Bounds For The Real And Complex Eigen Values Of Energy Of The Dominating Intuitionistic Fuzzy Graph


Abstract: We define the dominating energy for an intuitionistic fuzzy digraph \( G = (V, E, [\mu, \gamma]) \) and we derive the upper and lower bounds with real and complex eigenvalues for the energy of dominating intuitionistic fuzzy graph.

Index Terms: Intuitionistic fuzzy set (IFS), Fuzzy Graph (FG), Intuitionistic Fuzzy Graph (IFG).

I. INTRODUCTION

Domination in graphs is an extension area of graph theory. The most basic notion of domination happened in the game of chess. Jaenisch [2] tried to define the least member of queens essential to cover chess board. The study of domination in graphs was established by Ore [6]. The concept of edge domination was introduced by Mitchell and Hedetniemi [3]. Nagoorgani et al. [5] provided some exciting properties of fuzzy dominating set. The idea of perfect dominating set in IFG was presented by Mohioub [4]. Domination is a suitable tool for defining commercial networks. Study of domination concepts in IFG’s are more convenient than FG’s.

II. INTUITIONISTIC FUZZY GRAPH

Definition2.1. [7] An IFG is defined as \( G = (V, E, [\mu, \gamma]) \) where \( V \) is the set of vertices and \( E \) is the set of edges. Here \( \mu \) is an intuitionistic fuzzy membership value defined on \( V \times V \) and \( \gamma \) is an intuitionistic fuzzy non-membership value defined on \( V \times V \).

We denote \( \mu(v_i, v_j) \) by \( \mu_{ij} \) and \( \gamma(v_i, v_j) \) by \( \gamma_{ij} \) such that

\[
0 \leq \mu_{ij} + \gamma_{ij} \leq 1, \quad 0 \leq \mu_{ij} \gamma_{ij} \leq 1, \quad \text{where} \quad \gamma_{ij} = 1 - \mu_{ij} - \gamma_{ij}.
\]

Also we denote \( \mu_{ij} \) represents the strength of the relationship between \( v_i \) and \( v_j \) and \( \gamma_{ij} \) represents the strength of the non-relationship between \( v_i \) and \( v_j \) respectively.

Definition2.2. [7] Intuitionistic fuzzy adjacency matrix (IFAM) of an IFG is defined as \( A(G) = [a_{ij}] \) where \( a_{ij} = (\mu_{ij}, \gamma_{ij}) \).

Definition2.3. [7] An IFAM of an IFG can be written as two matrices one containing the entries as membership value and the other containing the entries as non-membership value. i.e.

\[
A(G) = \begin{bmatrix} (\mu_{ij}, \gamma_{ij}) \end{bmatrix}.
\]

We denote \( A_{\mu}(G) = [\mu_{ij}] \) is an IFAM of membership value and \( A_{\gamma}(G) = [\gamma_{ij}] \) is an IFAM of non-membership value.

Definition2.4. [7] Spectrum of an IFAM \( A(G) \) is defined as \( \{X, Y\} \) where \( X \) is the set eigenvalues (EV’s) of \( A_{\mu}(G) \) and \( Y \) is the set EV’s of \( A_{\gamma}(G) \). It is denoted by \( \text{spec}(A(G)) \).

Definition2.5. [7] Let \( \delta_1, \delta_2, \ldots, \delta_n \) be the EV’s of \( A_{\mu}(G) \) and let \( \delta_1, \delta_2, \ldots, \delta_n \) be the EV’s of \( A_{\gamma}(G) \). Then the energy of an IFG is defined as

\[
E(A(G)) = \left( \sum_{i=1}^{n} |\delta_i| \right) \left( \sum_{i=1}^{n} |\gamma_i| \right)
\]

where \( \sum_{i=1}^{n} |\delta_i| \) is the energy of membership value denoted by \( E(A_{\mu}(G)) \) and \( \sum_{i=1}^{n} |\gamma_i| \) is the energy of non-membership value denoted by \( E(A_{\gamma}(G)) \).

Theorem2.1. [7] Let \( G \) be an IFG with \( n \) vertices. If \( \lambda_1, \lambda_2, \ldots, \lambda_n \) are the EV’s (real or complex) of \( A_{\mu}(G) \) and \( \delta_1, \delta_2, \ldots, \delta_n \) are the EV’s (real or complex) of \( A_{\gamma}(G) \), then

\[
\left( \sum_{i=1}^{n} \lambda_i \right) \left( \sum_{i=1}^{n} \delta_i \right) = 2 \sum_{i=1}^{n} |\lambda_i| |\delta_i| \left( \sum_{i=1}^{n} \lambda_i \right) \left( \sum_{i=1}^{n} \delta_i \right) = 2 \sum_{i=1}^{n} |\gamma_i|^2.
\]

Theorem2.2. [1] Let \( G \) be an IFG (without loops) with \( n \) vertices. If \( \lambda_1, \lambda_2, \ldots, \lambda_n \) are the real eigenvalues (REV’s) of \( A_{\mu}(G) \) and \( \delta_1, \delta_2, \ldots, \delta_n \) are the REV’s of \( A_{\gamma}(G) \), then
Bounds For The Real And Complex Eigen Values Of Energy Of The Dominating Intuitionistic Fuzzy Graph

(i) $\left\lfloor 2 \sum_{i=1}^{n} \mu_i \mu_j + n(n-1) |A|^2 \right\rfloor \leq E(A_j(G)) \leq \left\lfloor 2n \sum_{i=1}^{n} \mu_i \mu_j \right\rfloor$

(ii) $\left\lfloor 2 \sum_{i=1}^{n} \gamma_i \gamma_j + n(n-1)|B|^2 \right\rfloor \leq E(A_j(G)) \leq \left\lfloor 2n \sum_{i=1}^{n} \gamma_i \gamma_j \right\rfloor$

Where $|A|$ is the determinant of $A_j(G)$ and $|B|$ is the determinant of $A_j(G)$.

**Theorem 2.3.** Let $G$ be IFG (without loops) with $n$ vertices and if $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the complex eigenvalues (CEV’s) of $A_j(G)$ and $\delta_1, \delta_2, \ldots, \delta_n$ are the CEV’s of $A_j(G)$, then

(i) $\left\lfloor \sum_{i=1}^{n} |\lambda|^2 \right\rfloor + n(n-1)|\lambda|^2 \leq E(A_j(G)) \leq \left\lfloor \sum_{i=1}^{n} |\lambda|^2 \right\rfloor - 2 \sum_{i=1}^{n} |\lambda_i|$

(ii) $\left\lfloor \sum_{i=1}^{n} |\delta|^2 \right\rfloor + n(n-1)|\delta|^2 \leq E(A_j(G)) \leq \left\lfloor \sum_{i=1}^{n} |\delta|^2 \right\rfloor - 2 \sum_{i=1}^{n} |\delta_i|$

Where $|A|$ is the determinant of $A_j(G)$ and $|B|$ is the determinant of $A_j(G)$.

**III. DOMINATING IFG (DIFG)**

Here we consider an IFG and we define $(\mu, \gamma) : V \rightarrow [0, 1]$ and we prove that $(\mu_i, \gamma_i)$ is an IFS. Then we determine the energy of DIFG $G=(V, E, \mu, \gamma, \mu_i, \gamma_i)$. 

**Definition 3.1.** Let $G$ be an IFG. An intuitionistic fuzzy vertex membership value $\mu_i$, is defined by $\mu_i : V \rightarrow [0, 1]$ and an intuitionistic fuzzy vertex non-membership value $\gamma_i$, is defined by $\gamma_i : V \rightarrow [0, 1]$ such that $0 \leq \mu_i(v) + \gamma_i(v) \leq 1$ and $\mu_i(v) \mu_j(v) \gamma_i(v) \gamma_j(v) \leq 1$ where $\gamma_i(v) = 1 - \mu_i(v) - \gamma_i(v)$.

Now we define the set

$(\mu, \gamma) : V \rightarrow [0, 1]$ for $\mu(v) = \max_{v \in V} (\mu(v), \gamma(v))$ and $\gamma(v) = \min_{v \in V} (\mu(v), \gamma(v))$ for any $v \in V$.

**Lemma 3.1.** Let $G$ be an IFG. If $\mu_i(v) = \max_{v \in V} (\mu(v), \gamma(v))$ and $\gamma_i(v) = \min_{v \in V} (\mu(v), \gamma(v))$, then $(\mu_i, \gamma_i)$ is an IFS.

[Further discussion, we call $G=(V, E, \mu, \gamma, \mu_i, \gamma_i)$ is a DIFG.]

**Definition 3.2.** An arc $(v_i, v_j)$ of a DIFG $G=(V, E, \mu, \gamma, \mu_i, \gamma_i)$ is called a strong arc if $\mu_i(v_i, v_j) \leq \mu_i(v_j) \land \mu_j(v_j) \land \gamma_i(v_i, v_j) \leq \gamma_i(v_j) \land \gamma_j(v_j)$. 

**Definition 3.3.** Let $G=(V, E, \mu, \gamma, \mu_i, \gamma_i)$ be a DIFG. Let $u, v \in V$, we say that $u$ dominates $v$ in $G$ if there exists a strong arc from $u$ to $v$. A subset $D \subseteq V$ is said to be dominating set in $G$ if for every $v \in V - D$, there exist $u$ in $D$ such that $u$ dominates $v$.

**Definition 3.4.** A dominating intuitionistic fuzzy adjacency matrix (DIFAM) of DIFG is defined as $D(G) = (d_{ij})$, where $$d_{ij} = \begin{cases} (\mu_i, \gamma_i) & \text{if } (v_i, v_j) \in E \\ 1, & \text{if } i = j \text{ and } v_i \in D \\ 0, & \text{otherwise} \end{cases}$$

This DIFAM $D(G)$ can be written as $D(G) = (D_\mu(G), D_\gamma(G))$.

**Definition 3.5.** Eigenvalues of DIFAM $D(G)$ is defined as $(X, Y)$ where $X$ is the set of EV’s of $D_\mu(G)$ and $Y$ is the set of EV’s of $D_\gamma(G)$.

**Definition 3.6.** Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the EV’s of $D_\mu(G)$ and $\delta_1, \delta_2, \ldots, \delta_n$ be the EV’s of $D_\gamma(G)$. Then the energy of DIFG $G=(V, E, \mu, \gamma, \mu_i, \gamma_i)$ is defined as

$$E(D(G)) = \left\lfloor \sum_{i=1}^{n} |\lambda_i| + \sum_{i=1}^{n} |\delta_i| \right\rfloor$$

where $\sum_{i=1}^{n} |\lambda_i|$ is the energy of membership value denoted by $E(D_\mu(G))$ and $\sum_{i=1}^{n} |\delta_i|$ is the energy of the non-membership value denoted by $E(D_\gamma(G))$.

**Theorem 3.1.** Let $G=(V, E, \mu, \gamma, \mu_i, \gamma_i)$ be a DIFG. Let $D$ be a dominating set. If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the real or complex EV’s of $D_\mu(G)$ and $\delta_1, \delta_2, \ldots, \delta_n$ are the real or complex EV’s of $D_\gamma(G)$ then

(i) $\sum_{i=1}^{n} \lambda_i = |D| \quad$ (ii) $\sum_{i=1}^{n} \lambda_i^2 = \sum_{i=1}^{n} \mu_i^2 + 2 \sum_{i=1}^{n} \mu_i \mu_j$

(iii) $\sum_{i=1}^{n} \delta_i = |D| \quad$ (iv) $\sum_{i=1}^{n} \delta_i^2 = \sum_{i=1}^{n} \gamma_i^2 + 2 \sum_{i=1}^{n} \gamma_i \gamma_j$

**IV. BOUNDS FOR ENERGY OF DIFG**

**Theorem 4.1.** Let $G=(V, E, \mu, \gamma, \mu_i, \gamma_i)$ be a DIFG with $n$ vertices and let $D$ be a dominating set. If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the REV’s of $D_\mu(G)$ and $\delta_1, \delta_2, \ldots, \delta_n$ are the REV’s of $D_\gamma(G)$ then...
Corollary 4.1. Let \( G = (V, E, \mu, \gamma, \mu_1, \gamma_1) \) be a DIFG with \( n \) vertices and let \( D \) be a dominating set. If \( \lambda_1, \lambda_2, ..., \lambda_n \) are the REV’s of \( D_\gamma(G) \) and if \( \delta_1, \delta_2, ..., \delta_n \) are the REV’s of \( D_\mu(G) \) and if \( E(D_\gamma(G)) \geq E(D_\mu(G)) \), then

\[
\sum_{i=1}^{n} \mu_i^2 + 2 \sum_{i=1}^{n} \mu_i \mu_j \geq \sum_{i=1}^{n} \lambda_i^2 + 2 \sum_{i=1}^{n} \lambda_i \lambda_j + n(n-1)\mu^2.
\]

**Proof.** From Theorem 4.1, we have

\[
\sqrt{\sum_{i=1}^{n} \gamma_i^2 + 2 \sum_{i=1}^{n} \gamma_i \gamma_j} \geq \sqrt{\sum_{i=1}^{n} \mu_i^2 + 2 \sum_{i=1}^{n} \mu_i \mu_j}.
\]

Theorem 4.2. Let \( G = (V, E, \mu, \gamma, \mu_1, \gamma_1) \) be a DIFG with \( n \) vertices and let \( D \) be a dominating set. If \( \lambda_1, \lambda_2, ..., \lambda_n \) are the CEV’s of \( D_\mu(G) \) and if \( \delta_1, \delta_2, ..., \delta_n \) are the CEV’s of \( D_\gamma(G) \) then

\[
\sum_{i=1}^{n} \lambda_i^2 \geq \sum_{i=1}^{n} \delta_i^2 + 2 \sum_{i=1}^{n} \delta_i \delta_j + n(n-1)\mu^2.
\]

**Proof.** (i) Upper bound: In Cauchy-Schwarz inequality

\[
\left( \sum_{i=1}^{n} a_i b_i \right)^2 \leq \left( \sum_{i=1}^{n} a_i^2 \right) \left( \sum_{i=1}^{n} b_i^2 \right)
\]

Put \( a_i = 1, \ b_i = |\lambda_i| \), we get

\[
\sum_{i=1}^{n} |\lambda_i|^2 \leq \left( \sum_{i=1}^{n} |\lambda_i|^2 \right) \left( \sum_{i=1}^{n} |\lambda_i|^2 \right)^{\frac{1}{2}}
\]

Substitute Eq. (2) in Eq. (1), we get
Bounds For The Real And Complex Eigen Values Of Energy Of The Dominating Intuitionistic Fuzzy Graph

Substitute Eq. (2) in Eq. (1), we get

$$E(D_e(G)) \geq \sqrt{\left( \sum_{i=1}^{n} |\lambda_i| \right)^2 - 2 \sum_{1 \leq i < j \leq n} |\lambda_i \lambda_j|}$$

Lower bound: By Definition 3.6, we have

$$E(D_e(G)) = \left( \sum_{i=1}^{n} |\lambda_i| \right)^2$$

$$= \sum_{i=1}^{n} |\lambda_i|^2 + 2 \sum_{1 \leq i < j \leq n} |\lambda_i \lambda_j|$$

$$= \left( \sum_{i=1}^{n} |\lambda_i| \right)^2 - 2 \sum_{1 \leq i < j \leq n} |\lambda_i \lambda_j| + \frac{n(n-1)}{2} \sum_{i=1}^{n} |\lambda_i|^2$$

$$\Rightarrow E(D_e(G)) \geq \sqrt{\left( \sum_{i=1}^{n} |\lambda_i| \right)^2 - 2 \sum_{1 \leq i < j \leq n} |\lambda_i \lambda_j| + n(n-1)A^2}$$

From Eq. (3) and Eq. (4) we get (i) and similarly we can prove (ii).

Corollary 4.3. Let $G = (V, E, \mu', \mu, \gamma, \gamma')$ be a DIFG with $n$ vertices and let $D$ be a dominating set. If $\lambda_1, \lambda_2, ..., \lambda_n$ are the CEV’s of $D_e(G)$ and if $\delta_1, \delta_2, ..., \delta_n$ are the CEV’s of $D_\gamma(G)$ and if $E(D_e(G)) \geq E(D_\gamma(G))$ then

$$E(D_e(G)) \geq \sqrt{\left( \sum_{i=1}^{n} |\lambda_i| \right)^2 - 2 \sum_{1 \leq i < j \leq n} |\lambda_i \lambda_j| + n(n-1)A^2}$$

Proof. From Theorem 4.2, we have

$$\sqrt{\left( \sum_{i=1}^{n} |\lambda_i| \right)^2 - 2 \sum_{1 \leq i < j \leq n} |\lambda_i \lambda_j|} \geq E(D_\gamma(G))$$

$$\Rightarrow \sqrt{\left( \sum_{i=1}^{n} |\lambda_i| \right)^2 - 2 \sum_{1 \leq i < j \leq n} |\lambda_i \lambda_j|} \geq E(D_e(G))$$

Again by Theorem 4.2, we have

$$E(D_e(G)) \geq \sqrt{\left( \sum_{i=1}^{n} |\lambda_i| \right)^2 - 2 \sum_{1 \leq i < j \leq n} |\lambda_i \lambda_j| + n(n-1)A^2}$$

Therefore we get

$$\sqrt{\left( \sum_{i=1}^{n} |\lambda_i| \right)^2 - 2 \sum_{1 \leq i < j \leq n} |\lambda_i \lambda_j| + n(n-1)A^2}$$

Theorem 4.3. Let $G = (V, E, \mu, \gamma)$ be an IFG and let $A(G) = (A_e(G), A_\gamma(G))$ be an IFAM of $G$. Let $G_i = (V, E, \mu_i, \gamma_i)$ be the DIFG of $G_i$ and let $D_i = (D_e(G_i), D_\gamma(G_i))$ be the DIFAM of $G_i$. If $\lambda_1, \lambda_2, ..., \lambda_n$ and $\delta_1, \delta_2, ..., \delta_n$ are the REV’s then

$$E(D_e(G_i)) \geq \sqrt{\left( \sum_{i=1}^{n} |\lambda_i| \right)^2 - 2 \sum_{1 \leq i < j \leq n} |\lambda_i \lambda_j| + n(n-1)A_i^2}$$

Proof. By Theorem 2.2, we have

$$\left( E(D_e(G_i)) \right)^2 \geq 2 \sum_{1 \leq i < j \leq n} \mu_i \mu_j + n(n-1)A_i^2$$

By Theorem 4.1, we have

$$E(D_e(G)) \leq n \left( \sum_{i=1}^{n} \mu_i^2 + 2 \sum_{1 \leq i < j \leq n} \mu_i \mu_j \right)$$

Corollary 4.4. Let $G = (V, E, \mu', \mu, \gamma, \gamma')$ be a DIFG with $n$ vertices and let $D$ be a dominating set. If $\lambda_1, \lambda_2, ..., \lambda_n$ are the CEV’s of...
Using Eq. (5), we get
\[
\left(E(D_\mu(G))\right)^2 \leq n\left(\sum_{i=1}^{n} |\lambda_i|^2 + E(A_\mu(G))^2\right)
\]
Similarly we can prove (ii).

**Theorem 4.** Let \( \mathcal{G}=(V,E,\mu,\gamma) \) be an IFG and let
\( \mathcal{M}(G)=(\lambda_1(G),\lambda_2(G)) \) be an IFAM of \( \mathcal{G} \).
Let \( \mathcal{G}_i=(V,E,\mu,\gamma,\mu_i,\gamma_i) \) be the DIFG of \( \mathcal{G}_i \) and let
\( \mathcal{D}(G)=(D_\mu(G),D_\gamma(G)) \) be the DIFAM of \( \mathcal{G}_i \). If
\( \lambda_1,\lambda_2,\ldots,\lambda_n, \delta_1,\delta_2,\ldots,\delta_n \) are the CEV’s then
\[
(i) \left(E(D_\mu(G))\right)^2 \leq n\left[E(A_\mu(G))^2\right] \leq n\left[E(A_\gamma(G))^2\right].
\]

**Proof.** By Theorem 2.3, we have
\[
\left(E(A_\mu(G))^2\right)^2 \geq \sum_{i=1}^{n} |\lambda_i|^2 + n(n-1)|A|^2 \geq \sum_{i=1}^{n} |\lambda_i|^2.
\]
By Theorem 4.2, we have
\[
\left(E(D_\mu(G))\right)^2 \leq n\left(\sum_{i=1}^{n} |\lambda_i|^2 - 2\sum_{i=1}^{n} |\lambda_i|^2\right)
\]
Using Eq. (6), we get
\[
\left(E(D_\mu(G))\right)^2 \leq n\left[E(A_\mu(G))^2\right]
\]
Similarly we can prove (ii).

**V. NUMERICAL EXAMPLE**

Let us demonstrate the above ideas in the following example.

**Example 5.1.**

Consider the DIFG given by \( \mathcal{G}=(V,E,\mu,\gamma,\mu_1,\gamma_1) \) which is shown in Figure 1, where \( V = \{v_1,v_2,v_3,v_4\} \) and \( \mu_1,\gamma_1 \) are given by
\[
\mu_1 : V \rightarrow [0,1] \text{ and } \gamma_1 : V \rightarrow [0,1].
\]
For the DIFG in Figure 1, we have
\[
\mu_1(v_1) = \max \left(\mu(v_1,v_j)\right) = \max \left(\mu(v_1,v_2),\mu(v_1,v_3),\mu(v_1,v_4)\right) = \max (0.6,0.3) = 0.6.
\]
Similarly,
\[
\mu_1(v_2) = 0.4, \mu_1(v_3) = 0.7, \mu_1(v_4) = 0.8
\]
and
\[
\gamma_1(v_1) = \min \left(\gamma(v_1,v_j)\right) = \min \left(\gamma(v_1,v_2),\gamma(v_1,v_3),\gamma(v_1,v_4)\right) = \min (0.2,0.1) = 0.1.
\]
Similarly,
\[
\gamma_1(v_2) = 0.1, \gamma_1(v_3) = 0.1, \gamma_1(v_4) = 0.1.
\]
Here \( V_1 \) dominates \( V_4 \) and \( V_2 \) dominates \( V_3 \) because it satisfies the following:
\[
\rho(v_1,v_3) \leq \rho(v_1,v_2), \rho(v_1,v_4) \leq \rho(v_1,v_3), \rho(v_2,v_3) \leq \rho(v_2,v_4), \rho(v_2,v_4) \leq \rho(v_2,v_3).
\]
Here
\[
V = \{v_1,v_2,v_3,v_4\} \text{ and } D = \{v_1,v_2\}, \quad V - D = \{v_3,v_4\}.
\]
(6) The DIFAM is
\[
D(G) = \begin{pmatrix}
(0.1) & (0.6,0.2) & 0 & (0.3,0.1) \\
(0.4,0.3) & (0.1) & (0.3,0.1) & 0 \\
(0.5,0.2) & (0.7,0.1) & 0 & (0.6,0.3) \\
(0.3,0.5) & (0.8,0.1) & (0.4,0.5) & 0
\end{pmatrix}
\]
Where
\[
D_\mu(G) = \begin{pmatrix}
1 & 0.6 & 0 & 0.3 \\
0.4 & 1 & 0.3 & 0 \\
0.5 & 0.7 & 0 & 0.6 \\
0.3 & 0.8 & 0 & 0.4
\end{pmatrix}
\]
and
\[
D_\gamma(G) = \begin{pmatrix}
1 & 0.2 & 0 & 0.1 \\
0.3 & 1 & 0.1 & 0 \\
0.2 & 0.1 & 0 & 0.3 \\
0.5 & 0.1 & 0.5 & 0
\end{pmatrix}
\]
The energies of \( A_\mu(G) \) and \( A_\gamma(G) \) are given by
\[
\text{spec}(A_\mu(G)) = \{1.0786, -0.0772, -0.5007 + 0.2354i, -0.5007 - 0.2354i\}
\]
\[
\text{spec}(A_\gamma(G)) = \{0.5391, 0.1028, -0.1928, -0.4491\}
\]
\[
E(A_\mu(G)) = 1.0786 + 0.0772 + 0.5533 + 0.5533 = 2.2625
\]
\[
E(A_\gamma(G)) = 0.5391 + 0.1028 + 0.1928 + 0.4491 = 1.2838
\]
Bounds For The Real And Complex Eigen Values Of Energy Of The Dominating Intuitionistic Fuzzy Graph

\[ \text{spec}(D_e(G)) = \{1.7605, -0.4261, 0.5025, 0.1631\} \]

\[ \text{spec}(D_r(G)) = \{1.2935, 0.7733, 0.3317, -0.3985\} \]

\[ E(D_e(G)) = 1.7605 + 0.4261 + 0.5025 + 0.1631 = 2.8523 \]

\[ E(D_r(G)) = 1.2935 + 0.7733 + 0.3317 + 0.3985 = 2.7970 \]

From Theorem 2.3, we have

(i) \( 1.9229 \leq 2.2625 \leq 2.6695 \) (ii) \( 1.1711 \leq 1.2838 \leq 1.4698 \)

From Theorem 4.1, we have

(i) \( 2.5565 \leq 2.8523 \leq 3.7736 \) (ii) \( 2.6274 \leq 2.7970 \leq 3.1875 \)

By Corollary 4.1, we have \( 3.7736 \geq 2.6274 \).

VI. CONCLUSION

In the directed graph we cannot expect the roots are always real but it has complex roots also so we derived the bounds for the energy of DIFG with real and complex eigenvalues of the DIFAM of the given graph.

REFERENCES


AUTHORS PROFILE

First Author. Dr. G. DEEPA, Assistant Professor, Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Tamil Nadu, India.

Second Author. Dr. B. PRABA, Professor, Department of Mathematics, SSN College of Engineering, Chennai, Tamil Nadu, India.

Third Author. Dr. V.M. CHANDRASEKARAN, Professor, Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Tamil Nadu, India.

Fourth Author. Dr. K. RAJAKUMAR, Associate Professor, School of Computer Science and Engineering, Vellore Institute of Technology, Tamil Nadu, India.