A Mathematical Model to Examination the Behavior of Two Competing Biological Species under the Effect of a Toxicant

Gauri Agrawal, Alok Agrawal, Anuj Kumar Agarwal, Piyush Kumar Tripathi

Abstract*: It is well known that the toxicants present in the environment affect the growth of any biological population living in that habitat. It also affects the carrying capacity of the environment with respect to that biological population. In this paper we are considering two logistically growing biological populations competing for a common resource under the effect of a toxicant and we've assumed that the first population discharges toxicant which is harmful to the second population only. Since, condition of the population and their habitat are limited therefore, keeping the above in the mind, here we've proposed a mathematical model to study the behaviour of the two competing population and observed that one species dies away as the time lapses due to the effect of the toxicants. It has been shown further that under certain conditions both the competing species can coexist in a long run.

***Keywords* : competing populations, toxicants, growth rate, carrying capacity,.**

I. INTRODUCTION

On the Earth, the behavior of every living thing can be studied using population biology. When researchers begin to evaluate a population of species, they use many tools to help them gather information such as: observing the growth of biological population, competition for resources between biological populations in same ecological niche and so on. Since, conditions of the population and their habitat are limited, so mathematical models are used to make predictions for growth and survival of biological population in the environment. Let's look some of the population biological studies based on mathematical models: the effect of one toxicant, simultaneous effects of two or more toxicants on biological population, allelopathic effect on two competing population, effect of toxicants in case of deformity (Agarwal et al, 2016; Agrawal et al, 2000; Deluna and Hallam, 1987; Dubey et al, 2010; Dubey and Hussain, 2000; Kumar et al, 2016; Shukla and Agrawal, 1999; Shukla and Dubey, 1996; Shukla et al 2001). In 1996, Shukla and Dubey considered the concurrent impact of two toxicants for the cases of instantaneous spill, constant and periodic emission of each of the toxicant into the environment. In 1999, Shukla and Agrawal proposed some mathematical model in the field of ecotoxicology for the impact of at least one toxicants on a single or two interacting organic species. In 2000, Dubey and Hussain proposed and analyzed a mathematical model to consider the coexistence of two contending plant species in same biological specialty. Shukla et al, 2001 considered the impact of a toxicant discharged from external sources on two contending biological species. Dubey et al 2010 studied an additional phenomenon of a chemical defense mechanism including two contending species. In this case both species emitted the toxicant in the environment and affecting the other. Further, In 2016, Agarwal et al and Kumar et al considered the effect of a solitary toxicant when toxicant discharged by organic species itself and discharged by some outer sources respectively. In these studies, they show that after-effect of toxicant some members loose their reproduction capability.

In population biology, there may exist some conditions where two biological populations living in same ecological niche make a competition for resources. Out of these two populations, one population produces a toxicant which affects the growth rate of other biological population. It also affects the carrying capacity of the environment regarding the second population.

For example, plants growing in the close proximity to black walnuts often develop mysterious illnesses without warning. Leaves on annuals yellow, wilt and die due to the allelopathic effect that the chemical juglone, produced by walnuts, has on vegetable plants like cabbage, peppers, tomatoes, potatoes etc. Allelopathy refers to the harmful effects of one plant on another plant, from the release of biochemicals, known as allelochemicals to inhibit germination or impede development of nearby plant life.

We consider two logistically growing biological species living in a habitat and competing for common resource. Let N_1 (t) and N_2 (t) be the population densities of the two biological species at time t . T(t) is the environmental concentration of the amount being emitted by the first population (with population density N_1 (T). U(T) is the uptaken amount of toxicant by the second population. This uptaken toxicant increases the mortality rate in second population. The amount of toxicant present in the environment decreases the carrying capacity of the environment with respect to the 2nd population. We assume that this toxicant is not harmful
to the 1st population.

Revised Manuscript Received on September 25, 2019
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Keeping the above in the mind, we propose the following mathematical model.

II. MATHEMATICAL MODEL

$$\frac{dN_1}{dt} = r_1 N_1 - r_1 \frac{N_1^2}{K_1} - \theta_1 N_1 N_2 \cdots\cdots\cdots (1)$$

$$\frac{dN_2}{dt} = r_2 N_2 - r_2 \frac{N_2^2}{K_2(T)} - dN_2 U - \theta_2 N_1 N_2 \dots\dots\dots (2)$$

$$\frac{dT}{dt} = \lambda N_1 - \delta T - \gamma T N_2 + \pi \nu N_2 U \dots\dots\dots (3)$$

$$\frac{dU}{dt} = \gamma T N_2 - \beta U - \nu N_2 U \dots\dots\dots (4)$$

Here the growth rates of the populations are r_1 , r_2. K_1 and K_2 represent the carrying capacities of the environment with respect to the populations. θ_1 , θ_2 are the competition coefficients. d is the mortality rate of the second species due to uptake of toxicant. λ is the rate by which first species is producing toxicant. The natural wash out rate of T is δ>0. γ>0 depletion rate coefficient of T(t) due to its uptake by members of second species . β>0 is the natural wash out rate coefficient of U(t). ν>0 is the depletion rate coefficient of U(t) due to decay of some members of N_2 (t) and a fraction π(0≤π≤1) of which may re-enter into the environment.

$$K_2(0) = K_{20} > 0, \frac{dK_2}{dT} < 0 \text{ for } T > 0$$

III. EQUILIBRIUM POINTS

The model given above has four equilibrium points namely

$E_1(0,0,0,0)$, $E_2(0, K_{20}, 0, 0)$, $E_3\left(K_1, 0, \frac{\lambda K_1}{\delta}, 0\right)$ and $E_4(N_1^*, N_2^*, T^*, U^*)$.

The existence of E_1, E_2, E_3 is obvious. In the following, we are giving a short proof for the existence and uniqueness of E_4.

$$N_1 = K_1\left(1 - \frac{\theta_1 N_2}{r_1}\right) = h_1(N_2) \quad (say) \cdots\cdots\cdots (5)$$

$$N_2 = K_2(T)\left[1 - \frac{dU + \theta_2 N_1}{r_2}\right] = h_2(N_2) \quad (say) \cdots\cdots\cdots (6)$$

$$T = \frac{\lambda K_1(r_1 - \theta_1 N_2) + r_1 \pi \nu N_2 U}{r_1(\delta + \gamma N_2)} = g(N_2) \quad (say) \cdots\cdots\cdots (7)$$

$$and\ U = \frac{\gamma \lambda K_1 (r_1 - \theta_1 N_2) N_2}{r_1 f(N_2)} = h(N_2) \quad (say) \cdots\cdots\cdots (8)$$

Where

$$f(N_2) = \beta\delta + (\beta\gamma + \nu\delta) N_2 + (1 - \pi)\gamma\nu N_2^2$$

Then we can see that $f(N_2) > 0 \quad for\ N_2 > o$

Now, it can be verified that,

$h_1(N_2) > 0$, $h_2(N_2) > 0$ and $\frac{dh_1}{dN_2} = -\frac{\theta_1 K_1}{r_1} < 0$

Let

$$F(N_2) = r_2 N_2 - K_2(h(N_2))[r_2 - d.g(N_2) - \theta_2 . h_1(N_2)] \dots\dots\dots (9)$$

then $F(0) = 0 - K_{20}[r_2 - d.0 - \theta_2 K_1]$

$= -K_{20}[r_2 - \theta_2 K_1]$

< 0, provided $r_2 - \theta_2 K_1 > 0$ $\dots\dots\dots (10)$

And $F(K_{20}) > 0$, provided $r_1 - \theta_1 K_{20} > 0$. $\dots\dots\dots (11)$

It can be seen that $F'(N_2) > 0$ in the interval $(0, K_{20})$

This shows that $F(N_2) = 0$ has a unique root say N_2^* in $(0, K_{20})$. Once N_2^* is found , N_1^*, T^*& U^*can be found using equations (5), (7), (8). Hence, existence and uniqueness of E_4 is established.

IV. LOCAL STABILITY ANALYSIS

We construct the variational matrix for finding the Local stability behaviour of the equilibrium points.

$$M = \begin{pmatrix} r_1 - \frac{2r_1 N_1}{K_1} - \theta_1 K_{20} & -\theta_1 N_1 & 0 & 0 \\ -\theta_2 N_2 & r_2 - \frac{2r_2 N_2}{K_2} - dU - \theta_2 N_2 & 0 & -dN_2 \\ \lambda & -\gamma T + \pi\nu U & -\delta - \gamma N_2 & \pi\nu N_2 \\ 0 & \gamma T - \nu U & \gamma N_2 & -\beta - \nu N_2 \end{pmatrix}$$

At $E_1(0,0,0,0)$,

$$M_1 = \begin{pmatrix} r_1 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 \\ \lambda & 0 & -\delta & 0 \\ 0 & 0 & 0 & -\beta \end{pmatrix}$$

It is clear that the first two eigen values are +ve whereas the last two eigen values are –ve, which shows that E_1 is a saddle point unstable in $N_1 - N_2$ plane and stable in $T - U$ plane.

At $E_2(0, K_{20}, 0, 0)$

$$M_2 = \begin{pmatrix} r_1 - \theta_1 K_{20} & 0 & 0 & 0 \\ -\theta_2 K_{20} & -r_2 & 0 & -dK_{20} \\ \lambda & 0 & -\delta - \gamma K_{20} & \pi\nu K_{20} \\ 0 & 0 & \gamma K_{20} & -\beta - \nu K_{20} \end{pmatrix}$$

$$|M_2 - xI| = 0 \Rightarrow$$

$$\begin{vmatrix} r_1 - \theta_1 K_{20} - x & 0 & 0 & 0 \\ -\theta_2 K_{20} & -r_2 - x & 0 & -dK_{20} \\ \lambda & 0 & -\delta - \gamma K_{20} - x & \pi\nu K_{20} \\ 0 & 0 & \gamma K_{20} & -\beta - \nu K_{20} - x \end{vmatrix} = 0$$

$$\Rightarrow (r_1 - \theta_1 K_{20} - x)\begin{vmatrix} -r_2 - x & 0 & -dK_{20} \\ 0 & -\delta - \gamma K_{20} - x & \pi\nu K_{20} \\ 0 & \gamma K_{20} & -\beta - \nu K_{20} \end{vmatrix} = 0$$

$$\Rightarrow (r_1 - \theta_1 K_{20} - x)(-r_2 - x)\begin{vmatrix} -\delta - \gamma K_{20} - x & \pi\nu K_{20} \\ \gamma K_{20} & -\beta - \nu K_{20} - x \end{vmatrix} = 0$$

$$\Rightarrow (r_1 - \theta_1 K_{20} - x)(-r_2 - x)\{(-\beta - \nu K_{20} - x)(-\delta - \gamma K_{20} - x) - \pi\nu\gamma K_{20}^2\} = 0$$

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Now for the other two eigen values, the expression in the curly bracket gives

$$(\beta + \nu K_{20} + x)(\delta + \gamma K_{20} + x) + \pi\nu\gamma K_{20}^2 = 0$$

$$\Rightarrow x^2 + (\beta + \nu K_{20} + \delta + \gamma K_{20})x + (\beta + \nu K_{20})(\delta + \gamma K_{20}) + \pi\nu\gamma K_{20}^2 = 0$$

$$\Rightarrow x^2 + bx + c = 0$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4c}}{2}$$

$$\Rightarrow b^2 - 4c \leq b^2$$

$or\ 4c \geq 0\ or\ c \geq 0$, which is satisfied.

Therefore, if $r_1 < \theta_1 K_{20}$, then all the eigen values are either -ve or having -ve real parts .

$\Rightarrow$ E_2 is Locally Asymptotically Stable.

At $E_3\left(K_1, 0, \frac{\lambda K_1}{\delta}, 0\right)$

$$M_3 = \begin{pmatrix} -r_1 & -\theta_1 K_1 & 0 & 0 \\ 0 & r_2 - \theta_2 K_1 & 0 & 0 \\ \lambda & \frac{-\gamma\lambda K_1}{\delta} & -\delta & 0 \\ 0 & \frac{\gamma\lambda K_1}{\delta} & 0 & -\beta \end{pmatrix}$$

$$|M_3 - xI| = 0$$

$$\Rightarrow \begin{vmatrix} -r_1 - x & -\theta_1 K_1 & 0 & 0 \\ 0 & r_2 - \theta_2 K_1 - x & 0 & 0 \\ \lambda & \frac{-\gamma\lambda K_1}{\delta} & -\delta - x & 0 \\ 0 & \frac{\gamma\lambda K_1}{\delta} & 0 & -\beta - x \end{vmatrix} = 0$$

$$\Rightarrow (-\beta - x)\begin{vmatrix} -r_1 - x & -\theta_1 K_1 & 0 \\ 0 & r_2 - \theta_2 K_1 - x & 0 \\ \lambda & \frac{-\gamma\lambda K_1}{\delta} & -\delta - x \end{vmatrix} = 0$$

$$\Rightarrow (-\beta - x)(-\delta - x)\begin{vmatrix} -r_1 - x & -\theta_1 K_1 \\ 0 & r_2 - \theta_2 K_1 - x \end{vmatrix} = 0$$

$$\Rightarrow (\beta + x)(\delta + x)(r_1 + x)(r_2 - \theta_2 K_1 - x) = 0$$

If $r_2 - \theta_2 K_1 < 0 \Rightarrow E_3$ is Locally Asymptotically Stable.

At $E_4(N_1^*, N_2^*, T^*, U^*)$,

$$M_4 = \begin{pmatrix} -\frac{r_1 N_1^*}{K_1} & -\theta_1 N_1^* & 0 & 0 \\ -\theta_2 N_2^* & -\frac{r_2 N_2^*}{K_2(T^*)} & 0 & -dN_2^* \\ \lambda & -(\gamma T^* - \pi\nu U^*) & -(\delta + \gamma N_2^*) & \pi\nu N_2^* \\ 0 & \gamma T^* - \nu U^* & \gamma N_2^* & -(\beta + \nu N_2^*) \end{pmatrix}$$

$$|M_4 - xI| = 0$$

$$\Rightarrow \left(-r_1\frac{N_1^*}{K_1} - x\right)\begin{vmatrix} -\frac{r_2 N_2^*}{K_2(T^*)} - x & 0 & -dN_2^* \\ -(\gamma T^* - \pi\nu U^* & -(\delta + \gamma N_2^*) - x & \pi\nu N_2^* \\ \gamma T^* - \nu U^* & \gamma N_2^* & -(\beta + \nu N_2^*) - x \end{vmatrix} + \theta_1 N_1^*\begin{vmatrix} -\theta_2 N_2^* & 0 & -dN_2^* \\ \lambda & -(\delta + \gamma N_2^*) - x & \pi\nu N_2^* \\ 0 & \gamma N_2^* & -(\beta + \nu N_2^*) - x \end{vmatrix} = 0$$

Or

$$\left(\frac{r_1 N_1^*}{K_1} + x\right)\left[\left(\frac{r_2 N_2^*}{K_2(T^*)} + x\right)\{(\delta + \gamma N_2^* + x)(\beta + \nu N_2^* + x) - \pi\nu\gamma N_2^{*2}\} + dN_2^*\{-(\gamma T^* - \pi\nu U^*)\gamma N_2^* + (\gamma T^* - \nu U^*)(\delta + \nu N_2^* + x)\}\right] + \theta_1 N_1^*\left[-\theta_2 N_2^*\{(\delta + \gamma N_2^* + x)(\beta + \nu N_2^* + x) - \pi\nu\gamma N_2^{*2}\} - d\lambda\gamma N_2^{*2}\right] = 0$$

$$or\left(\frac{r_1 N_1^*}{K_1} + x\right)\left[\left(\frac{r_2 N_2^*}{K_2(T^*)} + x\right)\{x^2 + (\delta + \gamma N_2^* + \beta + \nu N_2^*)x + \beta\delta + \delta\nu N_2^* + \beta\gamma N_2^* + (1-\pi)\nu\gamma N_2^*\} + dN_2^*(\gamma T^* - \nu U^*)(\delta + \nu N_2^* + x) - d\nu(\gamma T^* - \pi\nu U^*)N_2^{*2}\right] - \theta_1\theta_2 N_1^* N_2^*\{x^2 + (\delta + \gamma N_2^* + \beta + \nu N_2^*)x + \beta\delta + \delta\nu N_2^* + \beta\gamma N_2^* + (1-\pi)\nu\gamma N_2^{*2}\} - d\lambda\gamma N_2^{*2} = 0$$

$$or\left(\frac{r_1 N_1^*}{K_1} + x\right)\left[\left(\frac{r_2 N_2^*}{K_2(T^*)} + x\right)\{x^2 + (\delta + \gamma N_2^* + \beta + \nu N_2^*)x + \beta\delta + \delta\nu N_2^* + \beta\gamma N_2^* + (1-\pi)\nu\gamma N_2^*\} + dN_2^*(\gamma T^* - \nu U^*)x + dN_2^*(\delta\gamma T^* + \gamma^2 N_2^* T^* - \delta\nu U^* - \nu\gamma N_2^* U^*) - d\gamma^2 T^* N_2^* - (1-\pi)d\nu\gamma U^* N_2^*\right] - \theta_1\theta_2 N_1^* N_2^*\{x^2 + (\delta + \gamma N_2^* + \beta + \nu N_2^*)x + \beta\delta + \delta\nu N_2^* + \beta\gamma N_2^* + (1-\pi)\nu\gamma N_2^*\} - d\lambda\gamma N_2^{*2} = 0$$

$$or\left(\frac{r_1 N_1^*}{K_1} + x\right)\left[\left(\frac{r_2 N_2^*}{K_2(T^*)} + x\right)(x^2 + a_1 x + a_2) + \beta dU^* x + \beta\delta dU^* - (1-\pi)d\nu\gamma U^* N_2^{*2}\right] - \theta_1\theta_2 N_1^* N_2^*(x^2 + a_1 x + a_2) - d\lambda\gamma N_2^{*2} = 0$$

where
$a_1 = \delta + \beta + (\gamma + \nu)N_2^*$, $a_2 = \beta\delta + (\delta\nu + \beta\gamma)N_2^* + (1-\pi)\nu\gamma N_2^*$

$$or\left(\frac{r_1 N_1^*}{K_1} + x\right)\left[x^3 + \left(\frac{r_2 N_2^*}{K_2(T^*)} + a_1\right)x^2 + \left(\frac{a_1 r_2 N_2^*}{K_2(T^*)} + a_2 + \beta dU^*\right)x + \frac{a_2 r_2 N_2^*}{K_2(T^*)} + \beta\delta dU^* - (1-\pi)d\nu\gamma U^* N_2^{*2}\right] - \theta_1\theta_2 N_1^* N_2^* x^2 - a_1\theta_1\theta_2 N_1^* N_2^* x + a_2\theta_1\theta_2 N_1^* N_2^* - d\lambda\gamma N_2^{*2} = 0$$

$$x^4 + \left(\frac{r_1 N_1^*}{K_1} + \frac{r_2 N_2^*}{K_2(T^*)} + a_1\right)x^3 + \left\{\left(\frac{r_1 N_1^*}{K_1}\right)\left(\frac{r_2 N_2^*}{K_2(T^*)} + a_1\right) + \frac{a_1 r_2 N_2^*}{K_2(T^*)} + a_2 + \beta dU^* - \theta_1\theta_2 N_1^* N_2^*\right\}x^2 + \left\{\left(\frac{r_1 N_1^*}{K_1}\right)\left(\frac{a_1 r_2 N_2^*}{K_2(T^*)} + a_2 + \beta dU^*\right) + \frac{a_2 r_2 N_2^*}{K_2(T^*)} + \beta\delta dU^* - (1-\pi)d\nu\gamma U^* N_2^{*2} - a_1\theta_1\theta_2 N_1^* N_2^*\right\}x + \left[\left(\frac{r_1 N_1^*}{K_1}\right)\left(\frac{a_2 r_2 N_2^*}{K_2(T^*)} + \beta\delta dU^* - (1-\pi)d\nu\gamma U^* N_2^{*2}\right) + a_2\theta_1\theta_2 N_1^* N_2^* - d\lambda\gamma N_2^{*2}\right] = 0 \quad \ldots\ldots\ldots(12)$$

This can be written as,

$$\Rightarrow x^4 + b_1 x^3 + b_2 x^2 + b_3 x + b_4 = 0 \ldots\ldots\ldots (13)$$

Where

$b_1 = \frac{r_1 N_1^*}{K_1} + \frac{r_2 N_2^*}{K_2(T^*)} + a_1$,

$b_2 = \left(\frac{r_1 N_1^*}{K_1}\right)\left(\frac{r_2 N_2^*}{K_2(T^*)} + a_1\right) + \frac{a_1 r_2 N_2^*}{K_2(T^*)} + a_2 + \beta d U^* - \theta_1 \theta_2 N_1^* N_2^*$

,

$$b_3 = \left(\frac{r_1 N_1^*}{K_1}\right)\left(\frac{a_1 r_2 N_2^*}{K_2(T^*)} + a_2 + \beta d U^*\right) + \frac{a_2 r_2 N_2^*}{K_2(T^*)} + \beta\delta d U^* - (1-\pi) d v \gamma U^* N_2^{*2} - a_1 \theta_1 \theta_2 N_1^* N_2^*$$

$$b_4 = \left(\frac{r_1 N_1^*}{K_1}\right)\left(\frac{a_2 r_2 N_2^*}{K_2(T^*)} + \beta\delta d U^* - (1-\pi) d v \gamma U^* N_2^{*2}\right) + a_2 \theta_1 \theta_2 N_1^* N_2^* - d\lambda\gamma N_2^{*2}$$

By Routh – Hurwitz's Criterion , the roots of the polynomial are either -ve or with – ve real parts provided,

$$\left.\begin{array}{l} (i)\ \ b_1, b_2, b_3, b_4 > 0 \\ (ii)\ \ b_1 . b_2 > b_3 \\ (iii)\ \ b_1 b_2 b_3 > b_3^2 + b_1^2 b_4 \end{array}\right\} \ldots\ldots\ldots\ldots (14)$$

It can be seen that $b_1 > 0$. b_2 is +ve provided $r_2 - \theta_2 K_1 > 0$ and $r_1 - \theta_1 K_{20} > 0$. Since both of these conditions are used for establishing the existence and uniqueness of E_4, so these are obviously true.

Thus E_4 is Locally Asymptotically Stable provided condition (14) are satisfied.

V. NUMERICAL SIMULATION

To give a better prediction for the case when two competing biological species living in same environment, we present here numerical simulation of mathematical model (1-4) by assuming the carrying capacity function $K_2(T)$ corresponding to the second population N_2 as:

$$K_2(T) = K - \frac{b_1 T}{1 + b_2 T}$$

and set of parameters

$r_1 = 0.55,\quad d = 0.006,\quad K = 10.0,\quad K_1 = 10.0,\quad b_1 = 0.02,\quad b_2 = 1,$

$r_2 = 0.8,\quad \theta_1 = 0.01,\quad \theta_2 = 0.02,\quad \lambda = 0.05,\quad \delta = 0.08,\quad \gamma = 0.005,$

$\pi = 0.02,\quad v = 0.0002,\quad \beta = 0.08$

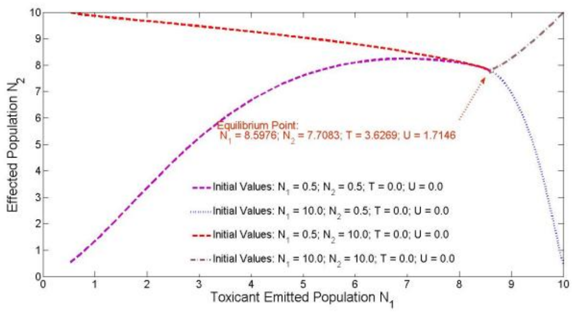

Figure 1 : Model system become stable at the equilibrium point

For the above set of parameters the model system (1-4) becomes stable at the equilibrium point $(N_1 = 8.5976; N_2 = 7.7083; T = 3.6269; U = 1.7146)$ for the different initial values.

In Figure 2 & 3, we have shown the effect of toxicant emission rate λ on both biological population (toxicant emitting population N_1 and toxicant effected population N_2 respectively). Figure 3 shows that as the toxicant emission rate increases, the density of toxicant effected biological population decreases. Since toxicant emitting population N_1 also make competition for resources with toxicant effected population N_2, therefore as the toxicant effected population decreases, toxicant emitting population N_1 increases due to increase of emission rate of toxicant (see Figure 2).

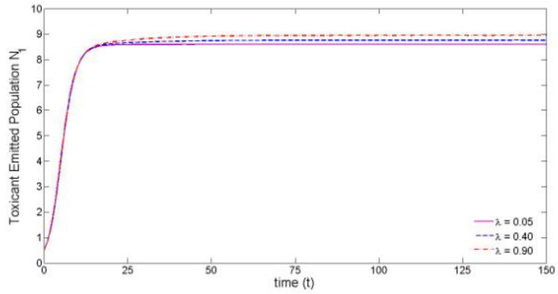

Figure 2: Variation in toxicant emitted population N_1 corresponding to the toxicant emission rate λ

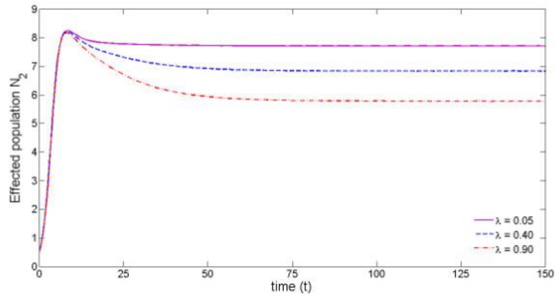

Figure 3: Variation in toxicant effected populationN_2 corresponding to the toxicant emission rate λ

In Figure 4 & 5, we have shown the effect of mortality rate d (due to uptake of the toxicant by the toxicant effected population) on both biological population (toxicant emitting population N_1 and toxicant effected population N_2 respectively). Same perception in the aboveparagraph, the toxicant emitting population N_1 increases and toxicant effected population decreases with increasing d.

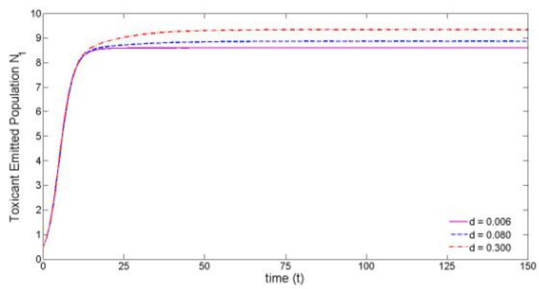

Figure 4: Variation in toxicant effected population N_1 corresponding to d.

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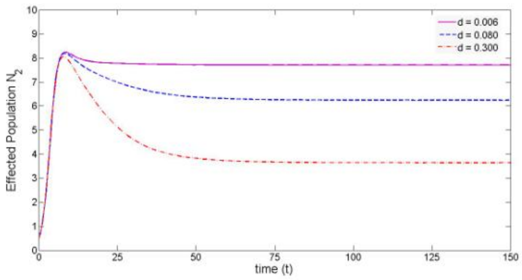

Figure 5: Variation in toxicant effected population N_2 corresponding to d

VI. CONCLUSION

In this work, we have considered a situation, where two biological species are competing for common resource under the effect of a toxicant. The toxicant is assumed to be produced by the first species and it is assumed to be harmful to the second species only.The model has four equilibrium pointsnamely $E_1(0,0,0,0)$, $E_2(0, K_{20}, 0,0)$, $E_3\left(K_1, 0, \frac{\lambda K_1}{\delta}, 0\right)$ and $E_4(N_1^*, N_2^*, T^*, U^*)$.The zero equilibrium E_1 is shown to be a saddle point whereas two non negative equilibrium points E_2 and E_3 are shown to be conditionally stable. In either of these two cases, one species dies away as the time lapses. The fourth equilibrium E_4 which is the interior equilibrium is shown to be stable under certain conditions. This means that both the competing species can coexist in a long run.

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