Stability Analysis of one Prey and Two Predators Model

B. Sita Ram Babu , Paparao. A. V , N. V. S. R. C. Murty Gamini

Abstract: *We projected a 3 species Ecological model with a Prey and 2 predators. Distributed kind of delay is incorporated within the interaction of prey and second predator is taken for investigation. The system dynamics is studied at its interior equilibrium purpose with exponential kind of delay kernels. The impact of your time delay on the energising behavior of the system is studied exploitation Numerical simulation. it's shown that the delay arguments with totally different delay parameters exhibit wealthy dynamics.*

Keywords: *co-existing state, local stability, global stability, Time delay, Numerical simulation.*

I. INTRODUCTION

Mathematical modeling in Ecology gains importance in recent decades. The stability analysis of ecosystems is quite intersecting and complex in nature. Differential equations are widely used in the stability analysis. Braun [8] and Simon's [9] explain the applications of differential equations in this area. Lokta [1] and Volterra [2] studied the different models in population ecology.Kapur [3, 4] discussed the models in biology, medicine, epidemiology, ecology etc. May [5], Freedman [6],Paul colinvaux [7] contributed a lot to this field. Time delays are natural in ecological phenomenon. The stability analysis of time delay models are widely explained by Cushing, J.M [10], Sreehari Rao [11], Gopalaswamy.K[12]. Time delay in interactions in three species models with a prey, predator and competitor models are discussed by paparao [13-17]. In spite of that a single prey with two predator model is taken for investigation. The model is represented by system of integro-differential equations and system dynamics is studied at co-existing state. Numerical simulation is allotted in support of stability analysis using MAT lab simulation.

II. MATHEMATICAL MODEL

A three species ecological model with a single prey and two predators are considered for investigation. Two predators namely first predator (N_2), second predator (N_3) are competing for the same prey (N_1) . A time delay is introduced in the interaction of prey and second predator (Gestation period of the prey) .Death rates of three populations are also considered for investigation.Keeping the above aspects in view, the model is characterized by the following system of integro- differential equations.

$$\frac{dN_1}{dt} = a_1N_1 - \alpha_{11}N_1^2 - \alpha_{12}N_1N_2 - \alpha_{13}N_1\int_{-\infty}^{t} k_1(t-u)N_3(u)du - d_1N_1$$

$$\frac{dN_2}{dt} = a_2N_2 - \alpha_{22}N_2{}^2 + \alpha_{21}N_2N_1 - \alpha_{23}N_2N_3 - d_2N_2$$

$$\frac{dN_3}{dt} = a_3N_3 - \alpha_{33}N_3^2 + \alpha_{31}N_3\int_{-\infty}^{t} k_2(t-u)N_1(u)du - \alpha_{32}N_2N_3 - d_3N_3$$

(2.1)

Where the parameters in the above model is described as follows

A. Nomenclature:

S.No	Parameter	Description
1	N_1, N_2 & N_3	Population strengths of three populations (prey, first predator and second predator) respectively
2	a_1, a_2, a_3	Growths rates of three populations
3	α_{ii} $(i=1,2,3)$	Inter species competitions rates of three species (negative values)
4	α_{12}	Prey and first predator interaction rate (negative value)
5	α_{21}	First predator and prey interaction rate (positive value)
6	α_{23}	First and second predators interaction rate (negative value)
7	α_{32}	Second and first predators interaction rate (negative value)
7	α_{13}	Prey and second predator interaction rate (negative value)
9	α_{31}	Second predator and prey interaction rate (positive value)
10	d_1, d_2, d_3	Death rates of three populations
11	$k_1(t-u)$ & $k_2(t-u)$	kernel weights

Put t-u = z , we get the following system of equations

Revised Version Manuscript Received on 16 September, 2019

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$$\frac{dN_1}{dt} = a_1N_1 - \alpha_{11}N_1^2 - \alpha_{12}N_1N_2 - \alpha_{13}N_1\int_0^\infty k_1(z)N_3(t-z)dz - d_1N_1$$
$$\frac{dN_2}{dt} = a_2N_2 - \alpha_{22}{N_2}^2 + \alpha_{21}N_2N_1 - \alpha_{23}N_2N_3 - d_2N_2$$
$$\frac{dN_3}{dt} = a_3N_3 - \alpha_{33}N_3^2 + \alpha_{31}N_3\int_0^\infty k_2(z)N_1(t-z)dz - \alpha_{32}N_2N_3 - d_3N_3$$
(2.2)

Kernels can be chosen of exponential type

III. EQUILIBRIUM STATES:

Solving the system of equations (2.1) by equating to zero we get the equilibrium point is given by

E_1 : Co-existing state

$$\overline{N}_1 = \frac{(a_1-d_1)(\alpha_{22}\alpha_{33}-\alpha_{23}\alpha_{32})+(a_2-d_2)(\alpha_{13}\alpha_{32}-\alpha_{12}\alpha_{33})+(a_3-d_3)(\alpha_{12}\alpha_{23}-\alpha_{13}\alpha_{22})}{\alpha_{11}(\alpha_{22}\alpha_{33}-\alpha_{23}\alpha_{32})+\alpha_{12}(\alpha_{21}\alpha_{33}-\alpha_{31}\alpha_{23})+\alpha_{13}(\alpha_{31}\alpha_{22}-\alpha_{21}\alpha_{32})}$$
$$\overline{N}_2 = \frac{(a_1-d_1)(\alpha_{21}\alpha_{33}-\alpha_{31}\alpha_{23})+(a_2-d_2)(\alpha_{11}\alpha_{33}+\alpha_{13}\alpha_{31})-(a_3-d_3)(\alpha_{11}\alpha_{23}+\alpha_{13}\alpha_{21})}{\alpha_{11}(\alpha_{22}\alpha_{33}-\alpha_{23}\alpha_{32})+\alpha_{12}(\alpha_{21}\alpha_{33}-\alpha_{31}\alpha_{23})+\alpha_{13}(\alpha_{31}\alpha_{22}-\alpha_{12}\alpha_{32})}$$
,
$$\overline{N}_3 = \frac{(a_1-d_1)(\alpha_{22}\alpha_{31}-\alpha_{21}\alpha_{32})-(a_2-d_2)(\alpha_{11}\alpha_{32}+\alpha_{12}\alpha_{31})+(a_3-d_3)(\alpha_{11}\alpha_{22}+\alpha_{12}\alpha_{21})}{\alpha_{11}(\alpha_{22}\alpha_{33}-\alpha_{23}\alpha_{32})+\alpha_{12}(\alpha_{21}\alpha_{33}-\alpha_{31}\alpha_{23})+\alpha_{13}(\alpha_{31}\alpha_{22}-\alpha_{12}\alpha_{32})}$$
. (3.1)

This equilibrium state exist only when, $\overline{N}_1 > 0, \overline{N}_2 => 0\ , \overline{N}_3 > 0$ (3.11)

IV. STABILITY OF THE EQUILIBRIUM POINT E1:

Theorem: The interior equilibrium point $E_1\left(\overline{N_1}, \overline{N_2}, \overline{N_3}\right)$ is locally asymptotically stable

Proof: Let the variational matrix is given by

$$J = \begin{bmatrix} -\alpha_{11}\overline{N_1} & -\alpha_{12}\overline{N_1} & -\alpha_{13}\overline{N_1}k_1^*(\lambda) \\ \alpha_{21}\overline{N_2} & -\alpha_{22}\overline{N_2} & -\alpha_{23}\overline{N_2} \\ \alpha_{31}\overline{N_3}k_2^*(\lambda) & -\alpha_{32}\overline{N_3} & -\alpha_{33}\overline{N_3} \end{bmatrix}$$
(4.1)

With The characteristic equation $\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0$ (4.2)

Where

$$b_1 = \left(\alpha_{11}\overline{N_1} + \alpha_{22}\overline{N_2} + \alpha_{33}\overline{N_3}\right)$$
$$b_2 = \left(\alpha_{11}\alpha_{22} + \alpha_{12}\alpha_{21}\right)\overline{N_1}\,\overline{N_2} + \left(\alpha_{11}\alpha_{33} + \alpha_{13}\alpha_{31}k_1(\lambda)k_2(\lambda)\right)\overline{N_1}\,\overline{N_3} + \left(\alpha_{22}\alpha_{33} - \alpha_{23}\alpha_{32}\right)\overline{N_2}\,\overline{N_3}$$
$$b_3 = \overline{N_1}\,\overline{N_2}\,\overline{N_3}\begin{pmatrix} \alpha_{11}\alpha_{22}\alpha_{33} + \alpha_{12}\alpha_{21}\alpha_{33} \\ +\alpha_{13}\alpha_{22}\alpha_{31}k_1(\lambda)k_2(\lambda) \\ -\alpha_{11}\alpha_{23}\alpha_{32} - \alpha_{12}\alpha_{23}\alpha_{31}k_2(\lambda) \\ -\alpha_{13}\alpha_{21}\alpha_{32}k_1(\lambda) \end{pmatrix}$$
(4.3)

By Routh-Hurwitz criteria, the system is stable if $b_1 > 0$, $\left(b_1b_2 - b_3\right) > 0$ and $b_3\left(b_1b_2 - b_3\right) > 0$.

Clearly

$$b_1 = \left(\alpha_{11}\overline{N_1} + \alpha_{22}\overline{N_2} + \alpha_{33}\overline{N_3}\right) > 0$$

By algebraic calculations

$$\begin{aligned}\left(b_1b_2 - b_3\right) = {} & \left(\alpha_{11}{}^2\alpha_{22} + \alpha_{11}\alpha_{12}\alpha_{21}\right)\overline{N_1}^2\overline{N_2} + \left(\alpha_{11}{}^2\alpha_{33} + \alpha_{11}\alpha_{13}\alpha_{31}k_1(\lambda)k_2(\lambda)\right)\overline{N_1}^2\overline{N_3} \\ & + \left(\alpha_{22}{}^2\alpha_{33} - \alpha_{22}\alpha_{23}\alpha_{32}\right)\overline{N_2}^2\overline{N_3} + \left(\alpha_{22}{}^2\alpha_{11} + \alpha_{22}\alpha_{12}\alpha_{21}\right)\overline{N_2}^2\overline{N_1} \\ & + \left(\alpha_{11}\alpha_{33}{}^2 + \alpha_{33}\alpha_{13}\alpha_{31}k_1(\lambda)k_2(\lambda)\right)\overline{N_3}^2\overline{N_1} + \left(\alpha_{22}\alpha_{33}{}^2 - \alpha_{33}\alpha_{23}\alpha_{32}\right)\overline{N_2}\,\overline{N_3}^2 \\ & + \overline{N_1}\,\overline{N_2}\,\overline{N_3}\left(2\alpha_{11}\alpha_{22}\alpha_{33} + \alpha_{12}\alpha_{23}\alpha_{31}k_2(\lambda) + \alpha_{13}\alpha_{21}\alpha_{32}k_1(\lambda)\right)\end{aligned}$$

$\left(b_1b_2 - b_3\right) > 0$ (Majority of the terms are positive)

(4.4)

Also $b_3\left(b_1b_2 - b_3\right) > 0$

Hence the interior equilibrium point $E_1\left(\overline{N_1}, \overline{N_2}, \overline{N_3}\right)$ is locally asymptotically stable

V. GLOBAL STABILITY

Theorem: Theinterior equilibrium point $E_1\left(\overline{N_1}, \overline{N_2}, \overline{N_3}\right)$ is globally asymptotically stable

Proof: Let the Lyapunov function be

$$V\left(N_1, N_2, N_3\right) = \sum_{i=1}^{3} N_i - \overline{N_i} - \overline{N_i}\log\left(\frac{N_i}{\overline{N_i}}\right)$$
$$+\frac{1}{2}\alpha_{13}\int_0^\infty k_1(z)\int_{t-z}^{t}[N_3 - \overline{N_3}]^2 dudz$$
$$+\frac{1}{2}\alpha_{31}\int_0^\infty k_2(z)\int_{t-z}^{t}[N_1 - \overline{N_1}]^2 dudz \qquad (5.1)$$

The time derivate of 'V' along the solutions of equations (2.1) is

$$V^1(t) = \sum_{i=1}^{3}\frac{\left[N_i - \overline{N_i}\right]}{N_i}N_i^1$$
$$+\frac{1}{2}\alpha_{13}\int_0^\infty k_1(z)\left[N_3 - \overline{N_3}\right]^2 dz$$
$$-\frac{1}{2}\alpha_{13}\int_0^\infty k_1(z)\left[N_3(t-z) - \overline{N_3}\right]^2 dz$$
$$+\frac{1}{2}\alpha_{31}\int_0^\infty k_2(z)\left[N_1 - \overline{N_1}\right]^2 dz \qquad (5.2)$$
$$-\frac{1}{2}\alpha_{31}\int_0^\infty k_2(z)\left[N_1(t-z) - \overline{N_1}\right]^2 dz$$

From the equation (2.1) we have

$$V^1(t)=\left[N_1-\overline{N_1}\right]\begin{pmatrix}a_1-\alpha_{11}N_1-\alpha_{12}N_2\\ -\alpha_{13}\int_0^\infty k_1(z)N_3(t-z)dz-d_1\end{pmatrix}$$

$$+\left[N_2-\overline{N_2}\right]\begin{pmatrix}a_2-\alpha_{22}N_2\\ +\alpha_{21}N_1-\alpha_{23}N_3-d_2\end{pmatrix}$$

$$+\left[N_3-\overline{N_3}\right]\begin{pmatrix}a_3-\alpha_{32}N_3\\ +\alpha_{31}\int_0^\infty k_2(z)N_1(t-z)dz-\alpha_{32}N_2-d_3\end{pmatrix}$$

$$+\frac{1}{2}\alpha_{31}\left[N_1-\overline{N_1}\right]^2$$

$$+\frac{1}{2}\alpha_{13}\left[N_3-\overline{N_3}\right]^2$$

$$-\frac{1}{2}\alpha_{13}\int_0^\infty k_1(z)\left[N_3(t-z)-\overline{N_3}\right]^2dz$$

$$-\frac{1}{2}\alpha_{31}\int_0^\infty k_2(z)\left[N_1(t-z)-\overline{N_1}\right]^2dz$$

By proper choice of $a_1, a_2 \,\&\, a_3$

$$a_1=\alpha_{11}\overline{N_1}+\alpha_{12}\overline{N_2}$$

$$+\alpha_{13}\int_0^\infty k_2(z)N_3(t-z)dz+d_1$$

$$a_2=-\alpha_{21}\overline{N_1}+\alpha_{22}\overline{N_2}$$

$$+\alpha_{23}\overline{N_3}+d_2$$

$$\&a_3=-\alpha_{31}\int_0^\infty k_2(z)N_1(t-z)dz$$

$$+\alpha_{33}\overline{N_3}+\alpha_{32}\overline{N_2}+d_3$$

$$=-\alpha_{11}\left(N_1-\overline{N_1}\right)^2-\alpha_{22}\left(N_2-\overline{N_2}\right)^2$$

$$-\alpha_{33}\left(N_3-\overline{N_3}\right)^2$$

$$-\left(\alpha_{32}+\alpha_{23}\right)\left(N_2-\overline{N_2}\right)\left(N_3-\overline{N_3}\right)$$

$$+\left(\alpha_{21}-\alpha_{12}\right)\left(N_2-\overline{N_2}\right)\left(N_1-\overline{N_1}\right)$$

$$+\frac{1}{2}\alpha_{31}\left[N_1-\overline{N_1}\right]^2+\frac{1}{2}\alpha_{13}\left[N_3-\overline{N_3}\right]^2$$

$$-\frac{1}{2}\alpha_{13}\int_0^\infty k_1(z)\left[N_3(t-z)-\overline{N_3}\right]^2dz$$

$$-\frac{1}{2}\alpha_{31}\int_0^\infty k_2(z)\left[N_2(t-z)-\overline{N_2}\right]^2dz \qquad (5.3)$$

Using the inequality $ab\le\frac{a^2+b^2}{2}$,

$$\int_0^\infty k_1(z)\left[N_3(t-z)-\overline{N_3}\right]^2\le\int_0^\infty k_1(z)dz=1$$

$$\int_0^\infty k_2(z)\left[N_2(t-z)-\overline{N_2}\right]^2\le\int_0^\infty k_2(z)dz=1,$$

$$=-\alpha_{11}\left(N_1-\overline{N_1}\right)^2-\alpha_{22}\left(N_2-\overline{N_2}\right)^2$$

$$-\alpha_{33}\left(N_3-\overline{N_3}\right)^2$$

$$-\frac{\left(\alpha_{32}+\alpha_{23}\right)}{2}\left[\left(N_2-\overline{N_2}\right)^2+\left(N_3-\overline{N_3}\right)^2\right]$$

$$+\frac{1}{2}\alpha_{31}\left[N_1-\overline{N_1}\right]^2$$

$$+\frac{1}{2}\alpha_{13}\left[N_3-\overline{N_3}\right]^2$$

$$+\frac{\left(\alpha_{21}-\alpha_{12}\right)}{2}\left[\left(N_2-\overline{N_2}\right)^2+\left(N_1-\overline{N_1}\right)^2\right]$$

$$-\frac{1}{2}\left(\alpha_{31}+\alpha_{13}\right)$$

$$\le-\left\|\left(\alpha_{11}+\frac{1}{2}\alpha_{31}+\frac{1}{2}\alpha_{21}-\frac{1}{2}\alpha_{12}\right)\right\|\left(N_1-\overline{N_1}\right)^2$$

$$-\left\|\left(\alpha_{22}-\frac{1}{2}\alpha_{12}+\frac{1}{2}\alpha_{21}-\frac{1}{2}\alpha_{32}-\frac{1}{2}\alpha_{23}\right)\right\|\left(N_2-\overline{N_2}\right)^2$$

$$-\left\|\left(\alpha_{33}+\frac{1}{2}\alpha_{13}-\frac{1}{2}\alpha_{32}-\frac{1}{2}\alpha_{23}\right)\right\|\left(N_3-\overline{N_3}\right)^2-\frac{1}{2}\left\|\left(\alpha_{31}+\alpha_{13}\right)\right\|$$

$$V^1(t)\le-\mu\sum_{i=1}^{3}\left[N_i-\overline{N_i}\right]^2<0$$

Where $\mu=\min\begin{pmatrix}\alpha_{11}+\alpha_{22}+\alpha_{33}+\frac{1}{2}\alpha_{13}\\ +\frac{1}{2}\alpha_{31}+\frac{1}{2}\alpha_{21}-\frac{1}{2}\alpha_{12}\\ -\frac{1}{2}\left(\alpha_{31}+\alpha_{13}\right)\end{pmatrix}$

$$\frac{dV}{dt}<0$$

Therefore the system is globally stable at interior equilibrium $E_1\left(\overline{N_1},\overline{N_2},\overline{N_3}\right)$

VI. NUMERICAL EXAMPLE

Graphs Description:

S.No	Figures	Description
1	The figures(A)	Shows the variation of N_1, N_2 and N_3 with respect to Time (t)
2	The figures(B)	The phase portrait of N_1, N_2 and N_3

.

Example 1: Let a_1=6, α_{11}=0.01, α_{12}=0.45, α_{13}=0.3, a_2=2.5, α_{21}=0.43, α_{22}=0.1, α_{23}=0.32, a_3=3, α_{31}=0.01, α_{32}=0.12, α_{33}=0.23, d_1 = 0.02, d_2 = 0.02, d_3 = 0.03, N1=15, N_2 =15, N_3 =15.

The systems of equations (2.1) are simulated using MATLAB using ode45.The system of equations without delay is solved with the same package we get the following results illustrated by the graphs 6.1(A), 6.1(B) for the following parametric values:

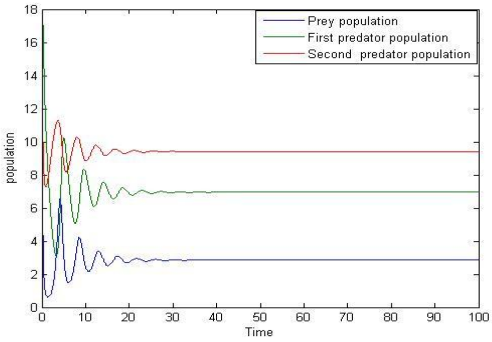


Fig:6. 1(A)

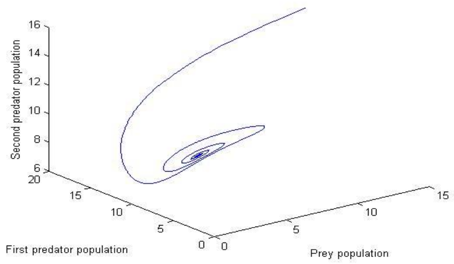


Fig: 6.1(B)

For the above mentioned parametric values, the three populations asymptotic to the fixed equilibrium point E (3, 7,10). Hence the system is asymptotically stable.

Exponential function is given by

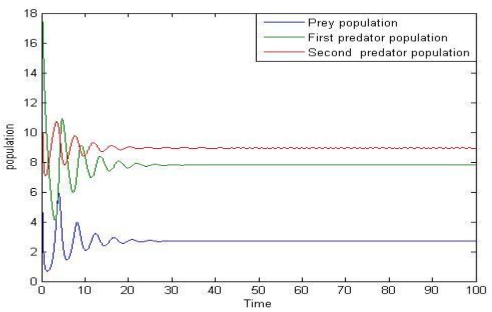
$$k_1(z) = k_2(z) = ae^{-az} \quad for \;\; a > 0$$

Then the Laplace transform of $k_1(z) \,\&\, k_2(z)$ are defined

as $k_1(\lambda) = k_2(\lambda) = \int_0^\infty e^{-\lambda t} a e^{-at} dt = \dfrac{a}{a+\lambda}$

The results are simulated for the above system of equations (2.3) Using MAT LAB simulation with the parameters shown in Example 1 with different kernel values are plotted below.

1. λ=0.5, a= 5 E (3, 8, 9)



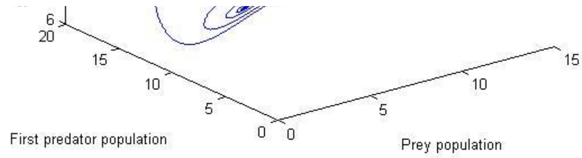


Fig: 6.2(A)

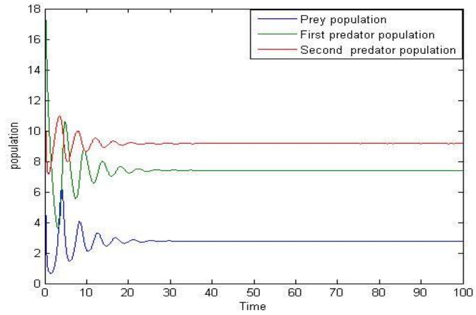


Fig: 6.2(B)

The system is asymptotically stable to a fixed equilibrium point E (3,8,9) .For fixed value of λ=0.5, as on a value increases from 1 to 100, still the system is asymptotically stable to the fixed equilibrium point. For λ=0.5, a= 150, the system is asymptotic to a fixed equilibrium E (3,7,10) .

2. λ=10, a= 0.5E (3, 8, 10)

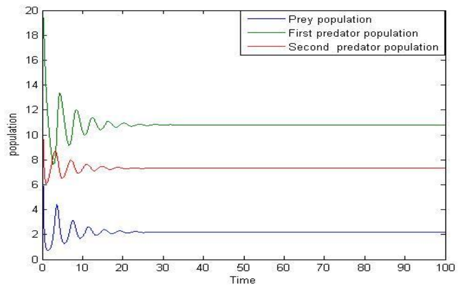
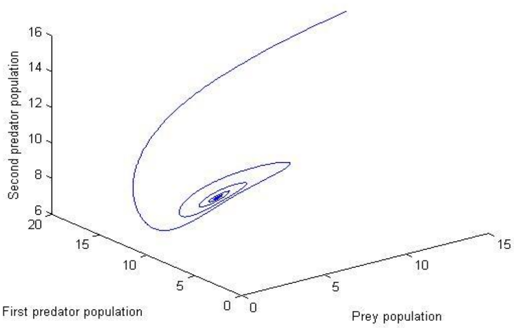


Fig: 6.3(A)

The system is asymptotically stable to a fixed equilibrium point E (3, 8, 10). For fixed value of λ=10, as on a value



increases from 1 to 100, still the system is asymptotically stable to the fixed equilibrium point. For λ=10, a= 150, the system is asymptotic to a fixed equilibrium E (2, 13, 7)

3. λ=0.5, a= 0.5 E (3, 11, 8)

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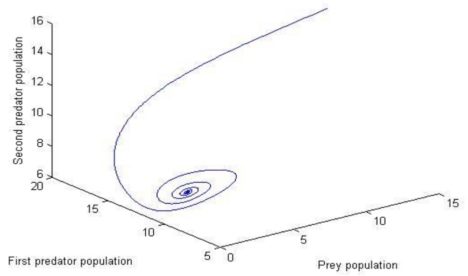

Fig: 6.4(B)

The system is asymptotically stable to a fixed equilibrium point E (3,11,8)

4. λ=0.5, a= 1 E (3, 9, 9)

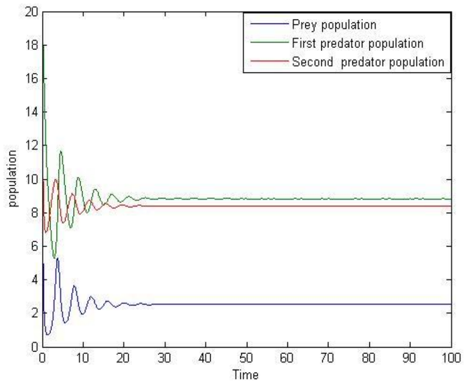

Fig: 6.5(A)

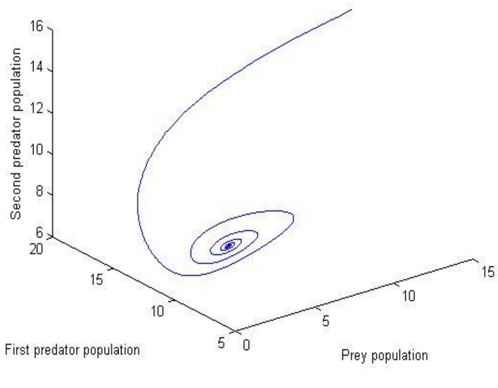

Fig: 6.5(B)

The system is asymptotically stable to a fixed equilibrium point E (3,9,9)

VII. CONCLUSION

A three species ecological model with a single prey and two Predators is considered for investigation. Here two predators are competing for the same Prey. The time delay is imposed on the prey and second predator species. The co-existing state is identified . The system is asymptotically stability . The global stability is studied by Lyapunov's function. The dynamics of the system is studied using numerical simulation in support of stability analysis. We consider numerical example with delay and without delay agreements. The system is asymptotically stable if there is no delay impact. The impact of delay with different kernel strength is studied and observed that thesystems are asymptotically stable, exhibit periodic solutions and limit cycles. For (i) λ=10, a= 150 (ii) λ=0.5 , a= 0.5 (iii) λ=0.5, a= 1 , there is a significant growth in the first predator population .So delay play a significant role in system dynamics.

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