

# Nano $\alpha\psi$ -Connectedness and Compactness in Nano Topological Spaces



R. Jeevitha, M. Parimala, R. Udhaya Kumar

**Abstract:** In this article, the concept of nano  $\alpha\psi$  connected,  $T_{N\alpha\psi}$  space in nano topological spaces (NTS) and entrenched few of their accompanying features. Further we investigate the  $N_{\alpha\psi}$  compact space in nano topological space. AMS (2010) Subject classification: 54A05,54C10,54D15.

**Keywords:**  $N_{\alpha\psi}$  connected,  $T_{N\alpha\psi}$  space and  $N_{\alpha\psi}$  compact.

## I. INTRODUCTION

Connectedness and disconnectedness in topology is introduced by A.V.Arhangelskii et al.,[2]. Compactness, in general is an essential part of the topological spaces (TS) with regard to the property of closed and bounded subsets. The idea of compactness and connectedness are beneficial for basic ideas of general topology as well as for advanced branches of mathematics. Benchalli and Priyanka M Bansali [3] investigated the properties of gb connectedness and gb compactness in topology. sg compact space introduced by Dontchev and Ganster [5]. E.Ekici [6] analyzed the properties of separated and connected space. b connectedness and b disconnectedness introduced by ABD EL et al.,[1]. The perception of  $\alpha$  and  $\psi$  closed sets in TS were introduced by O.Njastad [10] and M.K.R.S.Veerakumar[11] was initiate the idea of semi closed and semi pre closed sets. The notion  $\alpha\psi$  closed set in topology is established by R.Devi et.al., [4]. Lellis Thivagar [9] was introduced a new concept of nano topology, it was defined interms with reference to estimation and boundary section of a subset of the world via correspondence relation on it. The formulation of the notion of nano closed set, nano interior and ncl as well as the introduced of notion of nsc and nac set was done by Lellis Thivagar. On  $\alpha\psi$  compact spaces are introduced by V.Kokilavani et al.,[7]. S.Krishnaprakash et al.,[8] innovative some concept of nano compact space and nano connected in nano topology. The intention of the research work is to establish the conception of  $N_{\alpha\psi}$  connected,  $T_{N\alpha\psi}$  space closed set and find few of their features. It also established the conception of  $N_{\alpha\psi}$  compact. The current study is about few of associated theorems, results and attributes.

## II. PRELIMINARIES

Throughout this paper, we use the following symbol, nano open - no, nano closed - nc, nano closure - ncl, nano interior - nint, nano topological space -NTS, nano  $\alpha\psi$  open-  $n\alpha\psi o$ , nano  $\alpha\psi$  closed -  $n\alpha\psi c$ .

**Definition 2.1.** [9] Let  $V$  be a not empty limited set of substance namely the universe and  $R$  be a correspondence relation on  $V$  called as the indiscernibility relation. Hence  $V$  is separated into disjoint correspondence courses. Let  $Y$  is a subset of  $V$ , then

(a) The lower approximation :  $\underline{LR} = \bigcup_{y \in V} \{R(y) : R(y) \subseteq Y\}$ , where  $R(Y)$  intend the correspondence courses ascertained by  $y \in V$ .

(b) The upper approximation :  $\overline{UR} = \bigcup_{y \in V} \{R(y) : R(y) \cap Y \neq \emptyset\}$ , where  $R$  is the Situated of every element, which can be probably defined as  $Y$  in concern with  $R$ .

(c) The boundary region :  $B_R = \overline{UR} - \underline{LR}$ , where  $R$  is the Situated of every element, which can be probably defined as neither as  $Y$  nor as not  $Y$  in concern with  $R$ .

**Property 2.2.** [9] If  $(U,R)$  is a estimation space and  $X, Y \subseteq U$ . Then

1.  $\underline{LR} \subseteq X \subseteq \overline{UR}$
2.  $\underline{LR}(\emptyset) = \overline{UR}(\emptyset) = \emptyset$  and  $\underline{LR}(U) = \overline{UR}(U) = U$
3.  $\overline{UR}(X \cup Y) = \overline{UR}(X) \cup \overline{UR}(Y)$
4.  $\overline{UR}(X \cap Y) \subseteq \overline{UR}(X) \cap \overline{UR}(Y)$
5.  $\underline{LR}(X \cup Y) \supseteq \underline{LR}(X) \cup \underline{LR}(Y)$
6.  $\underline{LR}(X \cap Y) = \underline{LR}(X) \cap \underline{LR}(Y)$
7.  $\underline{LR}(X) \subseteq \underline{LR}(Y)$  and  $\overline{UR}(X) \subseteq \overline{UR}(Y)$  whenever  $X \subseteq Y$
8.  $\overline{UR}(X^c) = [\underline{LR}(X)]^c$  and  $\underline{LR}(X^c) = [\overline{UR}(X)]^c$
9.  $\underline{LR}(\underline{LR}(X)) = \overline{UR}(\underline{LR}(X)) = \underline{LR}(X)$
10.  $\overline{UR}(\overline{UR}(X)) = \underline{LR}(\overline{UR}(X)) = \overline{UR}(X)$

**Definition 2.3.** [8] A NTS  $V$  is nano compact if each no cover of  $V$  has a finite sub cover.

**Definition 2.4.** [9] Let  $V$  be an universe and  $R$  be corresponding relation on  $V$  and

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$\tau_R(X) = \{V, \phi, LR, \overline{UR}, B_R\}$  where  $X \subseteq V$ . Then

$\tau_R(X)$  fulfills the subsequent axioms:

1.  $V$  and  $\phi \in \tau_R(X)$
2. The union of the element of any sub collection of  $\tau_R(X)$  is in  $\tau_R(X)$
3. The intersection of the element of any finite sub collection of  $\tau_R(X)$  is in  $\tau_R(X)$  Then  $\tau_R(X)$  is a NTS on  $V$ , it is outlined by  $(V, \tau_R(X))$ . The element of  $\tau_R(X)$  are identified as no sets and complement of no sets is called nc. Then  $\tau_R(X)$  is a NTS on  $V$ , it is outlined by  $(V, \tau_R(X))$ . The element of  $\tau_R(X)$  are identified as no sets and complement of no sets is called nc.

**Definition 2.5.** [9] If  $(V, \tau_R(Y))$  be a NTS in concern with  $Y$  where  $Y \subseteq V$  and if  $S \subseteq V$ , then

1.  $Nint(S) = \cup \{\text{nano open set} \subseteq S\}$ .
2.  $Ncl(S) = \cap \{\text{nano closed set} \supseteq S\}$ .

**Definition 2.6.** [8] Let  $U$  be a NTS which are defined by nano connected if  $U$  can not be articulated as a separate unionized of not empty no sets. If it nano connected as a subspace then a subset of  $U$  is nano connected.

### III. NANO $\alpha\psi$ -CONNECTEDNESS

**Definition 3.1.** Let  $U$  be a NTS are said to be nano  $\alpha\psi$  connected if  $U$  can't be articulated as a separate unionized of not empty nano  $\alpha\psi$  open sets. If it nano  $\alpha\psi$  connected as a subspace then a subset of  $U$  is nano  $\alpha\psi$  connected.

**Example 3.2.** Let  $U = \{k, l, m\}$  with  $U/R = \{\{k\}, \{l, m\}\}$  and  $X = \{k\}$ . Then the nano topology  $\tau_R(X) = \{U, \phi, \{k\}\}$  and nano  $\alpha\psi$  Closed =  $\{U, \phi, \{l\}, \{m\}, \{l, m\}\}$ . Here  $(U, \tau_R(X))$  is nano  $\alpha\psi$  connected because it can not be explicit as a disjoint combination of not empty nano  $\alpha\psi$  open sets.

**Theorem 3.3.** A NTS  $U$ , the subsequent are equivalent.

- (a)  $U$  is  $N_{\alpha\psi}$  connected.
  - (b) The subset of  $U$  are  $U$  and  $\phi$  signifying nano  $\alpha\psi$  open and nano  $\alpha\psi$  closed.
  - (c) Every nano  $\alpha\psi$  continuous map of  $U$  in to a discrete space  $V$  with a minimum of two points leading to a invariable function.
- Proof.** (a)  $\Rightarrow$  (b) Assume  $U$  is  $N_{\alpha\psi}$  connected. If  $Y \subseteq U$  the some  $\text{nano}\phi$  and  $\text{nano}\psi c$  set. Now  $U = Y \cup (U - Y)$ , union of sets forced to disjoint of two non empty nano  $\alpha\psi$  open set which is disagree with (a). Hence  $Y = \phi$  or  $U$ .
- (b)  $\Rightarrow$  (a) Let  $S$  and  $T$  are disjoint non empty nano  $\alpha\psi$  subsets of  $U$  then  $U = S \cup T$ . Here  $S$  is both  $\text{nano}\phi$  and  $\text{nano}\psi c$ . By hypothesis  $S = \phi$  or  $U$ . Hence  $U$  is  $N_{\alpha\psi}$  connected.
- (b)  $\Rightarrow$  (c) Suppose  $f : S \rightarrow T$  be a nano  $\alpha\psi$  continuous.  $S$  is covered by  $\text{nano}\phi$  and  $\text{nano}\psi c$  covering  $\{f^{-1}(t) : t \in T\}$ . Let us

assume that  $f^{-1}(t) = \phi$  or  $U$  for every  $t \in T$ . If  $f^{-1}(t) = \phi$  for all  $t \in T$ , then  $f$  be unsuccessful to be a function. Hence there exist  $t \in T$  then  $f^{-1}(t) \not\subseteq \phi$ , therefore  $f^{-1}(t) \not\subseteq U$ . This proves that  $f$  is a invariable function.

(c)  $\Rightarrow$  (b) Suppose  $Y$  be a  $\text{nano}\phi$  and  $\text{nano}\psi c$  in  $U$ . Imagine  $X \not\subseteq \phi$ . Say  $f : S \rightarrow T$  be a nano  $\alpha\psi$  open map identified by  $f(X) = t$  and  $f(U - X) = y$  for few different points  $t$  and  $y$  in  $T$ . By (c)  $f$  is a invariable function. Hence  $U = X$ .

**Theorem 3.4.** Each  $N_{\alpha\psi}$  connected space is nano connected.

**Proof.** Suppose  $U$  is not  $N_{\alpha\psi}$  connected. Let us assume that  $U$  is not nano connected. Then  $H$  be a disjoint of two non empty subset of  $U$ , signifying no and nc in  $U$ . Therefore  $H$  is a not empty subset of  $U$  which is some  $\text{nano}\phi$  and  $\text{nano}\psi c$  in  $U$ . Then by [Theorem 3.3.],  $U$  is not  $N_{\alpha\psi}$  connected.

The nano connected space is not  $N_{\alpha\psi}$  connected space is proved by the succeeding example.

**Example 3.5.** Let  $U = \{l, m, n, o\}$  with  $U/R = \{\{l\}, \{m, n\}, \{o\}\}$  and  $X = \{m, n\}$ . Then the NTS  $\tau_R(X) = \{U, \phi, \{m\}, \{m, n, o\}, \{m, o\}\}$  and nano  $\alpha\psi$  Closed =  $\{U, \phi, \{l\}, \{o\}, \{l, n\}, \{m, n\}, \{m, o\}, \{n, o\}, \{l, m, o\}, \{m, n, o\}\}$ .

Here  $(U, \tau_R(X))$  is not  $N_{\alpha\psi}$  connected because  $U = \{l, m, o\} \cup \{n\}$ , here  $\{l, m, o\}$  and  $\{n\}$  are  $\text{nano}\phi$  sets. But it is nano connected because it can not be expressed as a disjoint union of non empty no sets.

**Theorem 3.6.** If  $S$  is a  $N_{\alpha\psi}$  connected set of a NTS  $U$  and  $X, Y$  are nano  $\alpha\psi$  separated set of  $U$  such that  $S \subseteq X \cup Y$ , then either  $S \subseteq X$  or  $S \subseteq Y$ .

**Proof.** Since  $S = (S \cap X) \cup (S \cap Y)$ , suppose  $(S \cap X) \cap N_{\alpha\psi}cl(S \cap Y) \subseteq X \cap N_{\alpha\psi}cl(Y) = \phi$ . Similarly  $(S \cap X) \cap N_{\alpha\psi}cl(S \cap Y) = \phi$ . If  $S \cap X$  and  $S \cap Y$  are non empty, therefore  $S$  is not  $N_{\alpha\psi}$  connected which is a contradiction. Hence, either  $S \cap X = \phi$  or  $S \cap Y = \phi$ . This gives that  $S \subseteq U$  or  $S \subseteq Y$ .

**Theorem 3.7.** Let  $S$  and  $T$  be a non empty subsets of NTS  $U$  are said to be  $N_{\alpha\psi}$  separated if  $S \cap N_{\alpha\psi}cl(T) = \phi = N_{\alpha\psi}cl(S) \cap T$ . If  $S$  is a  $N_{\alpha\psi}$  connected set of NTS  $U$  and  $S \subseteq T \subseteq N_{\alpha\psi}cl(S)$ , then  $T$  is  $N_{\alpha\psi}$  connected.

**Proof.** Let  $T$  is not  $N_{\alpha\psi}$  connected. Then there be  $N_{\alpha\psi}$  separated set  $U$  and  $V$  such that  $T = U \cup V$ . Hence  $U$  and  $V$  are non empty  $U \cap N_{\alpha\psi}cl(V) = \phi = N_{\alpha\psi}cl(U \cap V)$ . By [Theorem 3.3.] we obtain  $S \subseteq U$  or  $S \subseteq V$ .

(i) Assume that  $S \subseteq U$ . Then  $N_{\alpha\psi}cl(S) \subseteq N_{\alpha\psi}cl(U)$  and  $V \cap N_{\alpha\psi}cl(S) = \phi$ . But by hypothesis,  $V \subseteq T \subseteq N_{\alpha\psi}cl(S)$  and  $V = N_{\alpha\psi}cl(S) \cap V = \phi$ . This is contradiction since  $V$  is non empty.

(ii) Assume  $S \subseteq V$ . Similarly we get  $U$  is empty. This is contradiction. Therefore  $T$  is  $N_{\alpha\psi}$  connected.

**Result 3.8.** If  $S$  is a  $N_{\alpha\psi}$  connected subset of a NTS  $U$ , then  $N_{\alpha\psi}cl(S)$  is  $N_{\alpha\psi}$  connected.

**Theorem 3.9.** If  $S$  and  $T$  is a  $N_{\alpha\psi}$  connected and  $N_{\alpha\psi}$  separated  $U$  then  $S \cup T$  is  $N_{\alpha\psi}$  connected, where  $S$  and  $T$  be subsets of a NTS  $U$ .

**Proof.** Let us take  $S \cup T$  is not  $N_{\alpha\psi}$  connected. Then there exist  $N_{\alpha\psi}$  separated  $P, Q$  in  $U$  such that  $S \cup T = P \cup Q$ . Then  $S \subset P \cup Q$ . From [Theorem 3.6.], implies that  $S \subset P$  or  $S \subset Q$ . Similarly we get that  $T \subset P$  or  $T \subset Q$ . If  $S \subset P$  and  $T \subset P$ , then  $S \cup T = P$  and  $Q = \emptyset$ . This is a contradiction, therefore  $S \subset P$  and  $T \subset Q$ . Again  $S \subset Q$  and  $T \subset P$ . Hence we get  $N_{\alpha\psi}cl(S) \cap T \subset N_{\alpha\psi}cl(P) \cap Q = \emptyset$  and  $N_{\alpha\psi}cl(T) \cap S \subset N_{\alpha\psi}cl(P) \cap Q = \emptyset$ . Therefore  $S, T$  are  $N_{\alpha\psi}$  separated in  $U$ . This is a contradiction, hence  $S \cup T$  is  $N_{\alpha\psi}$  connected.

**Theorem 3.10.** Let  $S$  and  $T$  be two NTS and  $S \times T$  be with the product of nano topology. If  $S$  and  $T$  are  $N_{\alpha\psi}$  connected, then  $S \times T$  is  $N_{\alpha\psi}$  connected.

**Proof.** Let us take any points  $(s_1, t_1)$  and  $(s_2, t_2)$  in  $S \times T$ , then the subspace  $(S \times \{t_1\}) \cup (\{s_2\} \times T)$  contains the two points. This subspace is  $N_{\alpha\psi}$  connected, since it is the union of two  $N_{\alpha\psi}$  connected subspaces of  $S \times T$  with a point  $(s_1, t_1)$  is common. By [Theorem 3.11],  $S \times T$  is  $N_{\alpha\psi}$  connected.

**Definition 3.11.** Let  $U$  be a NTS is called  $T_{N_{\alpha\psi}}$  space, if each  $N_{\alpha\psi}$  set of  $U$  is nc subset of  $U$ .

**Definition 3.12.** A map  $f : (U, \tau_R(p)) \rightarrow (V, \tau_R(q))$  be nano  $\alpha\psi$ -continuous if  $f^{-1}(G)$  is a  $N_{\alpha\psi}$  set of  $(U, \tau_R(p))$  for every no set  $G$  in  $(V, \tau_R(q))$ .

**Definition 3.13.** A map  $f : (U, \tau_R(p)) \rightarrow (V, \tau_R(q))$  be nano  $\alpha\psi$ -irresolute if  $f^{-1}(G)$  is a  $N_{\alpha\psi}$  set of  $(U, \tau_R(p))$  for every  $N_{\alpha\psi}$  set  $G$  in  $(V, \tau_R(q))$ .

**Theorem 3.14.** Let  $f : S \rightarrow T$  be a  $N_{\alpha\psi}$  continuous onto and  $S$  is  $N_{\alpha\psi}$  connected, then  $T$  is nano connected.

**Proof.** Assume  $T$  is not nano connected. Take  $X$  and  $Y$  are disjoint non empty nano open set in  $T$  then  $T = X \cup Y$ . Since  $f$  is  $N_{\alpha\psi}$  continuous and onto,  $S = f^{-1}(X) \cup f^{-1}(Y)$  where  $f^{-1}(X)$  and  $f^{-1}(Y)$  are disjoint not empty  $N_{\alpha\psi}$  in  $U$ . This disagrees with the concept that  $S$  is  $N_{\alpha\psi}$  connected. Then  $T$  is nano connected.

**Theorem 3.15.** Suppose  $f : S \rightarrow T$  be a  $N_{\alpha\psi}$  irresolute and  $S$  is  $N_{\alpha\psi}$  connected, hence  $T$  is  $N_{\alpha\psi}$  connected.

**Proof.** Assume  $T$  is not  $N_{\alpha\psi}$  connected. Suppose  $T = X \cup Y$  where  $X$  and  $Y$  are separate non empty  $N_{\alpha\psi}$  set in  $T$ . Since  $f$  is  $N_{\alpha\psi}$  irresolute and onto,  $S = f^{-1}(X) \cup f^{-1}(Y)$  where  $f^{-1}(X)$  and  $f^{-1}(Y)$  are separate non empty  $N_{\alpha\psi}$  open in  $U$ . This disagrees with the concept that  $S$  is  $N_{\alpha\psi}$  connected. Then  $T$  is  $N_{\alpha\psi}$  connected.

**Theorem 3.16.** Let  $U$  a  $T_{N_{\alpha\psi}}$  space then  $U$  would be nano connected iff it is  $N_{\alpha\psi}$  connected.

**Proof.** Let us take  $U$  is nano connected. Then  $U$  can not be delivered as separated unionized of two non empty proper subsets of  $U$ . Assume  $U$  is not a  $N_{\alpha\psi}$  connected space. Let  $S$  and  $T$  be some two  $N_{\alpha\psi}$  subsets of  $U$  like that  $U = S \cup T$ , where  $S \cap T = \emptyset$  and  $S \subset U, T \subset U$ . Since  $U$  is  $T_{N_{\alpha\psi}}$

space and  $S, T$  are  $N_{\alpha\psi}$  open.  $S, T$  are nano open subsets of  $U$  which is disagree that  $U$  is nano connected. Hence  $U$  is  $N_{\alpha\psi}$  connected. Inversely, each no set is  $N_{\alpha\psi}$ . Hence each  $N_{\alpha\psi}$  connected space is nano connected.

#### IV. NANO $\alpha\psi$ -COMPACTNESS

**Definition 4.1.** A collection  $H_j$  for all  $j \in J$  of a  $N_{\alpha\psi}$  sets in a NTS  $U$  is called a  $N_{\alpha\psi}$  cover of a subset  $G$  of  $U$  if  $G \subset \cup_{j \in J} H_j$ .

**Definition 4.2.** A subset  $G$  of a NTS  $U$  is said to be  $N_{\alpha\psi}$  compact relative to  $U$  if for each collection  $H_j$  for all  $j \in J$  of  $N_{\alpha\psi}$  subsets of  $U$  such that  $G \subset \cup_{j \in J} H_j$  there exist a limited subset  $K$  of  $J$  hence  $G \subset \cup_{j \in J} H_j$ .

**Definition 4.3.** A NTS  $U$  is  $N_{\alpha\psi}$  compact if each  $N_{\alpha\psi}$  cover of  $U$  has a limited subcover.

**Definition 4.4.** A subset  $G$  of a NTS  $U$  is called to be  $N_{\alpha\psi}$  compact if  $G$  is  $N_{\alpha\psi}$  compact as a subspace of  $U$ .

**Theorem 4.5.** Each  $N_{\alpha\psi}$  subset of a  $N_{\alpha\psi}$  compact space is  $N_{\alpha\psi}$  compact relative to  $U$ .

**Proof.** Let  $H$  be  $N_{\alpha\psi}$  subset of  $N_{\alpha\psi}$  compact space  $U$ . So  $H^c$  is  $N_{\alpha\psi}$  in  $U$ . Assume that  $G = \{S_j : j \in J\}$  be a  $N_{\alpha\psi}$  cover of  $H$  where  $N_{\alpha\psi}$  set in  $U$ . Hence  $G^* = G \cup H^c$  is a  $N_{\alpha\psi}$  open cover of  $U$ . Because  $U$  is  $N_{\alpha\psi}$  compact and  $G^*$  is lower degree to a finite sub cover of  $U$ . That is  $U = S_{j_1} \cup S_{j_2} \cup \dots \cup S_{j_n} \cup H^c$ , since  $S_{j_n} \in G$ . But  $H$  and  $H^c$  are disjoint. Therefore  $H \subset S_{j_1} \cup S_{j_2} \cup \dots \cup S_{j_n}, S_{j_n} \in G$ . Hence any  $N_{\alpha\psi}$  open cover  $G$  of  $H$  has a limited sub cover. Hence  $H$  is  $N_{\alpha\psi}$  compact relative to  $U$ .

**Theorem 4.6.** Each  $N_{\alpha\psi}$  compact space is nano compact.

**Proof.** Assume that  $U$  be a  $N_{\alpha\psi}$  compact space. Let us take a no cover  $H_j$ , that is a collection of no set  $H_j$  in NTS. Then that no cover  $H_j$  is also a  $N_{\alpha\psi}$  cover of  $U$ , because we know that each no set is  $N_{\alpha\psi}$  open set. Here  $U$  is  $N_{\alpha\psi}$  compact, then  $N_{\alpha\psi}$  cover  $H_j$  of  $U$  contains a finite sub cover. Hence  $U$  is nano compact.

**Theorem 4.7.** If  $U$  is nano compact and  $T_{N_{\alpha\psi}}$  space then  $U$  is  $N_{\alpha\psi}$  compact. **Proof.** Let  $U$  be a nano compact and  $T_{N_{\alpha\psi}}$  space. Assume that there exist  $H_j$  be a  $N_{\alpha\psi}$  cover of  $U$ . Here  $U$  is  $T_{N_{\alpha\psi}}$  space, by hypothesis every  $N_{\alpha\psi}$  set is no. Therefore exist a no cover of  $U$ . This shows  $U$  is  $N_{\alpha\psi}$  compact.

**Theorem 4.8.** Let  $f : S \rightarrow T$  be surjective,  $N_{\alpha\psi}$  continuous function. If  $S$  is  $N_{\alpha\psi}$  compact then  $T$  is nano compact.

**Proof.** Let us assume that  $\{H_j : j \in J\}$  be a no cover of  $T$ . Now  $f$  is  $N_{\alpha\psi}$  continuous function then  $\{f^{-1}(H_j) : j \in J\}$  is  $N_{\alpha\psi}$  cover of  $S$  has a limited sub cover say  $\{f^{-1}(H_j) : j = 1, 2, \dots, n\}$ . Therefore  $S = \cup_{j=1}^n f^{-1}(H_j) \Rightarrow f(S) = \cup_{j=1}^n H_j$ . Since  $f$  is surjective, so  $T = \cup_{j=1}^n H_j$ . Hence  $\{H_1, H_2, \dots, H_n\}$  is a limited sub cover of  $\{H_j : j \in J\}$  for  $T$ . Therefore  $T$  is nano compact.

**Theorem 4.9.** If a function  $f : S \rightarrow T$  is  $N_{\alpha\psi}$  irresolute and  $G$  of  $S$  is  $N_{\alpha\psi}$  compact relative to  $S$ , then  $f(G)$  is  $N_{\alpha\psi}$  compact relative to  $T$ .



**Proof.** Let  $\{H_j : j \in J\}$  be any collection of  $N_{\alpha\psi}$  sets in  $T$  like that  $f(G) = \bigcup_{j=1}^n H_j$ . Hence  $G \subseteq \bigcup_{j=1}^n f^{-1}(H_j)$ , where  $\{f^{-1}(H_j) : j \in J\}$  is  $N_{\alpha\psi}$  open set in  $S$ . Since  $G$  is  $N_{\alpha\psi}$  compact relative to  $S$ , then there exist a limited sub collection  $\{H_1, H_2, \dots, H_n\}$  this implies that  $G \subseteq \bigcup_{j=1}^n f^{-1}(H_j)$ . Hence  $f(G) = \bigcup_{j=1}^n H_j$ . Therefore  $f(G)$  is  $N_{\alpha\psi}$  compact relative to  $T$ .



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### V. CONCLUSION

In this article it is unrepentant some the new thought of nano  $\alpha\psi$  connected and  $T_{N_{\alpha\psi}}$  space. Further the study carried out a nano  $\alpha\psi$  compact spaces and also derive some of their related attributes.

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