Modeling of Liquid Hot Metal Sloshing in Ladles During Transportation by Locomotives (A Bond Graph Method)

Abhijit Roy, Anup Kumar Saha

Abstract*: *Safe and secure transportation of liquid hot metal in steel plants is very challenging. About ninety percent of transportation is by means of locomotives. Sloshing is a common phenomenon in open container liquid transportation due to external excitation. Non-linear sloshing dynamics of liquid hot metal in ladle due to locomotive movement is the prime consideration of this paper. Liquid hot metal inside the ladle has been considered in the line of an equivalent mechanical system. Resulting forces and moments acting on the ladle inside wall are considered equal in all senses. An equivalent mechanical dynamic system representation of sloshing by bond graph modellinghas been formulated by keeping records in a satisfactory way.Future research scopes has been identifiedin parallel with an outline mapped. Hot metal liquid has two distinct components of the hydrodynamic pressure in consideration of rigid containershas been identified. Bottom segment of the molten metal column moves unison with the ladleandis directlyproportional with the acceleration of the ladle. Whereas the second one 'convective' pressure at the free surface, particularly experiences the sloshing tendency due to external forces.

Keywords: Bond graph Dynamics Locomotives Modeling Simulation Sloshing

I. INTRODUCTION

Ladle car as shown in Fig 1 is a wagon like structure on which there is a seat to rest the ladle [1]. The wheels rolls over the rail while pushed or pulled by locomotives. The buffer or stopper (that contains spring) of ladle car and locomotives while comes in contact, external excitation comes into play. This excitation transmitted to the Ladle body, refractory lining and ultimately liquid hotmetal inside the ladle. In rigid containers of hot liquidmetal, has two distinct components of the hydrodynamic pressure. Bottom segment of the molten metal column moves unison with the ladle and is in direct proportion with the acceleration of the locomotives. Whereas the second one at the free surface, also known to be ‘‘convective’’ pressure, particularly experiences the sloshing tendency due to external forces.

Momentum of locomotive, sudden brake or torque and joint in rail lines which are the causes of different external forces [6] that may cause sloshing e.g. due to [7]. The liquid hot metal dynamics inside the ladle can be approximated by a very close resemblance to mechanical system [8]. For the urpose of resemblance the assumption of equal resultant forces and that of the moments acting on the ladle wall is taken into consideration [9].

II. MATHEMATICAL MODELLING

An equivalent mechanical model as shown in Fig.2 consisting of a rigid mass moving in unison with the ladle, and a series of masses M_n, representing the equivalent mass of each sloshing mode. Stiffness K_n and damping R_n represents for each modal mass [3]. Two modal masses have been considered for simplicity.

Fig. 1 Ladle Car ready for transportation

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Nomenclature	
P	density of liquid metal
U	velocity vector in x direction
V	velocity vector in y direction
W	velocity vector in z direction
Φ	arbitrary conserved property of moving fluid particle
Q	generalized coordinates
Q	generalized forces
I_0	moment of inertia of total rigid mass
I_F	mass moment of inertia of the liquid metal about they-axis and passing through the centre of mass
L	Lagrangian
T	kinetic energy
U	potential energy
Λ	dissipation energy
ξ_{1n}	roots of determinants
M_0	rigid mass moving unison with the ladle
M_n	modal mass of n[th] number
K_n	spring stiffness of each modal masses
R_n	damping coefficient of modal masses
H_0	depth of rigid mass from CG
H_n	height of modal mass from CG
M_y	Pitching moment caused by hydrodynamic force
$\dot{x}_0$	velocity of rigid bottom liquid metal with ladle mass
$\dot{x}_n$	velocity of each modal mass
$\ddot{x}_0$	acceleration of rigid bottom liquid metal with ladle mass
$\ddot{x}_n$	acceleration of each modal mass
F	hydrodynamic force
Ω	angular frequency of oscillation
ω_n	angular frequency of n[th] mode of oscillation
β_n	damping factor of equivalent dashpot
Ω	frequency of external excitation
A_0	Integral constant
φ_0	rotational motion of the container about the z-axis through the center of gravity
R	radius of the ladle
x_n	displacement of the equivalent mass relative to the container wall
x_0	displacement of the container

Fluid total mass:

$$M_F = M_o + \sum_{n=1}^{\infty} M_n \tag{1}$$

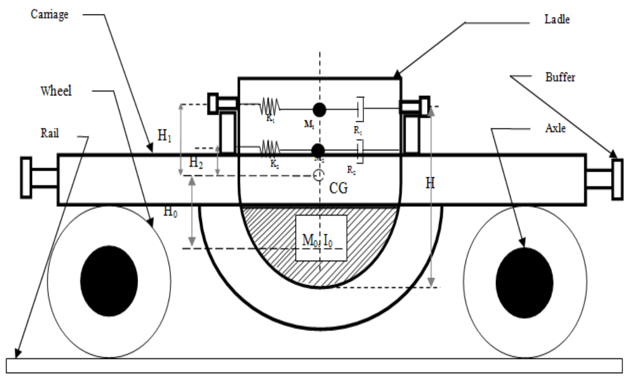


Fig. 2. Equivalent Mechanical Model of Ladle on Ladle Car.

Mass moment of inertia of the liquid metal about the x-axis and passing through the centre of mass of the molten metal:

$$I_F = I_o + M_o H_o^{2} + \sum_{n=1}^{\infty} M_n H_n^{2} \tag{2}$$

Preservation of Centre of mass should be as per following equation:

$$M_0 H_0 - \sum_{n=1}^{\infty} M_n H_n = 0 \tag{3}$$

The equations of motion of the equivalent model developed using Lagrange's equation may be derived as follows:

$$\frac{d}{dt}\left(\frac{\delta}{\delta \dot{p}} L\right) - \left(\frac{\delta}{\delta p} L\right) = -\frac{\delta}{\delta \dot{q}} \lambda + Q \tag{4}$$

Where the nomenclatures are, Lagrangian L=T-U, q is generalized coordinates, Q is generalized forces and λ is dissipation energy function. T and U are the kinetic and potential energies, respectively.

Kinematic Energy:

$$T = -\frac{1}{2} M_0 (\dot{x}_0 - H_0 \dot{\phi})^2 + \frac{1}{2} I_0 \dot{\phi}^2 + \frac{1}{2} \sum_{n=1}^{\infty} M_n (\dot{x}_n + \dot{x} + H_n \dot{\phi})^2 \tag{5}$$

Potential Energy:

$$U = \frac{1}{2} M_0 H_0 g \phi^2 - \frac{1}{2} g \phi^2 \sum M_n H_n - g\phi \sum_{n=1}^{\infty} M_n x_n + \frac{1}{2} \sum_{n=1}^{\infty} K_n x_n^2 \tag{6}$$

Dissipation energy:

$$\lambda = \frac{1}{2} \sum_{n=1}^{\infty} R_n \dot{x}_n^2 = \sum_{n=1}^{\infty} M_n \omega_n \beta_n \dot{x}_n^2 \tag{7}$$

The generalised coordinate {q} and force {Q} are represented by vectors

$$\{q\}=\{x \quad x_n \quad \phi\}^T \qquad \{Q\}=\{-F_x \quad 0 \quad M_y\}^T \tag{8}$$

Now applying Lagrange's from equation (4) and using expressions (5) to (8) following equations of motions can be derived. Force equation will be

$$M_0(\ddot{x}-H_0\ddot{\phi})+\sum_{n=1}^{\infty}\left[M_n(\ddot{x}+\ddot{x}_n+H_n\ddot{\phi})\right]=-F_x \tag{9}$$

The n^{th} mode of slosh equation will be as follows

$$M_n(\ddot{x}+\ddot{x}_n+H_n\ddot{\phi})+K_nx_n+2M_n\omega_n\beta_n\dot{x}_n-M_ng\phi=0 \tag{10}$$

Whereas moment equation becomes

$$I_0\ddot{\phi}+M_0H_0(\ddot{x}-H_0\ddot{\phi})-g\sum_{n=1}^{\infty}M_nx_n+\sum_{n=1}^{\infty}\left[M_nH_n(\ddot{x}+\ddot{x}_n+H_n\ddot{\phi})\right]=M_y \tag{11}$$

Equations (9) to (11) fully describe the translation and pitching of the model. Now let us derive the equation of lateral excitation for combined sway and pitching of the damped model

$$M_n\ddot{x}_n(t)+R_n\dot{x}_n(t)+K_nx_n(t)=M_n\Omega^2A_0\sin\Omega t+M_n(g+H_n\Omega^2)\phi\sin\Omega t \tag{12}$$

Liquid metal at the hemispherical bottom of ladle do not participate in sloshing and mass of this portion is unison with ladle mass (as mentioned earlier),sodiscussion here covered with cylindrical portion only and general solution for a cylindrical shaped ladle will be as follows:

Due to ladle car movement the following hydrodynamic force with combination of translational and pitching excitations (of liquid inside ladle):

$$F_x=M_FA_0\Omega^2\sin\Omega t\left\{1+\sum_{n=1}^{\infty}\frac{2R\Omega^2\tanh(\xi_{1n}H/R)}{\xi_{1n}H(\xi_{1n}^2-1)(\omega_n^2-\Omega^2)}\right\}+M_F\phi\Omega^2\sin\phi t\times\sum_{n=1}^{\infty}2R\tanh(\xi_{1n}H/R)\Omega^2[(H/2)\Omega^2$$

$$\left\{\frac{(1-(4R/\xi_{1n}H)\tanh(\xi_{1n}H/R)+g)]}{\xi_{1n}H(\xi_{1n}^2-1)(\omega_n^2-\Omega^2)}\right\} \tag{13}$$

Pitching moment caused by hydrodynamic force is

$$M_y=\phi\Omega^2\sin\Omega t\times\left\{\begin{array}{l}I_F+m_F\sum_{n=1}^{\infty}\frac{2RH^2\Omega^2\tanh(\xi_{1n}H/R)}{\xi_{1n}H(\xi_{1n}^2-1)}\frac{[(1-4R/\xi_{1n}H)\tanh(\xi_{1n}H/R)]^2}{4(\omega_n^2-\Omega^2)}+\\ M_F\sum_{n=1}^{\infty}\frac{2R\tanh(\xi_{1n}H/R)}{\xi_{1n}H(\xi_{1n}^2-1)}\left\{\frac{(g/\Omega)^2+gh[1-(4R/\xi_{1n}H)\tanh(\xi_{1n}H/R)]}{(\omega_n^2-\Omega^2)}\right\}\end{array}\right\}$$

$$+M_FA_0\Omega^2\sin\Omega t\times M_F\sum_{n=1}^{\infty}\frac{2R\tanh(\xi_{1n}H/R)}{\xi_{1n}H(\xi_{1n}^2-1)}\left\{\frac{g+(h\Omega^2/2)[1-(4R/\xi_{1n}H)\tanh(\xi_{1n}H/R)]}{(\omega_n^2-\Omega^2)}\right\} \tag{14}$$

Now we can derive the Moment of inertia from above three equations as follows:

$$I_f=I_0+M_0H_0^2+\sum_{n=1}^{\infty}M_nH_n^2=M_F\left\{\frac{H^2}{12}+\frac{R^2}{4}-8R^2\sum_{n=1}^{\infty}\frac{1-(2R/\xi_{1n}H)\tanh(\xi_{1n}H/2R)}{\xi_{1n}(\xi_{1n}^2-1)}\right\} \tag{15}$$

III. Bondgraph Modeling:

Here the oscillation of liquid metal inside the ladle along x-direction acts as a spring –dashpot combination, while ladle pushed or pulled by locomotives. Due to external excitation applied at buffernear the C.G. of the ladle the oscillation along x-axis is to a large extent, causing sloshing in x-direction i.e. Sway motion in x-direction. In the following Fig.3 a bond graph model of liquid metal sloshing in ladle while transported by locomotive is shown.

Bond graph model generated with only two mode of slosh taken into consideration while liquid metal sloshes due to external excitation during transportation. There are mainly two part, the upper part which is bond graph model of the ladle car with ladle, resembles with popular bond graph model of a car body while the lower part thetwo mode of sloshing of liquid hot metal. In upper part it can be seenC (19, 22)i.e. Kf, Kr are representing stiffness of front and rear wheel of ladle car and for damping of same isR (20, 23) i.e. Rf and Rr. SF25 and SF27 representing the rail reaction on front and rear wheels.I30 representing rotary inertia and I3 representing mass of ladle and ladle car. The $1_{\dot{z}}$ represents linear junction and $1_{\dot{\theta}1}$ rotary junctions respectively of the ladle car and flow from $1_{\dot{z}}$ junction meet $1_{\dot{x}}$ junction, whichrepresents the velocity of the liquid inside the ladle in x- direction. A transformer connected between $1_{\dot{z}}$ junction and $1_{\dot{x}}$ junction through two 0-junctions converts linear motion from z-direction to x-direction and modulus of it is derived from activated bonds C6 and C17. Now flow from transformer (TF) transmitted through 0-junction to $1_{\dot{x}}$ junction.The rotary inertia connected to $1_{\dot{\theta}1}$ junction denoted by I (30).The inertia (I) at this 0-junction though bond number 14 representing the mass of ladle along with liquid metal at bottom of ladle that doesn't participate in sloshing. As the simulation of the model is done with two modal masses, stiffness and damping property of first and second modal masses are represented by K1(38),K2(42) andR1(39), R2(42) whereas I1 (39) and I2 (44) are respective of 1stand 2ndmodal masses respectively. Two flow activated C bonds numbered 37 and 40 attaches at 1 junctions are the observation points to find the sloshing effect or deflection of the two modal masses respectively due to external excitation on ladle by locomotives represented by SF 12.

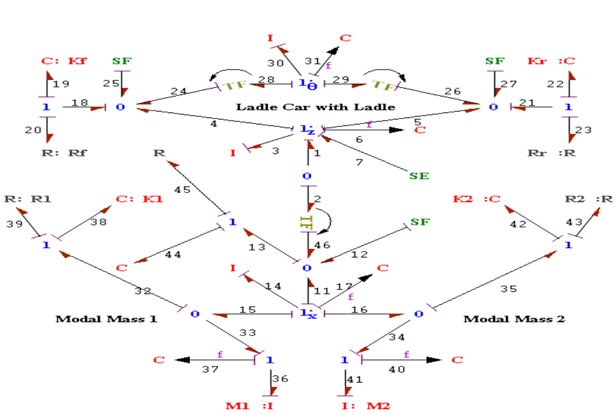


Fig.3. Bondgraph Model of hotmetal liquid slosh due to external excitation by locomotive.

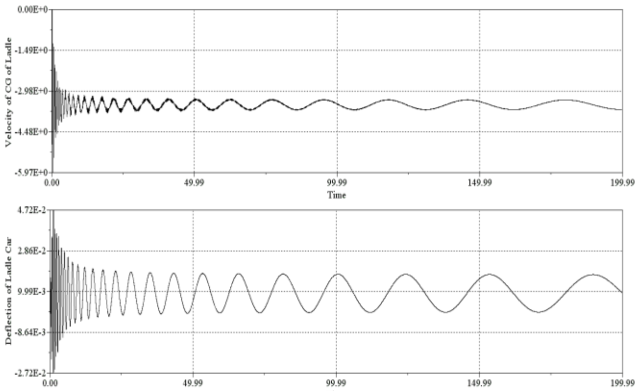
IV. RESULTS AND DISCUSSION

Simulation results obtained with parametric values as given in table 1 below.From the first graphs generated after simulation it can be seen in Figure 4 and Figure 5 that maximum velocity attained by first and second modal masses are 8.21e+0m/s and 7.68e+0m/s at 2.13623th and 2.1416th second respectively of simulation run for 200 seconds. Whereas simulation velocities are minimum for first as well as second modal masses which are 3.52e+0 m/s and 3.29e+0m/s respectively when speed of locomotive decreased at two times to that of previous one.Directions of velocity of both modal masses are opposite to that of locomotive movement. Sloshing responsepattern is similar to that numerically obtained by Patron and experimentally by Werner et al [7].

Table 1.Parametric Values.

Description	Values
Mass of the Ladle	$150x10^3$ Kg
Stiffness of 1st modal mass	180000 N
Stiffness of 2nd modal mass	150000 N
Damping of 1st modal mass	900 Ns/m
Damping of 2nd modal mass	1000Ns/m
Stiffness of Ladle car structure	$5.4x10^7$ N
Damping of Ladle car structure	$5.4\ x10^5$N
Stiffness of ladle car wheel	$5x10^7$Ns/m
Damping of ladle car wheel	$5x10^5$Kg-m
Moment of inertia of ladle car	50000 Kg
Mass of 1st modal mass	30000 Kg
Mass of 2nd modal mass	20000 m
Step-up height	0.06 m
Step-up length	0.2 m
Velocity of the locomotive	5.55m/s
Distance between two wheels	4m

The shape of the graph generated to find the deflection of ladle car CG as in figure 6 is 4.72e-2 m and –2.72e-2m in both direction. The maximum velocity attained bythe CG of liquid metal in ladle is -5.97e+0m/s. Parameters require to be changed as per optimum design needed.



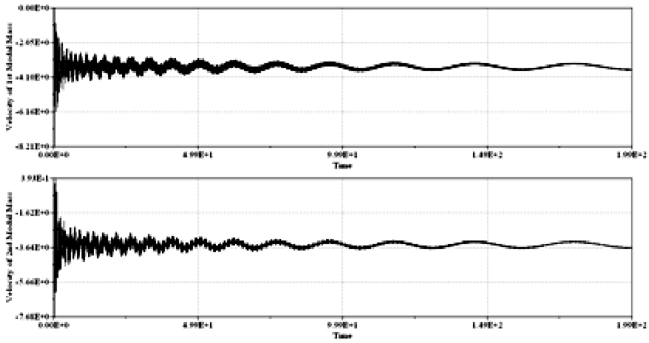


Fig.4.Velocity of Modal Masses at increased speed of locomotive.

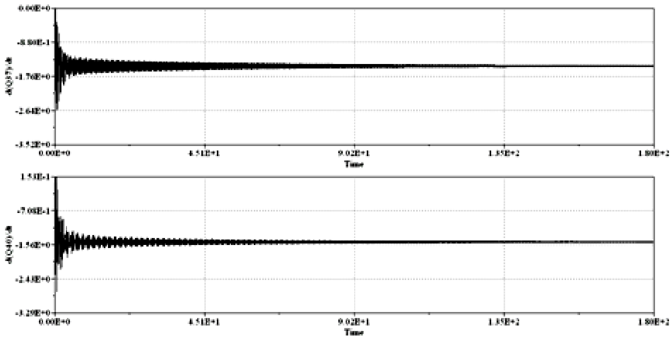


Fig.5.Velocity of Modal Masses at decreased speed of locomotive.

Fig.6.Velocity of CG of ladle and deflection of ladle car

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