Intuitionistic Fuzzy Translation INK-ideal in INK-algebra

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Abstract.; The ideas of intuitionistic fuzzy translation in INKI's in INK-algebras are presented. The view of intuitionistic fuzzy extensions (IF_ E) and (IF_ M) intuitionistic fuzzy multiplications of fuzzy INK-ideals per more than a few related assets are examined. As well, the associations sandwiched between IF_ translations, IF_ E and IF_ M of IF_ INKIs are studied.

Key words. INK-algebra, IF_INK-ideal, IF_ translation, IF_ extension, IF_ multiplication.

I. INTRODUCTION

Lee et al. as well Jun deliberated F_ translations, F_ extensions too F_ multiplications of F_ sub algebras in addition ideals in BCK/BCI-algebras. They well-thought-out kindred mid F_ T, F_ E and F_ M. In this paper, IF_ translations, IF_ E and IF_ M of F_INK-I in INK-algebras stay conferred. Relatives midst IF_ translations, IF_ M and IF_ M of FINK_I in INK-algebras stand as well examined.

II. PRELIMINARIES

An algebra (Ḱ; ⋄, 0) is named a INK-algebra uncertainty it mollifies the ensuing situations,

- ((x ⋄ □) ⋄ (x ⋄ z)) ⋄ (r ⋄ □) = 0
- ((x ⋄ z) ⋄ (□ ⋄ z)) ⋄ (x ⋄ □) = 0
- x ⋄ 0 = x
- x ⋄ □ = 0 and □ ⋄ x = 0 imply x = □, for all x, □, z in Ḱ

A F_S ʎ in a INK-algebra Ḱ is named a F_ INK-subalgebra of Ḱ, if

ʎ (x ⋄ y) ≥ min{ʎ(x), ʎ(y)}, ∀ x, q ∈ Ḱ.

Let (Ḱ, ⋄, 0) be a INK-algebra. A subclass □ of Ḱ is named a INK-subalgebra of Ḱ if the constant 0 of Ḱ is in □, and (□, ⋄, 0) itself procedures a INK-algebra.

A F_S ʎ in a INK-algebra Ḱ is named a F_ INK- ideal of Ḱ, if,

- ʎ (0) ≥ ʎ(x)

ʎ(x) ≥ min {ʎ (z ⋄ x) ⋄ (z ⋄ □), ʎ(□)}, ∀ x, □, z in Ḱ.

Definition 2.1 *An IF_ set A in a non-void set Ḱ is an object having the form A = {(x, $ʎ_A$(x), ξ_A(x)) / x ∈ Ḱ }, where $ʎ_A$: Ḱ → [0,1] and ξ_A: Ḱ → [0,1] denote (namely $ʎ_A$(x)) the degree of membership and (namely ξ_A(x)) the degree of non-membership of Ḱ to the set A correspondingly, and 0 ≤ $ʎ_A$(x) + ξ_A(x) ≤ 1, for all x ∈ Ḱ. For the take simplicity, then A= (x, $ʎ_A$, ξ_A) for the IF_ set A = {(x, $ʎ_A$(x), ξ_A(x)) / x in Ḱ}.*

Definition 2.2 *An IF_ set A= (x, $ʎ_A$, ξ_A) is named an IF_ sub algebra of Ḱ if it satisfies,*

i) $ʎ_A$ (x * □) ≥ min {$ʎ_A$ (x), $ʎ_A$ (□)},

ii) ξ_A (x * □) ≤ ma x {ξ_A (x), ξ_A (□)}, for all x, □ in Ḱ.

III. MAIN RESULTS

3.1. Intuitionistic Fuzz□ $\tilde{\alpha}$-Translation of IFINK-ideal.

Let A = ($ʎ_A$, ξ_A) for the IF_S is denoted by A= { ⟨x, $ʎ_A$(x), ξ_A (x)| x ∈ Ḱ ⟩ }. We yield Ț = 1– inf { ξ A (x)| p in Ḱ} for any A= ($ʎ_A$, ξ_A) of Ḱ.

Definition 3.1.1

Let A= ($ʎ_A$, ξ_A) be an IFS of Ḱ & let $\tilde{\alpha}$, in [0, T]. An object having the form

$A_\alpha{}^T = \left((ʎ_A)_\alpha{}^T, (\xi_A)_\alpha{}^T\right)$ is named an IF-$\tilde{\alpha}$-translation of A if $(ʎ_A)_\alpha{}^T(x) = ʎ_A(x) + \alpha$ and $(\xi_A)_\alpha{}^T(x) = \xi_A(x) - \alpha$, for every x in Ḱ.

Theorem.3.1.2 Let A = ($ʎ_A$, ξ_A) is an IFINK-ideal of Ḱ, the IF_ $\tilde{\alpha}$_ translation $A^T_{\tilde{\alpha}}$ of A is a IFINK-sub algebra of Ḱ.

Proof. Let x, □ ε Ḱ,

$(ʎ_A)^T_{\tilde{\alpha}}$(x ⋄ □) = $ʎ_A$(x ⋄ □) +α̃

= min {$ʎ_A$(z ⋄ (x ⋄ □)) ⋄ (z ⋄ □), $ʎ_A$(□)} +α̃

= min {$ʎ_A$((x ⋄ □)) ⋄ □), $ʎ_A$ (□)} + α̃

=min {$ʎ_A$(0), ʎ (□)} +α̃

≥ min {$ʎ_A$(x), ʎ (□)} +α̃

$(ʎ_A)^T_{\tilde{\alpha}}$(x ⋄ □) ≥ min { $(ʎ_A)^T_{\tilde{\alpha}}$(x), $(ʎ_A)^T_{\tilde{\alpha}}$(□)}

And

$(\xi_A)^T_{\tilde{\alpha}}$(x ⋄ □) = ξ_A(x ⋄ □) +α̃

= max {ξ_A(z ⋄ (x ⋄ □)) ⋄ (z⋄ □), ξ_A(□)}-α̃

= max {ξ_A((x ⋄ □)) ⋄ □), ξ_A (□)}-α̃

= max {ξ_A (0), ξ_A(□)}-α̃

≤ max {ξ_A (x), ξ_A (□)}-α̃

$(\xi_A)^T_{\tilde{\alpha}}$(x ⋄ □) ≤ ma x { $(\xi_A)^T_{\tilde{\alpha}}$(x), $(\xi_A)^T_{\tilde{\alpha}}$(□)

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Theorem.3.1.3 Let IRAT $A^T_{\tilde{\alpha}}$ of A be a IRID of Ḱ for $\tilde{\alpha}$ ε [0, I], then the ensuing is valid,

i) If $x \leq \square$, $(\lambda_A)^T_{\tilde{\alpha}}(x) \geq (\lambda_A)^T_{\tilde{\alpha}}(\square)$ & $(\xi_A)^T_{\tilde{\alpha}}(x) \leq (\xi_A)^T_{\tilde{\alpha}}(\square)$.

Proof. Let x, □ ε Ḱ, $x \leq \square$, $x \diamond \square = 0$, and this

$$(\lambda_A)^T_{\tilde{\alpha}}(x) \geq \min\{(\lambda_A)^T_{\tilde{\alpha}}(x \diamond \square), (\lambda_A)^T_{\tilde{\alpha}}(\square)\}$$
$$= \min\{(\lambda_A)^T_{\tilde{\alpha}}(0), (\lambda_A)^T_{\tilde{\alpha}}(a)\}$$
$$(\lambda_A)^T_{\tilde{\alpha}}(x) \geq (\lambda_A)^T_{\tilde{\alpha}}(a)$$

and

$$(\xi_A)^T_{\tilde{\alpha}}(x) \leq \max\{(\xi_A)^T_{\tilde{\alpha}}(x \diamond \square), (\xi_A)^T_{\tilde{\alpha}}(\square)\}$$
$$= \max\{(\xi_A)^T_{\tilde{\alpha}}(0), (\xi_A)^T_{\tilde{\alpha}}(a)\}$$
$$(\xi_A)^T_{\tilde{\alpha}}(x) \leq (\xi_A)^T_{\tilde{\alpha}}(a).$$

Theorem.3.1.4 If Let $A = (\lambda_A, \xi_A)$ is an IF_ Ss of Ḱ, such that the IF_ $\tilde{\alpha}$ – translation $A^T_{\tilde{\alpha}}=((\lambda_A)^T_{\tilde{\alpha}}, (\xi_A)^T_{\tilde{\alpha}})$ of A is an IFINK_ I of Ḱ for $\tilde{\alpha}$ ε [0, I], then $\tau_{\lambda A} = \{ x / x$ ε Ḱ and $(\lambda_A)^T_{\tilde{\alpha}}(x) = (\lambda_A)^T_{\tilde{\alpha}}(0)\}$ & $\tau_{\xi A} = \{ x / x$ ε Ḱ and $(\xi_A)^T_{\tilde{\alpha}}(x) = (\xi_A)^T_{\tilde{\alpha}}(0)\}$ are INK_ I of Ḱ .

Proof. $A^T_{\tilde{\alpha}} = ((\lambda_A)^T_{\tilde{\alpha}}, (\xi_A)^T_{\tilde{\alpha}})$ of A is an IF_ ideal of X. Then, $(\lambda_A)^T_{\tilde{\alpha}}$ and $(\xi_A)^T_{\tilde{\alpha}}$ are IFINK_ I of Ḱ. Clearly

0 in $\tau_{\lambda A}$, $\tau_{\Xi a}$

Let x, □, z ε Ḱ be $((z\diamond x) \diamond (z\diamond \square))$ ε $\tau_{\lambda A}$ □ ε $\tau_{\lambda A}$,

$$(\lambda_A)^T_{\tilde{\alpha}}(z\diamond x) \diamond (z\diamond \square) = (\lambda_A)^T_{\tilde{\alpha}}(0)$$
$$(\lambda_A)^T_{\tilde{\alpha}}(\square) = (\lambda_A)^T_{\tilde{\alpha}}(0)$$
$$(\lambda_A)^T_{\tilde{\alpha}}(x) \geq \min\{(\lambda_A)^T_{\tilde{\alpha}}(z\diamond x) \diamond (z\diamond \square), (\lambda_A)^T_{\tilde{\alpha}}(\square)\}$$
$$= \min\{(\lambda_A)^T_{\tilde{\alpha}}(0), (\lambda_A)^T_{\tilde{\alpha}}(0)\}$$
$$(\lambda_A)^T_{\tilde{\alpha}}(x) = (\lambda_A)^T_{\tilde{\alpha}}(0)$$

Since, $(\lambda_A)^T_{\tilde{\alpha}}$ is a fuzzy INK– ideal

$$(\lambda_A)^T_{\tilde{\alpha}}(0) \geq (\lambda_A)^T_{\tilde{\alpha}}(x)$$
$$(\lambda_A)^T_{\tilde{\alpha}}(x) = \{(\lambda_A)^T_{\tilde{\alpha}}(0)$$

This implies

$$\lambda_A(x) + \tilde{\alpha} = \lambda_A(0) + \tilde{\alpha} \text{ (or)}$$

$\lambda_A(x) = \lambda_A(0)$. So x ε $\tau_{\lambda A}$ therefore $\tau_{\lambda A}$ is an INK-ideal.

Yet again, $(a\diamond b) \diamond c$ ε $\tau_{\xi A}$ & b ε τ_{ξ},

$$(\xi_A)^T_{\tilde{\alpha}}(c\diamond a) \diamond (c\diamond b) = (\xi_A)^T_{\tilde{\alpha}}(0) \text{ and,}$$
$$(\xi_A)^T_{\tilde{\alpha}}(b) = (\xi_A)^T_{\tilde{\alpha}}(0) \text{ \&}$$
$$(\xi_A)^T_{\tilde{\alpha}}(a) \leq \max\{(\xi_A)^T_{\tilde{\alpha}}(c\diamond a) \diamond (c\diamond b), (v_A)^T_{\tilde{\alpha}}(b)\}$$
$$= \max\{(\xi_A)^T_{\tilde{\alpha}}(0), (\xi_A)^T_{\tilde{\alpha}}(0)\}$$
$$(\xi_A)^T_{\tilde{\alpha}}(x) = (\xi_A)^T_{\tilde{\alpha}}(0)$$

$(\xi_A)^T_{\tilde{\alpha}}$ is a F_ INK_ I

$$(\xi_A)^T_{\tilde{\alpha}}(0) \leq (\xi_A)^T_{\tilde{\alpha}}(x)$$
$$(\xi_A)^T_{\tilde{\alpha}}(x) = \{(\xi_A)^T_{\tilde{\alpha}}(0)$$

This implies $\xi_A(a) - \tilde{\alpha} = \xi_A(0) - \tilde{\alpha}$ (or)

$\xi_A(a) = \xi_A(0)$ so that a ε $\tau_{\xi A}$ therefore $\tau_{\lambda A}$ is an INK-ideal.

Theorem.3.1.5 Let the IF_ $\tilde{\alpha}$-translation $A^T_{\tilde{\alpha}} = ((\lambda_A)^T_{\tilde{\alpha}}, (\xi_A)^T_{\tilde{\alpha}})$ of A is an IFINK-ideal of Ḱ, for all $\tilde{\alpha}$ in [0, 1],

If $x \geq \square$, $(\lambda_A)^T_{\tilde{\alpha}}(x) \geq (\lambda_A)^T_{\tilde{\alpha}}(\square)$ & $(\xi_A)^T_{\tilde{\alpha}}(x) \leq (\xi_A)^T_{\tilde{\alpha}}(\square)$.

Proof. Let x, □ ε Ḱ.. $x \leq \square$, $x \diamond \square = 0$

we have,

$$(\lambda_A)^T_{\tilde{\alpha}}(x) \geq \min\{(\lambda_A)^T_{\tilde{\alpha}}(z\diamond x) \diamond (z\diamond \square), (\lambda_A)^T_{\tilde{\alpha}}(0)\}$$
$$= \min\{(\lambda_A)^T_{\tilde{\alpha}}(x \diamond \square), (\lambda_A)^T_{\tilde{\alpha}}(\square)\}$$
$$= \min\{(\lambda_A)^T_{\tilde{\alpha}}(0), (\lambda_A)^T_{\tilde{\alpha}}(\square)\}$$
$$(\lambda_A)^T_{\tilde{\alpha}}(x) = (\lambda_A)^T_{\tilde{\alpha}}(\square)$$

And,

$$(\xi_A)^T_{\tilde{\alpha}}(x) \leq \max\{(\xi_A)^T_{\tilde{\alpha}}(z\diamond x) \diamond (z\diamond \square), (\xi_A)^T_{\tilde{\alpha}}(0)\}$$
$$= \max\{(\xi_A)^T_{\tilde{\alpha}}(x \diamond \square), (\xi_A)^T_{\tilde{\alpha}}(\square)\}$$
$$= \max\{(\xi_A)^T_{\tilde{\alpha}}(0), (\xi_A)^T_{\tilde{\alpha}}(\square)\}$$
$$(\xi_A)^T_{\tilde{\alpha}}(x) = (\xi_A)^T_{\tilde{\alpha}}(\square).$$

3.2 PROPERTIES ON IFINK-IDEAL EXTENSION.

Definition.3.2.1 Let $A= (\lambda_A, \xi_A)$ and $B= (\lambda_B, \xi_B)$ be two IF_ sub set of Ḱ.. If $A \leq B$. $\lambda_A(x) \leq \lambda_B(x)$, $\xi_A(x) \geq \xi_B(x)$ for all x ε Ḱ, then B is IF_ extension of A.

Definition.3.3.2 Let $A= (\lambda_A, \xi_A)$ and $B= (\lambda_B, \xi_B)$ be two IF _ subsets of Ḱ. Then B is an IFINK_IE of A is the ensuing statements are effective.

i) B is an IF_ E of A

ii) If A is an IFINK- I of Ḱ, the B is an IFINK-I of Ḱ.

From the definition of IF_ $\tilde{\alpha}$-translation, $(\lambda_A)^T_{\tilde{\alpha}}(x) = \lambda_A(x)+\tilde{\alpha}$ and $(\xi_A)^T_{\tilde{\alpha}}(x) = \xi_A(x) - \tilde{\alpha}$ for each x ε Ḱ.

Theorem.3.3.3 Let $A = (\lambda_A, \xi_A)$ be an IFINK-ideal of Ḱ. and $\tilde{\alpha}$ in [0, τ]. Then the IF_ $\tilde{\alpha}$_ translation $A^T_{\tilde{\alpha}}=((\lambda_A)^T_{\tilde{\alpha}}, (\xi_A)^T_{\tilde{\alpha}})$ of A is an IFINK-ideal of A. An IFINK-ideal A ma□ not be denoted as an IF_ $\tilde{\alpha}$_ translation of A, that is the reverse of Theorem is not correct in universal as understood in the ensuing example.

Example.3.3.4 Let Ḱ. = {0, a, b, 1, 2} be a INK-algebra per the ensuing Cayley table

⋄	0	a	b	1	2
0	0	0	0	0	0
a	a	0	a	0	0
b	b	2	0	0	0
1	1	1	1	0	0
2	2	1	2	a	0

Let $A= (\lambda_A, \xi_A)$ be an IFSs of Ḱ, clearly by

Ḱ	0	a	b	1	2
μ_A	0.62	0.54	0.45	0.31	0.31
ξ_A	0.17	0.22	0.30	0.45	0.45

Then $A = (\lambda_A, \xi_A)$ be an IFINK-ideal of Ḱ.

Let $B = (\lambda_B, \xi_B)$ be an IF_ subset of Ḱ, defined b□

Ḱ	0	a	b	1	2
λ_B	0.64	0.60	0.48	0.35	0.35
ξ_B	0.15	0.18	0.27	0.41	0.41

$A= (\lambda_A, \xi_A)$ be an IFINK_ I extension of A. But it is not the IF_ $\tilde{\alpha}$-Translation $A^T_{\tilde{\alpha}}=((\lambda_A)^T_{\tilde{\alpha}}, (\xi_A)^T_{\tilde{\alpha}})$ of A for all $\tilde{\alpha}$ in [0, τ]. If $\tilde{\alpha} = 0.16$, $\tilde{\alpha} = 0.16 > 0.13= \lambda$. And the IF_ $\tilde{\alpha}$-translation $A^T_{\tilde{\alpha}}=(\lambda_A)^T_{\tilde{\alpha}}, (\xi_A)^T_{\tilde{\alpha}})$ of A is assumed as follows

Ḱ	0	a	b	1	2
$(\lambda_A)^T_{\tilde{\alpha}}$	0.71	0.58	0.50	0.58	0.50
$(\xi_A)^T_{\tilde{\alpha}}$	0.27	0.41	0.52	0.41	0.52

Therefore, $(\lambda_A)^T_{\tilde{\alpha}} + (\xi_A)^T_{\tilde{\alpha}} \neq 1$. Therefore, B is not the IF_ $\tilde{\alpha}$-translation $A^T_{\tilde{\alpha}}=(\lambda_A)^T_{\tilde{\alpha}}, (\xi_A)^T_{\tilde{\alpha}})$ of A for all $\tilde{\alpha}$ ε [0, τ].

Theorem.3.3.5 Let A= (λ_A, ξ_A) be an IFINK-I of Ḱ and $\tilde{\alpha}$ in [0, τ]. Then the IF_ $\tilde{\alpha}$-translation $A^T_{\tilde{\alpha}}=((\lambda_A)^T_{\tilde{\alpha}}, (\xi_A)^T_{\tilde{\alpha}})$ of A is an IFINK_ I extension of A. Then intersection of IFINK_ I of Ḱ is an IFINK_ I extension of A. But the union of IFI NK_ I extensions of an IFINK_I extension of as understood in the ensuing.

Example.3.3.6 Let A= (λ_A, ξ_A) be an IFINK-ideal of Ḱ and $\tilde{\alpha}$ ε [0, τ]. Then the IF_ $\tilde{\alpha}$-translation $A^T_{\tilde{\alpha}}=(\lambda_A)^T_{\tilde{\alpha}}, (\xi_A)^T_{\tilde{\alpha}})$ of A is an IFINK-ideal extension of Ḱ. Then intersection of IFINK-ideal extension of an IF_INK-ideal A of Ḱ is an IFI NK- ideal extension of A. But the union of IFINK-extensions of an IF_INK-A of Ḱ is not an IFINK-extension of a as seen in the following example.

Example.3.3.7 Let Ḱ = {0, 1, 2, 3, 4} be a INK-algebra by means of the succeeding Cayley table.

$\diamond$	0	a	b	1	2
0	0	0	0	0	0
a	a	0	a	0	0
b	b	2	0	0	b
1	1	2	a	0	b
2	2	a	2	a	0

Let A= (λ_A, ξ_A) be an IF_ Ss of Ḱ, well-defined by

Ḱ	0	a	b	1	2
λ_A	0.53	0.31	0.31	0.31	0.31
ξ_A	0.25	0.48	0.48	0.48	0.48

Before A= (λ_A, ξ_A) be an IFINK_ I of Ḱ

Let B = (λ_B, ξ_B) and C= (λ_C, ξ_C) be an IF_ Ss of Ḱ correspondingly by

Ḱ	0	a	b	1	2
λ_B	0.66	0.56	0.33	0.33	0.56
ξ_B	0.12	0.23	0.44	0.44	0.23

And

Ḱ	0	a	b	1	2
λ_C	0.68	0.61	0.47	0.41	0.41
ξ_C	0.11	0.35	0.31	0.35	0.35

Then B & C are IFINK-I of A

1) The intersection B∩C is an IF_ extension of A,

$\lambda_{B\cap C}(1) \geq \min\{\lambda_{B\cap C}(0\diamond 1)\diamond(0\diamond 1), \lambda_{B\cap C}(1)\}$

$\min\{\lambda_B(1), \lambda_C(1)\} \geq \min\{\lambda_{B\cap C}(0), \lambda_{B\cap C}(0)\}$

$\min\{0.33, 0.41\} \geq \min\{0.66, 0.33\}$

$0.33 = 0.33$

$\xi_{B\cap C}(1) \leq \max\{\xi_{B\cap C}(0\diamond 1)\diamond(0\diamond b), \xi_{B\cap C}(b)\}$

$\max\{0.44, 0.35\} \leq \max\{\xi_{B\cap C}(0), \xi_{B\cap C}(b)\}$

$\max\{0.44, 0.35\} \leq \max\{0.12, 0.44\}$

$0.44 = 0.44$

2) The union BUC is an IF_ E of A, it is not an IF_ E of A

$\lambda_{BUC}(1) \geq \min\{\lambda_{BUC}(0\diamond 1)\diamond(0\diamond b), \lambda_{BUC}(b)\}$

$\min\{0.33, 0.31\} \geq \min\{\lambda_{BUC}(0), \lambda_{BUC}(b)\}$

$\min\{0.41\} \geq \min\{0.68, 0.47\}$

$0.41 \not\geq 0.47$

$\xi_{BUC}(1) \leq \max\{\xi_{BUC}(0\diamond 1)\diamond(0\diamond b), \xi_{BUC}\, b)\}$

$\max\{0.35, 0.44\} \leq \max\{\xi_{BUC}(0), \xi_{BUC}(b)\}$

$\max\{0.35\} \leq \max\{0.11, 0.31\}$

$0.35 \not\leq 0.31$

Theorem.3.3.8 If A is an IFINK-ideal of Ḱ, formerly it is pure that $U_{\tilde{\alpha}}(\lambda_{\tilde{\alpha}}; t)$ and $L_{\tilde{\alpha}}(\xi_{\tilde{\alpha}}; s)$ are INK_ I of Ḱ for all t ε $I_m(\lambda_A)$ & s ε $I_m(\xi_A)$ with t ≥ $\tilde{\alpha}$. But, if we do not give a condition that A is an IFINK_ I of X, $U_{\tilde{\alpha}}(\lambda_{\tilde{\alpha}}; t)$ & $L_{\tilde{\alpha}}(\xi_{\tilde{\alpha}}; s)$ are not INK-ideal of Ḱ as perceived in the succeeding example.

Example.3.3.9 Let Ḱ = {0, 1, 2, 3, 4} be a INK-algebra in example 2 and A= (λ_A, ξ_A) be an IF_ Ss of Ḱ defined by

Ḱ	0	a	b	1	2
λ_A	0.56	0.39	0.21	0.39	0.39
ξ_A	0.23	0.38	0.57	0.38	0.38

$\lambda_A(1) \geq \min\{\lambda_A(1\diamond a)\diamond(1\diamond 0), \lambda_A(0)\}$

$0.39 \geq \min\{\lambda_A(b\diamond 1), \lambda_A(0)\}$

$0.39 \geq \min\{\lambda_A(0), \lambda_A(0)\}$

$0.39 \geq \min\{0.56, 0.56\}$

$0.39 \not\geq 0.56$

$\xi_A(3) \geq \min\{\xi_A(0), \xi_A(0)\}$

$0.28 \geq \min\{0.13, 0.13\}$

$0.28 \not\leq 0.13$

Therefore, A= (λ_A, ξ_A) is not an IFINK-I of Ḱ. For, $\tilde{\alpha}$ = 0.16, t = 0.60 and s = 0.45. We acquire $U_{\tilde{\alpha}}(\lambda_{\tilde{\alpha}}; t) = L_{\tilde{\alpha}}(\xi_{\tilde{\alpha}}; s)$ = {0,1,2,3,4} which are not INK -ideal of Ḱ. Level sets in IF_ $\tilde{\alpha}$-translations

Theorem.3.3.10 Let A= (λ_A, ξ_A) be an IF_ Ss of Ḱ & $\tilde{\alpha}$ ε [0, T], then the IF_ $\tilde{\alpha}$-translation $A^T_{\tilde{\alpha}}=(\lambda_A)^T_{\tilde{\alpha}}, (\xi_A)^T_{\tilde{\alpha}}$ of A is an IFINK-I of Ḱ if and only if $U_{\tilde{\alpha}}(\lambda_{\tilde{\alpha}}; t)$ and $L_{\tilde{\alpha}}(\xi_{\tilde{\alpha}}; s)$ are I NK-I of Ḱ, for t ε $I_m(\lambda_A)$ and s ε $I_m(\xi_A)$ with t ≥ $\tilde{\alpha}$.

Proof. Suppose that $A^T_{\tilde{\alpha}} = (\lambda_A)^T_{\tilde{\alpha}}, (\xi_A)^T_{\tilde{\alpha}}$ of A is an IFINK-ideal of Ḱ.

Then $(\lambda_A)^T_{\tilde{\alpha}}$ and $(\xi_A)^T_{\tilde{\alpha}}$ are IF_ INK of Ḱ, let, t ε $I_m(\lambda_A)$ and s ε $I_m(\xi_A)$ with ≥ $\tilde{\alpha}$.

Since, $(\lambda_A)^T_{\tilde{\alpha}}(0) \geq (\lambda_A)^T_{\tilde{\alpha}}(x)$

$\lambda_A(0) + \tilde{\alpha} = (\lambda_A)^T_{\tilde{\alpha}}(0)$

$\geq (\lambda_A)^T_{\tilde{\alpha}}(x)$

$= (\lambda_A)^T_{\tilde{\alpha}}(x) + \tilde{\alpha}$

≥ t, x ε $U_{\tilde{\alpha}}(\lambda_{\tilde{\alpha}}; t)$

σ ε $U_{\tilde{\alpha}}(\lambda_A; t)$

Let, x, □, z ε X, (z⋄ x) ⋄ (z⋄ □), □ ε $U_{\tilde{\alpha}}(\lambda_A; t)$

$\lambda_A((z\diamond x)\diamond(z\diamond \square)) \geq t-\tilde{\alpha}$ & $\lambda_A(\square) \geq t-\tilde{\alpha}$

$(\lambda_A)^T_{\tilde{\alpha}}((z\diamond x)\diamond(z\diamond \square)) = \lambda_A((z\diamond x)\diamond(z\diamond \square)) + \tilde{\alpha}$

≥ t

$(\lambda_A)^T_{\tilde{\alpha}}(\square) = \lambda_A(\square) + \tilde{\alpha}$

$(\lambda_A)^T_{\tilde{\alpha}}$ is a F_ INK_ I,

$\lambda_A(x) + \tilde{\alpha} = (\lambda_A)^T_{\tilde{\alpha}}(x)$

$\geq \min\{(\lambda_A)^T_{\tilde{\alpha}}((z\diamond x)\diamond(z\diamond \square)), (\lambda_A)^T_{\tilde{\alpha}}(\square)\}$

= min {t, t} ≥ t.

$\lambda_A((z\diamond x)\diamond(z\diamond \square)) \geq t-\tilde{\alpha}$ so that x ε $U_{\tilde{\alpha}}(\lambda_{\tilde{\alpha}}; t)$,

$U_{\tilde{\alpha}}(\lambda_{\tilde{\alpha}}; t)$ is a INK-I, of Ḱ.

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$(\xi_A)^T_{\tilde\alpha}(0) \le (\xi_A)^T_{\tilde\alpha}(x)$, x ε Ḱ, it becomes that

$\xi_A(0) - \tilde\alpha = (\xi_A)^T_{\tilde\alpha}(0)$

$\le (\xi_A)^T_{\tilde\alpha}(x)$

$= (\xi_A)^T_{\tilde\alpha}(x) - \tilde\alpha$

≤ s, x ε $L_{\tilde\alpha}$ (ξ_A; s)

0 ε $L_{\tilde\alpha}(\xi_A$; s).

Let, x, □, z ε Ḱ. such that (z ⋄ x) ⋄ (z ⋄ □), □ ε $L_{\tilde\alpha}(\xi_A$; s)

$\xi_A((z\diamond x)\diamond(z\diamond □)) \le s + \tilde\alpha$ and $\xi_A(□) \le s + \tilde\alpha$

$(\xi_A)^T_{\tilde\alpha}((z\diamond x)\diamond(z\diamond □)) = \xi_A((z\diamond x)\diamond(z\diamond □)) - \tilde\alpha \le s$

and

$(\xi_A)^T_{\tilde\alpha}(□) = \xi_A(□) - \tilde\alpha$

≤ s

Since, $(\xi_A)^T_{\tilde\alpha}$ is a F_ INK-I, therefore, it gives that.

$\xi_A(x) - \tilde\alpha = (\xi_A)^T_{\tilde\alpha}(x)$

$\le \max\{(\xi_A)^T_{\tilde\alpha}((z\diamond x)\diamond(z\diamond □)), (\xi_A)^T_{\tilde\alpha}(□)\}$

= max {t, t} ≤ s.

$\xi_A((z\diamond x)\diamond(z\diamond □)) \le s + \tilde\alpha$, x ε $L_{\tilde\alpha}(\xi_A$; s)

$L_{\tilde\alpha}(\xi_A$; s) is a INK-I, of Ḱ. Conversely, suppose that $U_{\tilde\alpha}(ƛ_A$; t) & $L_{\tilde\alpha}(\xi_A$; s) are INK- I of Ḱ for t ε $I_m(ƛ_A)$ and s ε $I_m(\xi_A)$ with t ≥ α̃. If them exist U ε Ḱ, such that $(ƛ_A)^T_{\tilde\alpha} < \psi \le (ƛ_A)^T_{\tilde\alpha}(u)$. Then, $ƛ_A(u) \ge \psi - \tilde\alpha$, but $ƛ_A(0) < \psi - \tilde\alpha$. This shows that u ε $U_{\tilde\alpha}(ƛ_A$; t) & 0 Ɇ $U_{\tilde\alpha}(ƛ_A$; t). This is contradiction, and $(ƛ_A)^T_{\tilde\alpha}(0) \ge (ƛ_A)^T_{\tilde\alpha}(x)$ for x ε Ḱ. Over, if them exist t ξ in Ḱ, such that, $(\xi_A)^T_{\tilde\alpha}(0) > X \ge (\xi_A)^T_{\tilde\alpha}(v)$, then $\xi_A(v) \le X + \tilde\alpha$, But, $\xi_A(v) \ge X + \tilde\alpha$. Show that ξ ε $L_{\tilde\alpha}(\xi_A$; s) and 0 Ɇ $L_{\tilde\alpha}(\xi_A$; s). This is contradiction & $\xi_A(0) \le (\xi_A)^T_{\tilde\alpha}(x)$ for x ε Ḱ.

We undertake that there exists a, b, c ε Ḱ, $(ƛ_A)^T_{\tilde\alpha}(a) \ge k_1 \le \min\{(ƛ_A)^T_{\tilde\alpha}(c\diamond a)\diamond(c\diamond b), (ƛ_A)^T_{\tilde\alpha}(b)\}$. Then, $(ƛ_A)^T_{\tilde\alpha}(c\diamond a)\diamond(c\diamond b) \ge k_1 - \tilde\alpha$ and $ƛ_A(b) \ge k_1 - \tilde\alpha$. But, $ƛ_A(a) < k_1 - \tilde\alpha$

Hence, (c ⋄ a) ⋄ (c ⋄ b) ε $U_{\tilde\alpha}(ƛ_A$; t) and b ε $L_{\tilde\alpha}(\xi_A$; s). But, a Ɇ $U_{\tilde\alpha}(ƛ_A$; t).

This stays contradiction.

$(ƛ_A)^T_{\tilde\alpha}(a) \ge \min\{(ƛ_A)^T_{\tilde\alpha}(z\diamond x)\diamond(z\diamond □), (ƛ_A)^T_{\tilde\alpha}(□)\}$ for all x, □, z ε Ḱ

Once more, adopt that there exist, x, □, z in Ḱ

$(ƛ_A)^T_{\tilde\alpha}(a) < \eta \le \max\{(\xi_A)^T_{\tilde\alpha}(z\diamond x)\diamond(z\diamond □), (\xi_A)^T_{\tilde\alpha}(□)\}$

Then,

$(\xi_A)^T_{\tilde\alpha}(x) < \eta + \tilde\alpha$ and $\xi_A(□) \le \eta + \tilde\alpha$

But, $\xi_A(v) \ge \eta + \tilde\alpha$

Hence,

(z ⋄ x) ⋄ (z ⋄ □) ε $L_{\tilde\alpha}(\xi_A$; s) and □ ε $L_{\tilde\alpha}(\xi_A$; s)

But, x Ɇ $L_{\tilde\alpha}(\xi_A$; s), which is a contradiction.

Therefore,

$(\xi_A)^T_{\tilde\alpha}(x) \le \min\{(\xi_A)^T_{\tilde\alpha}(z\diamond x)\diamond(z\diamond □), (\xi_A)^T_{\tilde\alpha}(y)\}$ for all x, □, z ε Ḱ.

Thus $A^T_{\tilde\alpha}$ is an IFINK- ideals of Ḱ.

Definition.3.3.11 Let A= $(ƛ_A, \xi_A)$ be an IFS of Ḱ and Let ɐ ε [0, 1]. An object having the from $A^M_ɐ = (ƛ_A)^M_ɐ, (\xi_A)^M_ɐ$ is called an IF_ ɐ-multiplication of A if, $(ƛ_A)^M_ɐ(x) = ɐ.ƛ_A(x)$ and $(\xi_A)^M_ɐ(x) = ɐ.\xi_A(x)$, for all x ε Ḱ . For an □ IFS A= $(ƛ_A, \xi_A)$ of Ḱ , an IR 0-multiplication $A^M_0 = (\mu_A)^M_0, (\xi_A)^M_0$ of A is an INK-I of Ḱ ,

for all ɐ ε [0, 1].

Theorem.3.3.12. Let A= $(ƛ_A, \xi_A)$ be an IF_ Ss of Ḱ such that IR ɐ-multiplication of $A^M_ɐ$ of Ḱ is an IFINK-ideal of A, for some ɐ ε [0,1], then A is an INK-ideal of Ḱ.

Proof. $(\mu_A)^M_ɐ$ & $(\xi_A)^M_ɐ$ is a IF_ INK of Ḱ for some ɐ ε [0, 1]. Let x, □, z ε Ḱ.

$ɐ.ƛ_A(0) \ge (ƛ_A)^M_ɐ(0) \ge (ƛ_A)^M_ɐ(x)$

$ɐ.ƛ_A(0) = ƛ_A(x).ɐ$

$ɐ.\xi_A(0) \le (\xi_A)^M_ɐ(0) \le (\xi_A)^M_ɐ(x)$

$ɐ.\xi_A(0) = \xi_A(x).ɐ$

so,

$ƛ_A(0) \ge ƛ_A(x)$

$\xi_A(0) \le \xi_A(x)$

Also,

$ɐ.ƛ_A(x) = (ƛ_A)^M_ɐ(x) \ge \min\{(ƛ_A)^M_ɐ(z\diamond x)\diamond(z\diamond □), (ƛ_A)^M_ɐ(□)\}$

$\ge \min\{ɐ.ƛ_A(z\diamond x)\diamond(z\diamond □), ɐ.ƛ_A(□)\}$

$ɐ.ƛ_A(x) \ge ɐ.\min\{ƛ_A(z\diamond x)\diamond(z\diamond □), ƛ_A(□)\}$

and,

$ɐ.\xi_A(x) = (\xi_A)^M_ɐ(x) \le \max\{(\xi_A)^M_ɐ(z\diamond x)\diamond(z\diamond □), (\xi_A)^M_ɐ(□)\}$

$\le \max\{ɐ.\xi_A(z\diamond x)\diamond(z\diamond □), ɐ.\xi_A(□)\}$

$ɐ.\xi_A(x) \le ɐ.\max\{\xi_A(z\diamond x)\diamond(z\diamond □), \xi_A(□)\}$

and so,

$ƛ_A(x) \ge \min\{ƛ_A(z\diamond x)\diamond(z\diamond □), ƛ_A(□)\}$

$\xi_A(x) \le \max\{\xi_A(z\diamond x)\diamond(z\diamond □), \xi_A(□)\}$

Hence A= $(ƛ_A, \xi_A)$ be an IFINK-ideal of Ḱ

Theorem.3.3.13 If A=$(ƛ_A, \xi_A)$ is an IR_ INK_ I of Ḱ then the INT-ɐ-multiplication $A^M_ɐ = (ƛ_A)^M_ɐ, (\xi_A)^M_ɐ$ of A is an INT- IF_ INK of Ḱ, for some ɐ ε [0, 1]

Proof. Let = $(ƛ_A, \xi_A)$ is an IR_ INK- I of Ḱ and let ɐ ε [0,1]

Then,

$(ƛ_A)^M_ɐ(0) = ɐ.ƛ_A(0)$

$\ge ɐ.ƛ_A(x)$

$(ƛ_A)^M_ɐ = (ƛ_A)^M_ɐ(x)$

And,

$(ƛ_A)^M_ɐ(x) = ɐ.ƛ_A(x)$

$\ge ɐ.\min\{ƛ_A(z\diamond x)\diamond(z\diamond □), ƛ_A(□)\}$

$\ge \min\{ɐ.ƛ_A(z\diamond x)\diamond(z\diamond □), ɐ.ƛ_A(□)\}$

$(ƛ_A)^M_ɐ(x) = \min\{(ƛ_A)^M_ɐ(z\diamond x)\diamond(z\diamond □), (ƛ_A)^M_ɐ(□)\}$

And,

$(\xi_A)^M_ɐ(0) = ɐ.\xi_A(0) \le ɐ.\xi_A(x)$

$(\xi_A)^M_ɐ = (\xi_A)^M_ɐ(x)$

And,

$(\xi_A)^M_ɐ(x) = ɐ.\xi_A(x)$

$\le ɐ.\max\{\xi_A(z\diamond x)\diamond(z\diamond □), \xi_A(□)\}$

$\le \max\{ɐ.\xi_A(z\diamond x)\diamond(z\diamond □), ɐ.\xi_A(□)\}$

$(\xi_A)^M_ɐ(x) = \max\{(\xi_A)^M_ɐ(z\diamond x)\diamond(z\diamond □), (\xi_A)^M_ɐ(□)\}$

Hence $A^M_ɐ = (ƛ_A)^M_ɐ, (\xi_A)^M_ɐ$ is an IF_ INK-ideal of Ḱ.

IV. CONCLUSION

In this paper, the conception of IFTR of IF_ INK_ I in INK-algebra are acquaint with and examined around of their valuable assets. We have given away that the IF_ α̃-translation of a intuitionistic F_ INK_ I is a intuitionistic F_ INK_ I extension nevertheless the converse is not one true

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