OPEN ACCESS

International Journal of Recent Technology and Engineering (IJRTE)
ISSN: 2277-3878 (Online), Volume-8 Issue-2, July 2019

Bounds for the Real and Complex Eigen Values of Energy of the Dominating Intuitionistic Fuzzy Graph

Deepa. G, Praba. B, Chandrasekaran. V.M, Rajakumar. Ks

Abstract: ***We define the dominating energy for an intuitionistic fuzzy di*** $G=(V,E,\mu,\gamma)$ ***and we derive the upper and lower bounds with real and complex eigenvalues for the energy of dominating intuitionistic fuzzy graph.***

Index Terms**:** ***Intuitionistic fuzzy set (IFS), Fuzzy Graph (FG), Intuitionistic Fuzzy Graph (IFG).***

I. INTRODUCTION

Domination in graphs is an extension area of graph theory. The most basic notion of domination happened in the game of chess. Jaenisch [2] tried to define the least member of queens essential to cover chess board. The study of domination in graphs was establi $n \times n$ by Ore [6]. The concept of edge domination was introduced by Mitchell and Hedetniemi [3]. Nagoorgani et al. [5] provided some exciting properties of fuzzy dominating set. The idea of perfect dominating set in IFG was presented by Mohioub [4]. Domination is a suitable tool for defining commercial networks. Study of domination concepts in IFG's are more convenient than FG's.

II. INTUITIONISTIC FUZZY GRAPH

Definition2.1. [7] An IFG is defined as $G=(V,E,\mu,\gamma)$ where V is the set of vertices and E is the set of edges. Here μ is an intuitionistic fuzzy membership value defined on $V \times V$ and γ is an intuitionistic fuzzy non-membership value defined on $V \times V$. We denote $\mu(v_i,v_j)$ by μ_{ij} and $\gamma(v_i,v_j)$ by γ_{ij} such that

(i) $0 \le \mu_{ij}+\gamma_{ij} \le 1$ (ii) $0 \le \mu_{ij},\gamma_{ij},\pi_{ij} \le 1$ where $\pi_{ij}=1-\mu_{ij}-\gamma_{ij}$.

Revised Manuscript Received on 30 July 2019.

* Correspondence Author

Dr. G. DEEPA*, Assistant Professor, Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Tamil Nadu, India.

Dr. B. PRABA, Professor, Department of Mathematics, SSN College of Engineering, Chennai, Tamil Nadu, India.

Dr. V.M. CHANDRASEKARAN, Professor, Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Tamil Nadu, India.

Dr. K. RAJAKUMAR, Associate Professor, School of Computer Science and Engineering, Vellore Institute of Technology, Tamil Nadu, India.

Also we denote μ_{ij} represents the strength of the relationship between v_i, v_j and γ_{ij} represents the strength of the non-relationship between v_i, v_j respectively.

Definition2.2. [7] Intuitionistic fuzzy adjacency matrix (IFAM) of an IFG is defined as $A(G)=[a_{ij}]$ where $a_{ij}=(\mu_{ij},\gamma_{ij})$.

Definition2.3. [7] An IFAM of an IFG can be written as two matrices one containing the entries as membership value and the other containing the entries as non-membership value. i.e. $A(G)=\left[(\mu_{ij},\gamma_{ij})\right]$. We denote $A_\mu(G)=\left[\mu_{ij}\right]$ is an IFAM of membership value and $A_\gamma(G)=\left[\gamma_{ij}\right]$ is an IFAM of non-membership value.

Definition2.4. [7] Spectrum of an IFAM $A(G)$ is defined as (X,Y) where X is the set eigenvalues (EV's) of $A_\mu(G)$ and Y is the set EV's of $A_\gamma(G)$. It is denoted by $spec(A(G))$.

Definition2.5. [7] Let $\lambda_1,\lambda_2,...,\lambda_n$ be the EV's of $A_\mu(G)$ and let $\delta_1,\delta_2,...,\delta_n$ be the EV's of $A_\gamma(G)$. Then the energy of an IFG is defined as

$$E(A(G))=\left(\sum_{i=1}^{n}|\lambda_i|,\sum_{i=1}^{n}|\delta_i|\right)$$

where $\sum_{i=1}^{n}|\lambda_i|$ is the energy of membership value denoted by $E(A_\mu(G))$ and $\sum_{i=1}^{n}|\delta_i|$ is the energy of non-membership value denoted by $E(A_\gamma(G))$.

Theorem2.1. [7] Let G be an IFG with n vertices. If $\lambda_1,\lambda_2,...,\lambda_n$ are the EV's (real or complex) of $A_\mu(G)$ and $\delta_1,\delta_2,...,\delta_n$ are the EV's (real or complex) of $A_\gamma(G)$, then

$$(i)\ \sum_{i=1}^{n}\lambda_i=0\ (ii)\ \sum_{i=1}^{n}\lambda_i^2=2\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji}\ (iii)\ \sum_{i=1}^{n}\delta_i=0\ (iv)\ \sum_{i=1}^{n}\delta_i^2=2\sum_{1\le i<j\le n}\gamma_{ij}\gamma_{ji}.$$

Theorem2.2. [1] Let G be an IFG (without loops) with n vertices. If $\lambda_1,\lambda_2,...,\lambda_n$ are the real eigenvalues (REV's) of $A_\mu(G)$ and $\delta_1,\delta_2,...,\delta_n$ are the REV's of $A_\gamma(G)$, then

$$(i)\ \sqrt{2\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji}+n(n-1)|A|^{\frac{2}{n}}}\le E(A_\mu(G))\le\sqrt{2n\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji}}$$

$$(ii)\ \sqrt{2\sum_{1\le i<j\le n}\gamma_{ij}\gamma_{ji}+n(n-1)|B|^{\frac{2}{n}}}\le E(A_\gamma(G))\le\sqrt{2n\sum_{1\le i<j\le n}\gamma_{ij}\gamma_{ji}}$$

Where $|A|$ is the determinant of $A_\mu(G)$ and $|B|$ is the determinant of $A_\gamma(G)$.

Theorem2.3. [1] Let G be IFG (without loops) with n vertices and if $\lambda_1,\lambda_2,...,\lambda_n$ are the complex eigenvalues (CEV's) of $A_\mu(G)$ and $\delta_1,\delta_2,...,\delta_n$ are the CEV's of $A_\gamma(G)$, then

$$(i)\ \sqrt{\sum_{i=1}^{n}|\lambda_i|^2+n(n-1)|A|^{\frac{2}{n}}}\le E(A_\mu(G))\le\sqrt{n\left[\left(\sum_{i=1}^{n}|\lambda_i|\right)^2-2\sum_{1\le i<j\le n}|\lambda_i\lambda_j|\right]}$$

$$(ii)\ \sqrt{\sum_{i=1}^{n}|\delta_i|^2+n(n-1)|B|^{\frac{2}{n}}}\le E(A_\gamma(G))\le\sqrt{n\left[\left(\sum_{i=1}^{n}|\delta_i|\right)^2-2\sum_{1\le i<j\le n}|\delta_i\delta_j|\right]}$$

Where $|A|$ is the determinant of $A_\mu(G)$ and $|B|$ is the determinant of $A_\gamma(G)$.

III. DOMINATING IFG (DIFG)

Here we consider an IFG and we define $(\mu_1,\gamma_1):V\to[0,1]$ and we prove that (μ_1,γ_1) is an IFS. Then we determine the energy of DIFG $G=(V,E,\mu,\gamma,\mu_1,\gamma_1)$.

Definition3.1. [8] Let G be an IFG. An intuitionistic fuzzy vertex membership value μ_1 is defined by $\mu_1:V\to[0,1]$ and an intuitionistic fuzzy vertex non-membership value γ_1 is defined by $\gamma_1:V\to[0,1]$ such that

(i) $0\le\mu_1(v_i)+\gamma_1(v_i)\le1$ (ii) $0\le\mu_1(v_i),\gamma_1(v_i),\pi_1(v_i)\le1$ where $\pi_1(v_i)=1-\mu_1(v_i)-\gamma_1(v_i)$.

Now we define the set

$(\mu_1,\gamma_1):V\to[0,1]$ by $\mu_1(v_i)=\max_{v_j}(\mu(v_i,v_j))$ and $\gamma_1(v_i)=\min_{v_j}(\gamma(v_i,v_j))$ for any $v_i\in V$.

Lemma3.1. [8] Let G be an IFG. If

$$\mu_1(v)=\max_{w}(\mu(v,w))\text{ and }\gamma_1(v)=\min_{w}(\gamma(v,w))$$

then (μ_1,γ_1) is an IFS.

[Further discussion, we call $G=(V,E,\mu,\gamma,\mu_1,\gamma_1)$ is a DIFG.]

Definition3.2. [8] An arc (v_i,v_j) of a DIFG $G=(V,E,\mu,\gamma,\mu_1,\gamma_1)$ is called a strong arc if

$$\mu(v_i,v_j)\le\mu_1(v_i)\wedge\mu_1(v_j)\text{ and }\gamma(v_i,v_j)\le\gamma_1(v_i)\wedge\gamma_1(v_j).$$

Definition3.3. [8] Let $G=(V,E,\mu,\gamma,\mu_1,\gamma_1)$ be a DIFG. Let $u,v\in V$, we say that u dominates v in G if there exists a strong arc from u to v. A subset $D\subseteq V$ is said to be dominating set in G if for every $v\in V-D$, there exist u in D such that u dominates v.

Definition3.4. [8]. A dominating intuitionistic fuzzy adjacency matrix (DIFAM) of DIFG is defined as $D(G)=(d_{ij})$, where

$$d_{ij}=\begin{cases}(\mu_{ij},\gamma_{ij}), & \text{if } (v_i,v_j)\in E\\ (1,1), & \text{if } i=j\ \&\ v_i\in D\\ 0, & \text{otherwise}\end{cases}$$

This DIFAM $D(G)$ can be written as $D(G)=(D_\mu(G),D_\gamma(G))$ where

$$D_\mu(G)=\begin{cases}\mu_{ij}, & \text{if } (v_i,v_j)\in E\\ 1, & \text{if } i=j\ \&\ v_i\in D\\ 0, & \text{otherwise}\end{cases}\text{ and }D_\gamma(G)=\begin{cases}\gamma_{ij}, & \text{if } (v_i,v_j)\in E\\ 1, & \text{if } i=j\ \&\ v_i\in D\\ 0, & \text{otherwise}\end{cases}$$

Definition3.5. [8] Eigenvalues of DIFAM $D(G)$ is defined as (X, Y) where X is the set of EV's of $D_\mu(G)$ and Y is the set of EV's of $D_\gamma(G)$

Definition3.6.[8] Let $\lambda_1,\lambda_2,...,\lambda_n$ be the EV's of $D_\mu(G)$ and $\delta_1,\delta_2,...,\delta_n$ be the EV's of $D_\gamma(G)$. Then the energy of DIFG $G=(V,E,\mu,\gamma,\mu_1,\gamma_1)$ is defined as

$$E(D(G))=\left(\sum_{i=1}^{n}|\lambda_i|,\sum_{i=1}^{n}|\delta_i|\right)$$

where $\sum_{i=1}^{n}|\lambda_i|$ is the energy of membership value denoted by $E(D_\mu(G))$ and $\sum_{i=1}^{n}|\delta_i|$ is the energy of the non-membership value denoted by $E(D_\gamma(G))$.

Theorem3.1. Let $G=(V,E,\mu,\gamma,\mu_1,\gamma_1)$ be a DIFG. Let D be a dominating set. If $\lambda_1,\lambda_2,...,\lambda_n$ are the real or complex EV's of $D_\mu(G)$ and if $\delta_1,\delta_2,...,\delta_n$ are the real or complex EV's of $D_\gamma(G)$ then

$$(i)\ \sum_{i=1}^{n}\lambda_i=|D|\quad(ii)\ \sum_{i=1}^{n}\lambda_i^2=\sum_{i=1}^{n}\mu_{ii}^2+2\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji}$$

$$(iii)\ \sum_{i=1}^{n}\delta_i=|D|\quad(iv)\ \sum_{i=1}^{n}\delta_i^2=\sum_{i=1}^{n}\gamma_{ii}^2+2\sum_{1\le i<j\le n}\gamma_{ij}\gamma_{ji}.$$

IV. BOUNDS FOR ENERGY OF DIFG

Theorem4.1. Let $G=(V,E,\mu,\gamma,\mu_1,\gamma_1)$ be a DIFG with n vertices and let D be a dominating set. If $\lambda_1,\lambda_2,...,\lambda_n$ are the REV's of $D_\mu(G)$ and if $\delta_1,\delta_2,...,\delta_n$ are the REV's of $D_\gamma(G)$ then

$$(i)\sqrt{\sum_{i=1}^{n}\mu_{ii}^2+2\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji}+n(n-1)|A|^{\frac{2}{n}}}\le E(D_\mu G)\le\sqrt{n\left(\sum_{i=1}^{n}\mu_{ii}^2+2\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji}\right)}$$

$$(ii)\sqrt{\sum_{i=1}^{n}\gamma_{ii}^2+2\sum_{1\le i<j\le n}\gamma_{ij}\gamma_{ji}+n(n-1)|B|^{\frac{2}{n}}}\le E(D_\gamma G)\le\sqrt{n\left(\sum_{i=1}^{n}\gamma_{ii}^2+2\sum_{1\le i<j\le n}\gamma_{ij}\gamma_{ji}\right)}$$

where $|A|$ is the determinant of $D_\mu(G)$ and $|B|$ is the determinant of $D_\gamma(G)$

Corollary4.1. Let $G=(V,E,\mu,\gamma,\mu_1,\gamma_1)$ be a DIFG with n vertices and let D be a dominating set. If $\lambda_1,\lambda_2,\ldots,\lambda_n$ are the REV's of $D_\mu(G)$ and if $\delta_1,\delta_2,\ldots,\delta_n$ are the REV's of $D_\gamma(G)$ and if $E(D_\mu(G))\ge E(D_\gamma(G))$, then

$$n\left(\sum_{i=1}^{n}\mu_{ii}^2+2\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji}\right)\ge\sum_{i=1}^{n}\gamma_{ii}^2+2\sum_{1\le i<j\le n}\gamma_{ij}\gamma_{ji}+n(n-1)|B|^{\frac{2}{n}}.$$

Proof. From Theorem 4.1, we have

$$\sqrt{n\left(\sum_{i=1}^{n}\mu_{ii}^2+2\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji}\right)}\ge E(D_\mu(G))$$

$$\Rightarrow\sqrt{n\left(\sum_{i=1}^{n}\mu_{ii}^2+2\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji}\right)}\ge E(D_\gamma(G))$$

$$\Rightarrow\sqrt{n\left(\sum_{i=1}^{n}\mu_{ii}^2+2\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji}\right)}\ge\sqrt{\sum_{i=1}^{n}\gamma_{ii}^2+2\sum_{1\le i<j\le n}\gamma_{ij}\gamma_{ji}+n(n-1)|B|^{\frac{2}{n}}}$$

$$\Rightarrow n\left(\sum_{i=1}^{n}\mu_{ii}^2+2\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji}\right)\ge\sum_{i=1}^{n}\gamma_{ii}^2+2\sum_{1\le i<j\le n}\gamma_{ij}\gamma_{ji}+n(n-1)|B|^{\frac{2}{n}}.$$

Corollary4.2. Let $G=(V,E,\mu,\gamma,\mu_1,\gamma_1)$ be a DIFG with n vertices and let D be a dominating set. If $\lambda_1,\lambda_2,\ldots,\lambda_n$ are the REV's of $D_\mu(G)$ and if $\delta_1,\delta_2,\ldots,\delta_n$ are the REV's of $D_\gamma(G)$ and if $E(D_\gamma(G))\ge E(D_\mu(G))$, then

$$n\left(\sum_{i=1}^{n}\gamma_{ii}^2+2\sum_{1\le i<j\le n}\gamma_{ij}\gamma_{ji}\right)\ge\sum_{i=1}^{n}\mu_{ii}^2+2\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji}+n(n-1)|A|^{\frac{2}{n}}.$$

Proof. From Theorem 4.1, we have

$$\sqrt{n\left(\sum_{i=1}^{n}\gamma_{ii}^2+2\sum_{1\le i<j\le n}\gamma_{ij}\gamma_{ji}\right)}\ge E(D_\gamma(G))$$

$$\Rightarrow\sqrt{n\left(\sum_{i=1}^{n}\gamma_{ii}^2+2\sum_{1\le i<j\le n}\gamma_{ij}\gamma_{ji}\right)}\ge E(D_\mu(G))$$

$$\Rightarrow\sqrt{n\left(\sum_{i=1}^{n}\gamma_{ii}^2+2\sum_{1\le i<j\le n}\gamma_{ij}\gamma_{ji}\right)}\ge\sqrt{\sum_{i=1}^{n}\mu_{ii}^2+2\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji}+n(n-1)|A|^{\frac{2}{n}}}$$

$$\Rightarrow n\left(\sum_{i=1}^{n}\gamma_{ii}^2+2\sum_{1\le i<j\le n}\gamma_{ij}\gamma_{ji}\right)\ge\sum_{i=1}^{n}\mu_{ii}^2+2\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji}+n(n-1)|A|^{\frac{2}{n}}.$$

Theorem4.2. Let $G=(V,E,\mu,\gamma,\mu_1,\gamma_1)$ be a DIFG with n vertices and let D be a dominating set. If $\lambda_1,\lambda_2,\ldots,\lambda_n$ are the CEV's of $D_\mu(G)$ and if $\delta_1,\delta_2,\ldots,\delta_n$ are the CEV's of $D_\gamma(G)$ then

$$(i)\sqrt{\left(\sum_{i=1}^{n}|\lambda_i|\right)^2-2\sum_{1\le i<j\le n}|\lambda_i\lambda_j|+n(n-1)|A|^{\frac{2}{n}}}\le E(D_\mu(G))\le\sqrt{n\left[\left(\sum_{i=1}^{n}|\lambda_i|\right)^2-2\sum_{1\le i<j\le n}|\lambda_i\lambda_j|\right]}$$

$$(i)\sqrt{\left(\sum_{i=1}^{n}|\delta_i|\right)^2-2\sum_{1\le i<j\le n}|\delta_i\delta_j|+n(n-1)|B|^{\frac{2}{n}}}\le E(D_\gamma(G))\le\sqrt{n\left[\left(\sum_{i=1}^{n}|\delta_i|\right)^2-2\sum_{1\le i<j\le n}|\delta_i\delta_j|\right]}$$

where $|A|$ is the determinant of $D_\mu(G)$ and $|B|$ is the determinant of $D_\gamma(G)$.

Proof. (i) Upper bound: In Cauchy_Schwarz inequality

$$\left(\sum_{i=1}^{n}a_ib_i\right)^2\le\left(\sum_{i=1}^{n}a_i^2\right)\left(\sum_{i=1}^{n}b_i^2\right)$$

Put $a_i=1,\ b_i=|\lambda_i|$, we get

$$\left(\sum_{i=1}^{n}|\lambda_i|\right)^2\le\left(\sum_{i=1}^{n}1\right)\left(\sum_{i=1}^{n}|\lambda_i|^2\right)$$

$$\Rightarrow\sum_{i=1}^{n}|\lambda_i|\le\sqrt{n\left(\sum_{i=1}^{n}|\lambda_i|^2\right)} \qquad (1)$$

But we know that

$$\left(\sum_{i=1}^{n}|\lambda_i|\right)^2=\sum_{i=1}^{n}|\lambda_i|^2+2\sum_{1\le i<j\le n}|\lambda_i\lambda_j|$$

$$\Rightarrow\sum_{i=1}^{n}|\lambda_i|^2=\left(\sum_{i=1}^{n}|\lambda_i|\right)^2-2\sum_{1\le i<j\le n}|\lambda_i\lambda_j| \qquad (2)$$

Substitute Eq. (2) in Eq. (1), we get

Retrieval Number F2403037619/19©BEIESP
DOI: 10.35940/ijrte.F2403.078219
Journal Website: www.ijrte.org

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

Substitute Eq. (2) in Eq. (1), we get

$$E\left(D_\mu(G)\right) \le \sqrt{n\left[\left(\sum_{i=1}^{n}|\lambda_i|\right)^2 - 2\sum_{1\le i<j\le n}|\lambda_i\lambda_j|\right]} \quad (3)$$

Lower bound: By Definition 3.6, we have

$$\left(E\left(D_\mu(G)\right)\right)^2 = \left(\sum_{i=1}^{n}|\lambda_i|\right)^2$$

$$= \sum_{i=1}^{n}|\lambda_i|^2 + 2\sum_{1\le i<j\le n}|\lambda_i||\lambda_j|$$

$$= \left[\left(\sum_{i=1}^{n}|\lambda_i|\right)^2 - 2\sum_{1\le i<j\le n}|\lambda_i\lambda_j|\right] + 2\frac{n(n-1)}{2}\underset{1\le i<j\le n}{AM}\left\{|\lambda_i||\lambda_j|\right\}$$

$$\Rightarrow E\left(D_\mu(G)\right) \ge \sqrt{\left(\sum_{i=1}^{n}|\lambda_i|\right)^2 - 2\sum_{1\le i<j\le n}|\lambda_i\lambda_j| + n(n-1)|A|^{\frac{2}{n}}} \quad (4)$$

From Eq. (3) and Eq. (4) we get (i) and similarly we can prove (ii).

Corollary4.3. Let $G=(V,E,\mu,\gamma,\mu_1,\gamma_1)$ be a DIFG with n vertices and let D be a dominating set. If $\lambda_1,\lambda_2,\ldots,\lambda_n$ are the CEV's of $D_\mu(G)$ and if $\delta_1,\delta_2,\ldots,\delta_n$ are the CEV's of $D_\gamma(G)$ and if $E\left(D_\mu(G)\right)\ge E\left(D_\gamma(G)\right)$ then

$$n\left[\left(\sum_{i=1}^{n}|\lambda_i|\right)^2 - 2\sum_{1\le i<j\le n}|\lambda_i\lambda_j|\right] \ge \left(\sum_{i=1}^{n}|\delta_i|\right)^2 - 2\sum_{1\le i<j\le n}|\delta_i\delta_j| + n(n-1)|B|^{\frac{2}{n}}.$$

Proof. From Theorem 4.2, we have

$$\sqrt{n\left[\left(\sum_{i=1}^{n}|\lambda_i|\right)^2 - 2\sum_{1\le i<j\le n}|\lambda_i\lambda_j|\right]} \ge E\left(D_\mu(G)\right)$$

$$\Rightarrow \sqrt{n\left[\left(\sum_{i=1}^{n}|\lambda_i|\right)^2 - 2\sum_{1\le i<j\le n}|\lambda_i\lambda_j|\right]} \ge E\left(D_\gamma(G)\right)$$

Again by Theorem 4.2, we have

$$E\left(D_\gamma(G)\right) \ge \sqrt{\left(\sum_{i=1}^{n}|\delta_i|\right)^2 - 2\sum_{1\le i<j\le n}|\delta_i\delta_j| + n(n-1)|B|^{\frac{2}{n}}}$$

Therefore we get

$$n\left[\left(\sum_{i=1}^{n}|\lambda_i|\right)^2 - 2\sum_{1\le i<j\le n}|\lambda_i\lambda_j|\right] \ge \left(\sum_{i=1}^{n}|\delta_i|\right)^2 - 2\sum_{1\le i<j\le n}|\delta_i\delta_j| + n(n-1)|B|^{\frac{2}{n}}.$$

Corollary4.4. Let $G=(V,E,\mu,\gamma,\mu_1,\gamma_1)$ be a DIFG with n vertices and let D be a dominating set. If $\lambda_1,\lambda_2,\ldots,\lambda_n$ are the CEV's of $D_\mu(G)$ and if $\delta_1,\delta_2,\ldots,\delta_n$ are the CEV's of $D_\gamma(G)$ and if $E\left(D_\gamma(G)\right)\ge E\left(D_\mu(G)\right)$ then

ute Eq. (2) in Eq. (1), we get

$$n\left[\left(\sum_{i=1}^{n}|\delta_i|\right)^2 - 2\sum_{1\le i<j\le n}|\delta_i\delta_j|\right] \ge \left(\sum_{i=1}^{n}|\lambda_i|\right)^2 - 2\sum_{1\le i<j\le n}|\lambda_i\lambda_j| + n(n-1)|A|^{\frac{2}{n}}.$$

Proof. From Theorem 4.2, we have

$$\sqrt{n\left[\left(\sum_{i=1}^{n}|\delta_i|\right)^2 - 2\sum_{1\le i<j\le n}|\delta_i\delta_j|\right]} \ge E\left(D_\gamma(G)\right)$$

$$\Rightarrow \sqrt{n\left[\left(\sum_{i=1}^{n}|\delta_i|\right)^2 - 2\sum_{1\le i<j\le n}|\delta_i\delta_j|\right]} \ge E\left(D_\mu(G)\right)$$

Again by Theorem 4.2, we have

Substitute Eq. (2) in Eq. (1),

$$E\left(D_\mu(G)\right) \ge \sqrt{\left(\sum_{i=1}^{n}|\lambda_i|\right)^2 - 2\sum_{1\le i<j\le n}|\lambda_i\lambda_j| + n(n-1)|A|^{\frac{2}{n}}}$$

Therefore we get

$$n\left[\left(\sum_{i=1}^{n}|\delta_i|\right)^2 - 2\sum_{1\le i<j\le n}|\delta_i\delta_j|\right] \ge \left(\sum_{i=1}^{n}|\lambda_i|\right)^2 - 2\sum_{1\le i<j\le n}|\lambda_i\lambda_j| + n(n-1)|A|^{\frac{2}{n}}.$$

Theorem4.3. Let $G=(V,E,\mu,\gamma)$ be an IFG and let $A(G)=\left(A_\mu(G),A_\gamma(G)\right)$ be an IFAM of G. Let $G_1=(V,E,\mu,\gamma,\mu_1,\gamma_1)$ be the DIFG of G and let $D(G)=\left(D_\mu(G),D_\gamma(G)\right)$ be the DIFAM of G_1. If $\lambda_1,\lambda_2,\ldots,\lambda_n$ and $\delta_1,\delta_2,\ldots,\delta_n$ are the REV's then

$$(i)\ \left(E\left(D_\mu(G)\right)\right)^2 \le n\left(\sum_{i=1}^{n}\mu_{ii}^2 + \left(E\left(A_\mu(G)\right)\right)^2\right)\quad (ii)\ \left(E\left(D_\gamma(G)\right)\right)^2 \le n\left(\sum_{i=1}^{n}\gamma_{ii}^2 + \left(E\left(A_\gamma(G)\right)\right)^2\right).$$

Proof. By Theorem 2.2, we have

$$\left(E\left(A_\mu(G)\right)\right)^2 \ge 2\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji} + n(n-1)|A|^{\frac{2}{n}}$$

$$\left(E\left(A_\mu(G)\right)\right)^2 \ge 2\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji} \Rightarrow 2\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji} \le \left(E\left(A_\mu(G)\right)\right)^2 \quad (5)$$

By Theorem4.1, we have

$$\left(E\left(D_\mu(G)\right)\right)^2 \le n\left(\sum_{i=1}^{n}\mu_{ii}^2 + 2\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji}\right)$$

we get

Using Eq. (5), we get

$$\left(E\left(D_\mu(G)\right)\right)^2 \le n\left(\sum_{i=1} G=(V,E,\mu,\gamma,\mu_1,\gamma_1)^2\right)$$

Similarly we can prove (ii).

Theorem4.4. Let $G=(V,E,\mu,\gamma)$ be an IFG and let $A(G)=\left(A_\mu(G),A_\gamma(G)\right)$ be an IFAM of G.

Let $G_1=(V,E,\mu,\gamma,\mu_1,\gamma_1)$ be the DIFG of G and let $D(G)=\left(D_\mu(G),D_\gamma(G)\right)$ be the DIFAM of G_1. If $\lambda_1,\lambda_2,\ldots,\lambda_n$ and $\delta_1,\delta_2,\ldots,\delta_n$ are the CEV's then

$$(i)\ \left(E\left(D_\mu(G)\right)\right)^2 \le n\left[\left(E\left(A_\mu(G)\right)\right)^2\right]\ (ii)\ \left(E\left(D_\gamma(G)\right)\right)^2 \le n\left[\left(E\left(A_\gamma(G)\right)\right)^2\right].$$

Proof. By Theorem 2.3, we have

$$\left(E\left(A_\mu(G)\right)\right)^2 \ge \sum_{i=1}^{n}|\lambda_i|^2 + n(n-1)|A|^{\frac{2}{n}} \ge \sum_{i=1}^{n}|\lambda_i|^2$$

$$\Rightarrow \sum_{i=1}^{n}|\lambda_i|^2 \le \left(E\left(A_\mu(G)\right)\right)^2 \qquad (6)$$

By Theorem4.2, we have

$$\left(E\left(D_\mu(G)\right)\right)^2 \le n\left[\left(\sum_{i=1}^{n}|\lambda_i|\right)^2 - 2\sum_{1\le i<j\le n}|\lambda_i\lambda_j|\right]$$

$$\Rightarrow \left(E\left(D_\mu(G)\right)\right)^2 \le n\left(\sum_{i=1}^{n}|\lambda_i|^2\right)$$

Using Eq. (6), we get

$$\left(E\left(D_\mu(G)\right)\right)^2 \le n\left[\left(E\left(A_\mu(G)\right)\right)^2\right]$$

Similarly we can prove (ii).

V. NUMERICAL EXAMPLE

Let us demonstrate the above ideas in the following example.

Example5.1.

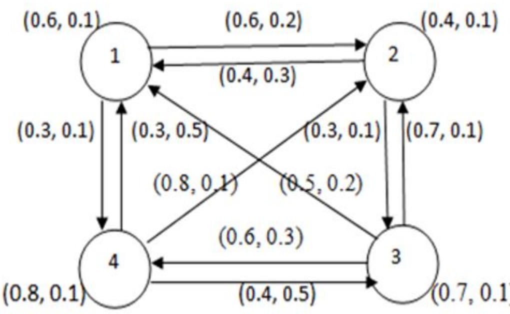

Figure 1 dominating intuitionistic fuzzy graph

Consider the DIFG given by which is shown in Figure 1, where $V=\{v_1,v_2,v_3,v_4\}$ and μ_1, γ_1 are given by $\mu_1: V\to[0,1]$ and $\gamma_1: V\to[0,1]$.

For the DIFG in Figure 1, we have

$$\mu_1(v_1)=\max_{v_j}\left(\mu(v_1,v_j)\right)=\max\left(\mu(v_1,v_2),\mu(v_1,v_4)\right)=\max(0.6,0.3)=0.6.$$

Similarly,

$$\mu_1(v_2)=0.4,\ \mu_1(v_3)=0.7,\ \mu_1(v_4)=0.8$$

and

$$\gamma_1(v_1)=\min_{v_j}\left(\gamma(v_1,v_j)\right)=\min\left(\gamma(v_1,v_2),\gamma(v_1,v_4)\right)=\min(0.2,0.1)=0.1.$$

Similarly,

$$\gamma_1(v_2)=0.1,\ \gamma_1(v_3)=0.1,\ \gamma_1(v_4)=0.1.$$

Here V_1 dominates V_4 and V_2 dominates V_3 because it satisfies the following:

$$\mu(v_1,v_4)\le\mu_1(v_1)\wedge\mu_1(v_4),\ \gamma(v_1,v_4)\le\gamma_1(v_1)\wedge\gamma_1(v_4)\ \mu(v_2,v_3)\le\mu_1(v_2)\wedge\mu_1(v_3),\ \gamma(v_2,v_3)\le\gamma_1(v_2)\wedge\gamma_1(v_3).$$

Here

$$V=\{v_1,v_2,v_3,v_4\} \text{ and } D=\{v_1,v_2\},\ V-D=\{v_3,v_4\}.$$

The DIFAM is

$$D(G)=\begin{pmatrix}(0,1) & (0.6,0.2) & 0 & (0.3,0.1)\\ (0.4,0.3) & (0,1) & (0.3,0.1) & 0\\ (0.5,0.2) & (0.7,0.1) & 0 & (0.6,0.3)\\ (0.3,0.5) & (0.8,0.1) & (0.4,0.5) & 0\end{pmatrix}$$

Where

$$D_\mu(G)=\begin{pmatrix}1 & 0.6 & 0 & 0.3\\ 0.4 & 1 & 0.3 & 0\\ 0.5 & 0.7 & 0 & 0.6\\ 0.3 & 0.8 & 0.4 & 0\end{pmatrix}$$

and

$$D_\gamma(G)=\begin{pmatrix}1 & 0.2 & 0 & 0.1\\ 0.3 & 1 & 0.1 & 0\\ 0.2 & 0.1 & 0 & 0.3\\ 0.5 & 0.1 & 0.5 & 0\end{pmatrix}$$

The energies of $A_\mu(G)$ $A_\gamma(G)$, $D_\mu(G)$ and $D_\gamma(G)$ are given by

$$spec\left(A_\mu(G)\right)=\{1.0786,-0.0772,-0.5007+0.2354i,-0.5007-0.2354i\}$$

$$spec\left(A_\gamma(G)\right)=\{0.5391,0.1028,-0.1928,-0.4491\}$$

$$E\left(A_\mu(G)\right)=1.0786+0.0772+0.5533+0.5533=2.2625$$

$$E\left(A_\gamma(G)\right)=0.5391+0.1028+0.1928+0.4491=1.2838$$

$$spec\left(D_\mu(G)\right)=\{1.7605,-0.4261,0.5025,0.1631\}$$

Retrieval Number F2403037619/19©BEIESP
DOI: 10.35940/ijrte.F2403.078219
Journal Website: www.ijrte.org

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

$$spec\left(D_{\gamma}\left(G\right)\right)=\left\{1.2935, 0.7733, 0.3317, -0.3985\right\}$$

$$E\left(D_{\mu}\left(G\right)\right)=1.7605+0.4261+0.5025+0.1631=2.8523$$

$$E\left(D_{\gamma}\left(G\right)\right)=1.2935+0.7733+0.3317+0.3985=2.7970$$

From Theorem 2.3, we have

(i) $1.9229 \le 2.2625 \le 2.6695$ (ii) $1.1711 \le 1.2838 \le 1.4698$

From Theorem 4.1, we have

(i) $2.5565 \le 2.8523 \le 3.7736$ (ii) $2.6274 \le 2.7970 \le 3.1875$

By Corollary 4.1, we have $3.7736 \ge 2.6274$.

VI. CONCLUSION

In the directed graph we cannot expect the roots are always real but it has complex roots also so we derived the bounds for the energy of DIFG with real and complex eigenvalues of the DIFAM of the given graph.

REFERENCES

1. G. Deepa, B. Praba, V.M. Chandrasekaran, Bounds of energy with real and complex complex roots of an intuitionistic fuzzy directed graph, *International Journal of Mathematical Modelling and Numerical Optimisation*, 2017, 8(1), PP. 42-61
2. C.F. Jaenisch de., Applications de l Analyze Mathematique a Jen des Echecs, *Saint Pétersbourg*, Petrograd, 1862.
3. S. Mitchell and S.T. Hedetniemi, Edge domination in trees, *Congr.*, 19, 1977, pp. 489-509
4. M.Q. Mohioub, Perfect dominating set in intuitionistic fuzzy graphs, *United States of America Research Journal*, 2014, 2(3), pp. 14-21.
5. A. Nagoor Gani, M. Akram and P. Vijayalakshmi, Certain type of fuzzy set in a fuzzy graph, *Int. J.Mach. Learn. Cyber*, DOI 10.1007/s13042-014-0267-8, 2014.
6. O. Ore, Theory of graphs, *Amer. Math. Soc. Colloq. Publ*., 38, 1962.
7. B. Praba, V.M. Chandrasekaran and G. Deepa, Energy of an intuitionistic fuzzy graph, *Italian journal of pure and applied mathematics*, 32, 2014, pp. 431-444.
8. B. Praba, G. Deepa, and V.M. Chandrasekaran, Dominating energy in an intuitionistic fuzzy graph, *International journal of pharmacy and technology*, 7(2), 2015, pp. 8876-8886.

AUTHORS PROFILE

First Author. Dr. G. DEEPA, Assistant Professor, Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Tamil Nadu, India.

Second Author. Dr. B. PRABA, Professor, Department of Mathematics, SSN College of Engineering, Chennai, Tamil Nadu, India.

Third Author. Dr. V.M. CHANDRASEKARAN, Professor, Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Tamil Nadu, India.

Fourth Author. Dr. K. RAJAKUMAR, Associate Professor, School of Computer Science and Engineering, Vellore Institute of Technology, Tamil Nadu, India.