$$

$$\text{LHS}=\begin{bmatrix} -s_1s_5-c_1s_{234}c_5 & -s_1c_5+c_1s_{234}s_5 & \mathbf{c_1c_{234}} & \mathbf{c_1(a_1+a_2c_2+a_3c_{23}+c_{234}d_5)} \\ c_1s_5-s_1s_{234}c_5 & c_1c_5+s_1s_{234}s_5 & \mathbf{s_1c_{234}} & \mathbf{s_1(a_1+a_2c_2+a_3c_{23}+c_{234}d_5)} \\ -c_{234}c_5 & c_{234}s_5 & -s_{234} & d_1-a_2s_2-a_3s_{23}-s_{234}d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$, $c_{i\,jk} = \cos(\theta_i + \theta_j + \theta_k)$ and $s_{ijk} = \sin(\theta_i + \theta_j + \theta_k)$

$$\text{RHS} = \begin{bmatrix} n_x & o_x & \mathbf{a_x} & \mathbf{p_x} \\ n_y & o_y & \mathbf{a_y} & \mathbf{p_y} \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

θ_1 can be isolated by dividing the second and first elements of fourth column and found out.

$$s_1\,(a_1 + a_2c_2 + a_3c_{23} + c_{234}d_5) = p_y$$

$$c_1\,(a_1 + a_2c_2 + a_3c_{23} + c_{234}d_5) = p_x$$

$$s_1 / c_1 = t_1 = p_y / p_x$$

$$\theta_1 = \text{atan2}\,(p_y, p_x) \qquad (1)$$

Alternatively

θ_1 can be isolated by dividing the second element and first element of third column and found out.

$$c_1c_{234} = a_x$$

$$s_1c_{234} = a_y$$

$$s_1 / c_1 = t_1 = a_y / a_x$$

$$\theta_1 = \text{atan2}\,(a_y, a_x) \qquad (2)$$

Wrist Roll Joint Angle

The overall complex homogeneous matrix of transformation is modified to find the wrist roll joint θ_5. ${}^oT_1\,{}^1T_2\,{}^2T_3\,{}^3T_4\,{}^4T_5 = {}^oT_5$

${}^1T_2 \times {}^2T_3 \times {}^3T_4 \times {}^4T_5 = {}^1T_5 = [{}^oT_1]^{-1} \times {}^oT_5$ $[{}^oT_1]^{-1} \times {}^oT_5 = {}^1T_5$

$$\text{LHS} = \begin{bmatrix} c_1n_x + s_1n_y & c_1o_x+s_1o_y & c_1a_x+s_1a_y & c_1p_x+s_1p_y-a_1 \\ -n_z & -o_z & -a_z & -p_z+d_1 \\ \mathbf{-s_1n_x+c_1n_y} & \mathbf{-s_1o_x+c_1o_y} & -s_1a_x+c_1a_y & -s_1p_x+c_1p_y \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{RHS} = \begin{bmatrix} -s_{234}c_5 & s_{234}s_5 & c_{234} & c_{234}d_5+a_3c_{23}+a_2c_2 \\ c_{234}c_5 & -c_{234}s_5 & s_{234} & s_{234}d_5+a_3s_{23}+a_2s_2 \\ \mathbf{s_5} & \mathbf{c_5} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

θ_5 can be isolated by dividing the first and second elements of third row and found out.

$$s_5 = -s_1n_x + c_1n_y$$

$$c_5 = -s_1o_x + c_1o_y$$

$$t_5 = s_5 / c_5 = (-s_1n_x + c_1n_y) / (-s_1o_x + c_1o_y)$$

$$\theta_5 = \text{atan2}\,(-s_1n_x + c_1n_y, -s_1o_x + c_1o_y) \qquad (3)$$

Sum of Shoulder, Elbow and Wrist Pitch Joint Angles

From the same matrix equality the sum of shoulder, elbow and wrist pitch joint angles can be found in the following steps.

$$[{}^oT_1]^{-1} \times {}^oT_5 = {}^1T_5$$

$$\text{LHS}=\begin{bmatrix} c_1n_x + s_1n_y & c_1o_x+s_1o_y & \mathbf{c_1a_x+s_1a_y} & c_1p_x+s_1p_y-a_1 \\ -n_z & -o_z & \mathbf{-a_z} & -p_z+d_1 \\ -s_1n_x+c_1n_y & -s_1o_x+c_1o_y & -s_1a_x+c_1a_y & -s_1p_x+c_1p_y \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{RHS} = \begin{bmatrix} -s_{234}c_5 & s_{234}s_5 & \mathbf{c_{234}} & c_{234}d_5+a_3c_{23}+a_2c_2 \\ c_{234}c_5 & -c_{234}s_5 & \mathbf{s_{234}} & s_{234}d_5+a_3s_{23}+a_2s_2 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$(\theta_2+\theta_3+\theta_4)$ can be isolated by dividing the second and first elements of third column and found out.

$$s_{234} = -a_z$$

$$c_{234} = c_1a_x + s_1a_y$$

$$t_{234} = s_{234} / c_{234} = -a_z / (c_1a_x + s_1a_y)$$

$$(\theta_2+\theta_3+\theta_4) = \text{atan2}\,(-a_z, c1ax + s_1a_y) \qquad (4)$$

Elbow Joint Angle

Elbow joint angle θ_3 can be found as follows.

$[{}^oT_1]^{-1} \times {}^oT_5 = {}^1T_5$

$$\text{LHS} = \begin{bmatrix} c_1n_x + s_1n_y & c_1o_x+s_1o_y & c_1a_x+s_1a_y & \mathbf{c_1p_x+s_1p_y-a_1} \\ -n_z & -o_z & -a_z & \mathbf{-p_z+d_1} \\ -s_1n_x+c_1n_y & -s_1o_x+c_1o_y & -s_1a_x+c_1a_y & -s_1p_x+c_1p_y \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{RHS} = \begin{bmatrix} -s_{234}c_5 & s_{234}s_5 & c_{234} & \mathbf{c_{234}d_5+a_3c_{23}+a_2c_2} \\ c_{234}c_5 & -c_{234}s_5 & s_{234} & \mathbf{s_{234}d_5+a_3s_{23}+a_2s_2} \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

θ_{23} and θ_2 are separated from the first and second elements of fourth column for convenience.

$c_{234}d_5 + \mathbf{a_3}c_{23} + \mathbf{a_2}c_2 = c_1p_x + s_1p_y - a_1$
$s_{234}d_5 + \mathbf{a_3}s_{23} + \mathbf{a_2}s_2 = -p_z + d_1$

We know that in this case $a_2 = a_3$

$c_{234}d_5 + a_3\,(c_{23}+c_2) = c_1p_x + s_1p_y - a_1$
$s_{234}d_5 + a_3\,(s_{23}+s_2) = -p_z + d_1$

Rearranging the Equations in the following steps,

$a_3\,(c_{23}+c_2) = c_1p_x + s_1p_y - a_1 - c_{234}d_5$
$a_3\,(s_{23}+s_2) = -p_z + d_1 - s_{234}d_5$
$c_{23}+c_2 = (c_1p_x+s_1p_y-a_1-c_{234}d_5)/a_3 \qquad (5)$
$s_{23}+s_2 = (-p_z+d_1-s_{234}d_5)/a_3 \qquad (6)$

To eliminate $c_{3,}$ squaring and adding Equation (5) and Equation (6) we get,

$(s_{23}+s_2)^2 + (c_{23}+c_2)^2 = ((c_1p_x + s_1p_y-a_1-c_{234}d_5)/\,a_3)^2 + ((-p_z+d_1-s_{234}d_5)/\,a_3)^2$

$s_{23}^2+s_2^2+2s_{23}s_2+c_{23}^2+c_2^2+2c_{23}\,c_2 = ((c_1p_x + s_1p_y-a_1-c_{234}d_5)/\,a_3)^2 + ((-p_z+d_1-s_{234}d_5)/\,a_3)^2$ $s_{23}^2+c_{23}^2+s_2^2+c_2^2+2(s_{23}\,s_2+c_{23}\,c_2) = ((c_1p_x + s_1p_y-a_1-c_{234}d_5)/\,a_3)^2 + ((-p_z+d_1-s_{234}d_5)/\,a_3)^2$

$2+2(s_{23}\,s_2+c_{23}\,c_2) = ((c_1p_x + s_1p_y-a_1-c_{234}d_5)/\,a_3)^2 + ((-p_z+d_1-s_{234}d_5)/\,a_3)^2$

$s_{23}s_2+c_{23}\,c_2 = \{((c_1p_x + s_1p_y-a_1-c_{234}d_5)/\,a_3)^2 + ((-p_z+d_1-s_{234}d_5)/\,a_3)^2-2\}/2$

We know that,

Cos (A-B) = Cos A Cos B + Sin A Sin B

Let us consider $A = \theta_2 + \theta_3$, $B = \theta_2$

$\mathrm{Cos}\,(\theta_2+\theta_3-\theta_2) = \mathrm{Cos}\,(\theta_2+\theta_3)\,\mathrm{Cos}\,\theta_2 + \mathrm{Sin}\,(\theta_2+\theta_3)\,\mathrm{Sin}\,\theta_2$

$c_3 = \{((c_1p_x + s_1p_y-a_1-c_{234}d_5)/\,a_3)^2+ ((-p_z+d_1-s_{234}d_5)/\,a_3)^2-2\}/2$

$\theta_3=a*\cos\ (\{((c_1p_x+s_1p_y-a_1-c_{234}d_5)/a_3)^2+\ ((-p_z+d_1-s_{234}d_5)/a_3)^2-2\}/2)$ (7)

Shoulder Joint Angle

Shoulder joint angle θ_2

We know that,

cos (A+B) = cos A cos B – sin A sin B

sin (A+B) = sin A cos B + cos A sin B

Let us consider $A = \theta_2,\ B = \theta_3$

$c_{23} = c_2c_3 - s_2s_3$

$s_{23} = s_2c_3 + c_2s_3$

From equations (5) and (6) we get,

$c_2c_3 - s_2s_3+c_2= (c_1p_x + s_1p_y-a_1-c_{234}d_5)/\,a_3$

$s_2c_3+c_2s_3+s_2 = (-p_z+d_1-s_{234}d_5)/\,a_3$

$(c_3+1)\,c_2 - s_3s_2= (c_1p_x+s_1p_y-a_1-c_{234}d_5)/a_3$ (8)

$s_3c_2+ (c_3+1)\,s_2 = (-p_z+d_1-s_{234}d_5)/a_3$ (9)

Reduce equations (8) and (9) to find s_2,

(8) x a_3s_3

$a_3s_3\,(c_3+1)\,c_2 - a_3s_3^2s_2=s_3\,(c_1p_x + s_1p_y-a_1-c_{234}d_5)$ (10)

(9) x $a_3\,(c_3+1)$:

$a_3s_3\,(c_3+1)c_2+a_3\,(c_3+1)^2s_2=(c_3+1)(-p_z+d_1-s_{234}d_5)$ (11)

To eliminate s_2, subtract equation (10) from Equation (11)

$((c_3+1)^2+s_3^2)\ a_3s_2= (c_3+1)\ (-p_z+d_1-s_{234}d_5)-\ s_3\ (c_1p_x + s_1p_y-a_1-c_{234}d_5))$

$s_2= (c_3+1)\ (-p_z+d_1-s_{234}d_5)\ -\ s_3\ (c_1p_x + s_1p_y-a_1-c_{234}d_5)/\ a_3\ ((c_3+1)^2+s_3^2)$ again reduce Equations (8) and (9) to find c_2

(8) x $a_3\,(c_3+1)$:

$a_3\,(c_3+1)^2c_2 - a_3\,(c_3+1)\,s_3s_2 = (c_3+1)\,(c_1p_x + s_1p_y-a_1-c_{234}d_5)$ (12)

(9) x a_3s_3:

$a_3s_3^2c_2+ a_3\,(c_3+1)\,s_2s_3 \quad =s_3(-p_z+d_1-s_{234}d_5)$ (13)

To eliminate c_2, add Equation (12) and Equation (13)

$a_3\ c_2\ ((c_3+1)^2+\ s_3^2) = (c_3+1)\ (c_1p_x + s_1p_y-a_1-c_{234}d_5) + s_3\ (-p_z+d_1-s_{234}d_5)$

$c_2 = ((c_3+1)\ (c_1p_x + s_1p_y-a_1-c_{234}d_5) + (s_3\ (-p_z+d_1-s_{234}d_5))/\ a_3\ ((c_3+1)^2+ s_3^2)$

$t_2 = s_2\,/\,c_2= (c_3+1)\ (-p_z+d_1-s_{234}d_5)\ -\ s_3\ (c_1p_x + s_1p_y-a_1-c_{234}d_5)/\ ((c_3+1)\ (c_1p_x + s_1p_y-a_1-c_{234}d_5) + (s_3\ (-p_z+d_1-s_{234}d_5))$

$\theta_2=\ \mathrm{atan2}\ ([((c_3+1)\ (-p_z+d_1-s_{234}d_5)/\ a_3\ )-\ (s_3\ (c_1p_x + s_1p_y-a_1-c_{234}d_5)/\ a_3)/\ \ ((c_3+1)^2+s_3^2)\ ],[(\ (c_3+1)\ (c_1p_x + s_1p_y-a_1-c_{234}d_5)/\ a_3)+\ (s_3\ (-p_z+d_1-s_{234}d_5)/\ a_3)\ /((c_3+1)^2+s_3^2)])$ (14)

Wrist Pitch Joint Angle

Wrist pitch joint angle is calculated as follows:

$\theta_4= \theta_{234}-\theta_2-\theta_3$

VI. RESULT INVERSE KINEMATICS

Thus Forward and inverse kinematics Analysis of Scorbot ER 5u Plus using "Labview" and using Matlab. It compared value is validated with Robocell software and compared result with Matlab GUI.

We have taken reading with 10 random end-effectors position and orientation & link length parameters we can find joint angles are taken from standard Robocell software and the result was compared with Labview results along with Matlab GUI.

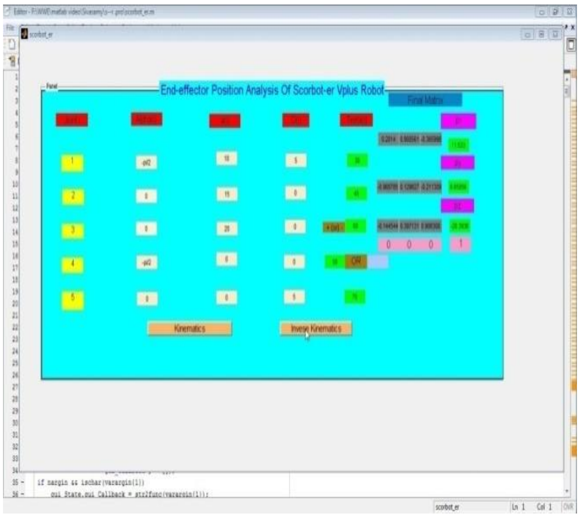

Figure 9: Inverse Kinamatics of Scorbot ER 5u Plus

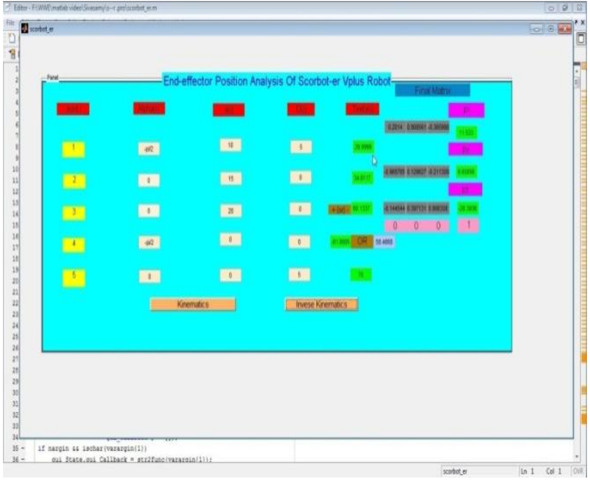

Figure 10: Joint Angle Values of Scorbot ER 5u Plus

VII. CONCLUSION

Thus Scorbot ER Vu Plus Forwarded and inverse kinematics analysis perform in mathematically and experimental result analysis of forward kinematics and inverse kinematics using Mat lab GUI and result was validate with Rob cell and also compared with Lab view result. Finally we concluded that Mat lab GUI modelled Scorbot ER Vu Plus result good compare to Lab view modelled. It's graphically designed and easier to use, and has the ability to solve any type of value, the person unknown and new to robotics the can easily utilising this software.

REFERENCES

1. Anurag Verma & Vivek A. Deshpande, 2011. "End-effector Position Analysis of Scorbot ER Vu Plus Robot", International Journal of Smart Home, Vol.5, No. 1, January, 2011.
2. Alba Perez, J.M. McCarthy, "Geometric design of RRP, RPR and PRR Serial Chains", Mechanism and Machine Theory, Vol.40, Issue.11, pp.1294–1311, November 2005
3. Cai, H.M. and T.T. Xing, "Kinematics simulation of industrial robot based on MATLAB". Advanced Materials and Research, pp.415-417: 690-696, 2012.
4. Ceccarelli M. and Lanni C., A Multi-objective Optimum Design of General 3R Manipulators for Prescribed Workspace Limits, Mechanism and Machine Theory, Vol. 39, pp. 119-132, 2004.

5. Chiang-Ju Chien a, Abdelhamid Tayebi, “Technical Communiqué Further Results on Adaptive Iterative Learning Control of Robot Manipulators”, Automatica, Vol.44, pp. 830–837, 2008
6. Chaudhary, H. and R. Prasad, “Intelligent Inverse Kinematics Control of Scorbot- ER Vu Plus Robot Manipulator”, International Journal of Advanced Engineering and Technology, Vol.1, Issue 5, pp.158-169, 2011.
7. Chaudhary, H., R. Prasad and N. Sukavanum, “Trajectory Tracking Control of Scorbot ER Vu Plus Robot Manipulator Based on Kinematical Approach”. International Journal of Advanced Engineering and Technology, Vol 4: pp.1174-1182, 2012.
8. Chris T. Freeman, “Constrained Point-to-Point Iterative Learning Control with Experimental Verification”, Control Engineering Practice, Vol. 20, pp.489–498,2012
9. Ch Ravi Kumar, Dr.K R Sudha, D V Pushpalatha, “Modelling and Control of 5DOF Robot Arm Using Neuro-Fuzzy Controller”, International Journal of Engineering Research & Technology (IJERT) ISSN: 2278-0181 Vol. 1 Issue 7, September – 2012.
10. Denavit, J., R. S. Hartenberg, “A Kinematic Notation for Lower-Pair Mechanisms Based on Matrices,” ASME Journal of Applied Mechanics, June 1955, 215-221.
11. Deshpande, V.A. and P.M. George, “Analytical solution for inverse kinematics of Scorbot ER Vu Plus, International Journal of Emerging Technology Advanced. Engineering, Vol. 2, pp. 478-481, 2012.
12. Duffy J., Analysis of Mechanism and Robot Manipulators. New York, USA: Wiley, 1980
13. Feng, Y., W. Yao-Nan and Y. Yi-Min, “Inverse Kinematics Solution for Robot Manipulator Based on Neural Network under Joint Subspace”, International Journal of Computers Communication and Control, Vol.7, pp. 460-473, 2012.
14. Featherstone R, "Position and Velocity Transformation between Robot End-Effector Coordinate and Joint Angle," International Journal of Robotics, Vol. 2, No. 2, pp. 35-45, 1983.
15. Gan, J.Q, Gan, J.Q., E. Oyama, E.M. Rosales and H. Hu, 2005. A Complete Analytical Solution to the Inverse Kinematics of the Pioneer 2 Robotic Arm, Robotica, Vol. 23, pp. 123-129, 2005.
16. Hanming Cai, and Tingting Xing, 2012. “Kinematics Simulation of Industrial Robot Based on Matlab” Advanced Materials Research, Volumes. 415-417, Page Number 690-696.
17. Robocell Software, Intelitek (Formally Eshed Robotec), 2007. http://www.intelitek.com.
18. S. Navaneetha Santhakumar “Forward Kinematics Analysis of SCORBOT ER Vu Plus Using Labview”, EJSR, ISBN; 1450-216X.Vol:72, No: 4, (2012), pp.549-557.
19. S. Navaneetha Santhakumar [Ph. D, Thesis] Anna University Chennai “Forwarded Kinematics and Inverse Kinematics Analysis of Scorbot ER Vu Plus using Labview” 2013.
20. Prabhu and Dev Anand. M “Forward and Inverse Kinematic Analysis of Scorbot-ER Vu Plus Manipulator” 2012.
21. Prabhu & Dev Anand. M “Complete Dynamic Analysis of Scorbot-ER Manipulator using Labview” 2013
22. RP Paul, B. Shimano, and GE Mayer, "Kinematic Control Equations for Simple Manipulators," IEEE Transaction on System, Man and Cybernetics, IEEE, Vol. SMC-11, No. 6, pp. 66-72, 1982.
23. S. B. Niku, Introduction to Robotics Analysis, Systems, Applications, Upper Saddle River, NJ: Prentice Hall, 2001, pp 67-71, 92-94.
24. Scorbot-ER Vu plus User’s Manual, 3rd Edition, Intelitek Inc., Catalog#100016 Rev. C, February 1996.
25. The MATLAB Toolbox for the Intelitek Scorbot (MTIS), http://www.usna.edu/Users/weapsys/esposito/scorbot.Matlab/.