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Retreat: Fuzzy Multi-Criteria Decision Making Based on Pros and Cons

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***Abstract**: We introduce a new fuzzy multi-criteria decision making method using interval type-2 pentagonal fuzzy numbers. We propose to incorporate pros and cons of an issue to arrive at better decision. The necessary concepts and terminologies are defined. The proposed decision method consists of three stages. We use a prioritized interval type-2 pentagonal fuzzy aggregation operator to arrive at a tentative decision in the second stage. We use the existing concept of synthesis value and define a new suitable formula in order to accommodate fuzzy decision environment with pros and cons of an issue. A formula to combine the de-fuzzified pro-value and con-value of an alternative is proposed and the same is used to rank the alternatives. We further introduce concept of consolation score to align the final decision with the gut level of a decision maker. The fuzzy concepts such as fuzzy relations and fuzzy trace theory are employed to arrive at consolation score. This newly proposed fuzzy multi-criteria decision making method is a comprehensive and complete approach to a decision problem particularly concerning life-issues.*

***Keywords**: Fuzzy Multi-Criteria Decision Making, Fuzzy trace theory, Interval type-2 pentagonal fuzzy numbers, Prioritized interval type-2 fuzzy aggregation operator, Pros and Cons.*

I. INTRODUCTION

Decision making is much beyond selecting the best alternative. It requires prediction about future, recollection of past and analysis of the present. Every decision is impacted by past experiences, present conditions and future expectations. The purpose of decision making is to solve some conflicts and problems. The problem could be of different nature like financial, psychological, social, organizational, business etc. For different problems decision makers take different approaches. In most of the cases, multi-criteria decision making methods are followed. In case of fuzzy environment fuzzy multi-criteria decision making method is more suitable.

Decision making concerning life-issues is much more complex as it involves criteria related to object of decision as well as subject of decision. In most of the decisions concerning life-issues, criteria affecting the decision maker are given priority over other criteria. Such decisions, involve goals, values, emotions, subjective feelings and personal aspirations of a decision maker [3]. Therefore, incorporation of psychological theories of decision making along with mathematical/classical theories of decision making are very significant to make the decision process more meaningful, complete and comprehensive. There have been very few approaches, which combine different theories (marginal theories, psychological theories and mathematical theories) [1] of decision making explicitly. Ignatius of Loyola, [2] a spiritual leader and founder of the Jesuit congregation suggested a decision procedure known as ‘discernment’, which incorporates various theories – marginal theories (in terms of pros and cons), psychological theories (in terms of being aware of inner movements and in particular considering the level of consolation/desolation) and mathematical approach (weighing pros and cons) [2]. We propose a fuzzy multi-criteria decision making process considering some principles of Ignatian discernment procedure [8] and using fuzzy concepts and principles. We particularly make use of fuzzy numbers as tools to represent opinions/truth values and fuzzy principles of aggregation, ranking and prioritization for mathematical computations. We also use the principles of fuzzy trace theory [4], [6] and fuzzy relation to confirm our decision.

II. PRELIMINARY CONCEPTS

A. Interval Type-2 Pentagonal Fuzzy Numbers

An interval type-2 pentagonal fuzzy number (IT2PFN) [5] is an interval type-2 fuzzy set on R. It is defined as $\underset{\sim}{A}_{ITP} = \left[\underset{\sim}{A}^U_{ITP}, \underset{\sim}{A}^L_{ITP} \right]$, where $\underset{\sim}{A}^U_{ITP} = (a_1^U, a_2^U, a_3^U, a_4^U, a_5^U; h^U)$ and $\underset{\sim}{A}^L_{ITP} = (a_1^L, a_2^L, a_3^L, a_4^L, a_5^L; h^L)$ are upper and lower pentagonal fuzzy numbers [7] respectively such that $\underset{\sim}{A}^L_{ITP} \subseteq \underset{\sim}{A}^U_{ITP}$.

B. Fuzzy Trace Theory

Fuzzy Trace Theory (FTT) [4], [6] basically is a theory of cognition suggested by the psychologist Charles Brainerd in 1990’s. According to this theory, when a decision maker is presented with information, the information processing takes place through two types of representations namely verbatim representation and gist representation. The former concerns surface level, exact details of the issue whereas the latter concerns the bottom-line meaning of information. Gist representation takes into account goals, subjective feelings and ultimate values of a decision maker.

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The fuzzy gist representations, which are given precedence over verbatim representation, are fuzzy guidelines that reflect a construal or interpretation of choice option. The fuzzy gists are basically representations of core social and cultural values. FTT affirms that the predictive validity of measures that is based on gist principles is higher than that of measures relying on verbatim processing [4].

C. Synthesized value

T.Y. Chen (2017), introduced the concept of synthesized value for two or more interval type-2 trapezoidal fuzzy numbers [9]. If an interval type-2 trapezoidal fuzzy number A_{ij}^k denotes an evaluative rating of alternative $a_i \in A$ with respect to a criterion $x_j^k \in X^k$, where $i = 1,2,...m$; $j = 1,2...n_k$; $k = 1,2...\eta$. Then the synthesis value Q_i^k is defined as follows:

$$Q_i^k = \begin{cases} [(1,1,1,1;1),(1,1,1,1;1)] \text{ if } k = 0, \\ \left[\left(\prod_{j=1}^{n_k} a_{1ij}^{kL}, \prod_{j=1}^{n_k} a_{1ij}^{kL}, \prod_{j=1}^{n_k} a_{1ij}^{kL}, \prod_{j=1}^{n_k} a_{1ij}^{kL}; \min_{j=1}^{n_k} h_{ij}^{kL} \right), \left(\prod_{j=1}^{n_k} a_{1ij}^{kU}, \prod_{j=1}^{n_k} a_{1ij}^{kU}, \prod_{j=1}^{n_k} a_{1ij}^{kU}, \prod_{j=1}^{n_k} a_{1ij}^{kU}; \min_{j=1}^{n_k} h_{ij}^{kU} \right) \right] \end{cases} \quad (1)$$

where $k = 1,2...\eta - 1$.

D. Priority-based weight

The Priority-based weight W_i^k of k^th class for an alternative $a_i \in A$ is defined in terms of synthesized value as follows [9]:

$$W_i^k = \left[\left(\prod_{\varphi=1}^{k} q_{1i}^{\varphi-1,L}, \prod_{\varphi=1}^{k} q_{2i}^{\varphi-1,L}, \prod_{\varphi=1}^{k} q_{3i}^{\varphi-1,L}, \prod_{\varphi=1}^{k} q_{4i}^{\varphi-1,L}; \min_{\varphi=1}^{k} h_{qi}^{\varphi-1,L} \right), \left(\prod_{\varphi=1}^{k} q_{1i}^{\varphi-1,U}, \prod_{\varphi=1}^{k} q_{2i}^{\varphi-1,U}, \prod_{\varphi=1}^{k} q_{3i}^{\varphi-1,U}, \prod_{\varphi=1}^{k} q_{4i}^{\varphi-1,U}; \min_{\varphi=1}^{k} h_{qi}^{\varphi-1,U} \right) \right] \quad (2)$$

where $k = 1,2,...\eta$ and $i = 1,2,...m$.

E. Normalized Priority-based Weight

The normalized priority-based weight $W_i^{'k}$ of k^th class for an alternative $a_i \in A$ is defined as follows:

$$W_i^{'k} = W_i^k \oslash \left(\bigoplus_{\gamma=1}^{\eta} \left(n_\gamma \cdot W_i^\gamma \right) \right) \quad (3)$$

$\oslash, \oplus$ symbolize division and addition respectively.

for $k = 1,2,...\eta$ and $i = 1,2,...m$. $W_i^{'k}$ is denoted as:

$$W_i^{'k} = W_i^k \oslash \left(\bigoplus_{\gamma=1}^{\eta} \left(n_\gamma \cdot W_i^\gamma \right) \right) \quad (4)$$

where $W_i^{'kL} = \left(w_{1i}^{'kL}, w_{2i}^{'kL}, w_{3i}^{'kL}, w_{4i}^{'kL}; h_{wi}^{kL} \right)$ and $W_i^{'kU} = \left(w_{1i}^{'kU}, w_{2i}^{'kU}, w_{3i}^{'kU}, w_{4i}^{'kU}; h_{wi}^{kU} \right)$; $k = 1,2,...\eta$ and $i = 1,2,...m$.

F. A prioritized interval type-2 fuzzy Aggregation Operator

If an interval type-2 trapezoidal fuzzy number A_{ij}^k denotes an evaluative rating of alternative $a_i \in A$ with respect to criterion $x_i \in X$, where $i = 1,2,...m$, $j = 1,2,...n$. For $i = 1,2,...m$, the prioritized IT2F aggregation operator is defined as follows:

$$PIT2FA(A_{i1}, A_{i1}, ...A_{in}) = \bigoplus_{k=1}^{\eta} \left(\bigoplus_{j=1}^{n_k} \left(W_i^{'k} \otimes A_{ij}^k \right) \right) \quad (5)$$

It is denoted as $PIT2PFA(A_{i1}, A_{i1}, ...A_{in}) = \left[P_i^L, P_i^U \right]$

$$\left[P_i^L, P_i^U \right] = \left[\left(p_{1i}^L, p_{2i}^L, p_{3i}^L, p_{4i}^L; h_{pi}^L \right), \left(p_{1i}^L, p_{2i}^L, p_{3i}^L, p_{4i}^L; h_{pi}^L \right) \right] \quad (6)$$

where $p_{\xi i}^L = \sum_{k=1}^{\eta} \sum_{j=1}^{n_k} w_{\xi i}^{'kL} . a_{\xi ij}^{kL}$ and $p_{\xi i}^U = \sum_{k=1}^{\eta} \sum_{j=1}^{n_k} w_{\xi i}^{'kU} . a_{\xi ij}^{kU}$ for $\xi \in \{1,2,3,4\}$; $h_{pi}^L = \min_{k-1}^{\eta} \min_{j=1}^{n_k} \left(\min \left\{ h_{wi}^{kL}, h_{ij}^{kL} \right\} \right)$ and $h_{pi}^U = \min_{k-1}^{\eta} \min_{j=1}^{n_k} \left(\min \left\{ h_{wi}^{kU}, h_{ij}^{kU} \right\} \right)$.

G. Spiritual Guide/Special Expert

In his classical book *Spiritual Exercises* Ignatius of Loyola speaks of the role of a guide [2], [8] in decision making, where the spiritual guide helps the retreatant to clarify and interpret the subjective feelings and possibly biased interpretation to a more indifferent interpretation before making a decision. In the process of guidance and dialogue there is a shift from an old understanding to a new understanding.

H. Consolation

Consolation [2], [8] is a technical term used in the book *Spiritual Exercises*. This implies emotional state of satisfaction. According to the philosopher Locke (1976), satisfaction is a positive emotional state, resulting from the cognitive as well as emotional appraisal. Thus, satisfaction involves both cognitive and affective components. Cognitive component refers to appraisal and affective component refers to various emotions such as happiness, confidence, gratitude, calm, at peace. In the context of decision making, positive emotion is a result of positive appraisal i.e. when the decision is goal congruent and significantly important for well-being of the decision-maker.

III. RETREAT: FUZZY MULTI-CRITERIA DECISION MAKING BASED ON PROS AND CONS

In this section, we propose a fuzzy multi-criteria decision making model which is motivated by three significant principles of Discernment proposed by Ignatius of Loyola namely:

1. Interaction helps in clarifying, interpreting and depth understanding
2. Consideration of pros and cons in decision making results in better decision
3. Final decision has to be confirmed with one's value system and ultimate goal of decision

The model comprises of three major stages which are constituted of different sub-stages namely *R*ealization, *E*rror-elimination, *T*entative choice, *Re*-evaluation, *A*cting on decision and *T*ertiary observation/*T*hanksgiving (RETReAT). At the first stage data are gathered in and through the process of observation, reflection and interaction. The principle at this stage is that 'clearer and greater the number of information, the better the decision.' Realization means becoming aware of the facts about the object and subject of decision.

This concerns realization of own ultimate goal, values and subjective experience along with the gathering of factual data regarding the options. Error-elimination means clarification of confused and possibly biased information which is done by means of interaction with a special Expert/Guide. Realization of values, feelings and goals of a decision maker helps to have a deeper and greater number of information. Error-Elimination through interaction helps to clarify and thereby deepen and enlarge the space of understanding. The process of interaction results in a transition from $x_i \in S$ to $x_{i+1} \in S$; where x_i is initial state, x_{i+1} is final state and S is individual interaction space. The whole process of interaction can be modeled mathematically in the following manner:

$$x_i; F(x_i) \xrightarrow{I_1} x_{i+1}; F(x_{i+1}) \xrightarrow{I_2} \ldots \xrightarrow{I_k} x_{i+k}; F(x_{i+k})$$

The flow chart of the RETReAT model is illustrated in the figure below:

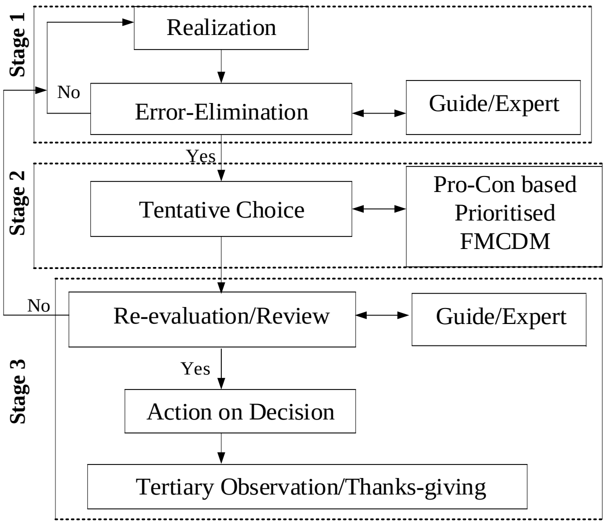

Fig. 1. Flow chart of RETReAT Model of decision making

A. Optimal clarity level

If there is no further change in the preference space occurring as the process of interaction, we consider it as optimal clarity level. We call fuzzy preference value attained at this level Universal Preference Value (UPV).

$$F(x_i) \xrightarrow{I_1} F(x_{i+1}) \xrightarrow{I_2} \ldots F(x_{i+k}) \xrightarrow{I_w} F(x_{i+k}) \xrightarrow{I_{w+1}} F(x_{i+k})$$

$F(x_{i+k})$ is universal preference value.

B. Universal Preference Value

Interaction helps us to reach universal preference value for an alternative with respect to sub-criteria. Universal preference means preference comparatively free from biases, ego-centric preferences and more aligned with goal-centric and value-centric preferences. This universal preference is taken as data in the matrix for mathematical computation.

C. Linguistic variable with corresponding IT2PFNs

We use the following interval type-2 pentagonal fuzzy scale to represent the linguistic variables:

Table- I: Linguistic variables with corresponding IT2PFNs

Linguistic Variables	Interval-valued pentagonal fuzzy numbers
Extremely false (EF)	[(0, 0, 0, 0, 0; 1), (0, 0, 0, 0, 0; 1)]
Very false (VF)	[(0, 0.015, 0.05, 0.085, 0.12; 1), (0, 0.005, 0.05, 0.095, 0.14; 1)]
False (F)	[(0.13, 0.165, 0.2, 0.235, 0.27; 1), (0.11, 0.155, 0.2, 0.245, 0.29; 1)]
Medium false (MF)	[(0.28, 0.315, 0.35, 0.385, 0.42; 1), (0.26, 0.305, 0.35, 0.345, 0.44; 1)]
Neutral (N)	[(0.43, 0.465, 0.5, 0.535, 0.57; 1), (0.41, 0.455, 0.5, 0.545, 059; 1)]
Medium true (MT)	[(0.58, 0.615, 0.65, 0.685, 0.72; 1), (0.56, 0.605, 0.65, 0.695, 0.74; 1)]
True (T)	[(0.73, 0.765, 0.8, 0.835, 0.87; 1), (0.71, 0.755, 0.8, 0.845, 0.89; 1)]
Very true (VT)	[(0.88, 0.915, 0.95, 0.985, 1; 1), (0.86, 0.905, 0.95, 0.995, 1; 1)]
Extremely true (ET)	[(1, 1, 1, 1, 1; 1), (1, 1, 1, 1, 1; 1)]

D. Pros and Cons based Multi-Criteria Decision with Prioritized Aggregation Operator using Interval Type-2 Pentagonal Fuzzy Numbers

The second stage concerns mathematical approach to analyze the data obtained in the first stage. We propose Interval Type-2 Pentagonal Prioritized Fuzzy Multi-criteria Decision Making based on Pros and Cons. We define the following concepts and terms which are used in the paper:

1) Pros: It provides the information in what way an alternative is advantageous and conducive to the goal or objectives.

2) Cons: It provides the information in what way an alternative is disadvantageous and not conducive to the goal or objectives.

3) Pro-value (p_{ij}): It provides the information to what degree an alternative is advantageous and congruent to the goal or objectives in relation to a particular pros.

4) Con-value (c_{ij}): It provides the information to what degree an alternative is disadvantageous and incongruent to the goal or objectives in relation to a particular cons.

5) Gist Representation of Decision: It is a fuzzy propositional guideline that reflects a construal and interpretation of choice option [4]. It represents what a decision maker values ultimately while taking his decision. For example, "struggle now enjoy later", is a gist value guiding a student for 'delay of gratification,' which adds value to the life of a student [4].

6) Alternative-Gist Matrix: Matrix formed with the alternatives taken as row and gist of decision in column is called alternative-gist matrix. The element a_{ij} represents the gist value of j^{th} gist with reference to i^{th} alternative.

7) Consolation Score: The Consolation score is defined as maximum of minimum of each row in the alternative-gist matrix.

E. Ranking Score

Pro-value and con-value for an alternative are decision aids in decision making. They are used in the following ways:

(i) Pro-value > con-value then decision is in FAVOR

(ii) Pro-value < con-value then decision is NOT in FAVOR

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(iii) Pro-value = con-value then decision is NEUTRAL.

The pro-value and con-value for an alternative are contradictory in nature and not compensatory. Considering these above cases we can introduce the concept of ranking score $R(A)$ which we define as follows:

$$R(A)=\alpha(pro\,value)-(\alpha-1)(con\,value) \qquad (7)$$

Where $\alpha\in[0,1]$ and $\alpha-1$ are degrees of optimism and pessimism respectively.

F. The Procedure in the newly proposed decision method

The model follows the following steps:

1. The alternative set A = {a_1, a_2…a_n} is formed

2. List of opinions/statements in terms of both pros and cons for each of the alternatives is prepared.

3. All the opinions/statements are reviewed, and redundancy removed with the help of experts and a criteria set is formed. We call it global criteria set C.

$$C=\left\{c_1^{+1}...c_p^{+1},c_1^{-1},...c_{p'}^{-1},c_{p+1}^{+2},...c_q^{+2}...,c_{p'+1}^{-2},...c_{q'}^{-2}...,c_{q+1}^{+k}...c_r^{+k},c_{q'+1}^{-k}...c_{r'}^{-k}\right\} \qquad (8)$$

4. Criteria in global set are categorized into categorized criteria sets. For the above global criteria set in (8) we can have the following categorized criteria set with both pros and cons with two conditions:

$$c^1=\left\{c_1^{+1},c_2^{+1},c_3^{+1}...,c_p^{+1}\right\}\cup\left\{c_1^{-1},c_2^{-1},c_3^{-1}...,c_{p'}^{-1}\right\}$$

$$c^2=\left\{c_{p+1}^{+2},c_{p+2}^{+2},c_{p+3}^{+2}...,c_q^{+2}\right\}\cup\left\{c_{p'+1}^{-2},c_{p'+2}^{-2},c_{p'+3}^{-2}...,c_{q'}^{-2}\right\} \qquad (9)$$

$$c^k=\left\{c_{q+1}^{+k},c_{q+2}^{+k},c_{q+3}^{+k}...,c_r^{+k}\right\}\cup\left\{c_{q'+1}^{-k},c_{q'+2}^{-k},c_{q'+3}^{-k}...,c_{q'}^{-k}\right\}$$

such that $c^i\cap c^j=\phi$ for $i\neq j;i,j=1,2...k$ and $\bigcup_{i=1}^{k}c^i=C$.

5. The categorized criteria set $C=\left\{c^1,c^2,...c^k\right\}$ is arranged according to the priority given to them as $C=\left\{c^{'1},c^{'2},...c^{'k}\right\}$ where $c^{'1},c^{'2},...c^{'k}$ are first, second and k[th] priority classes respectively.

6. We form criteria-wise pros-cons matrix with the universal preference values in the following manner:

Table- II: Criteria-wise pro-con matrix

Criteria	Pros	Cons
C_1	p_{11}	c_{11}
	p_{12}	c_{12}
	p_{13}…	c_{13}…
C_2	p_{21}	c_{21}
	p_{22}	c_{22}
	p_{23}…	c_{23}…
.	.	.
.	.	.
C_n	p_{nm}	c_{nk}

G. Mathematical procedure

1. The corresponding IT2PFNs is assigned using table 1. Then the synthesized value is computed by using the following newly suggested formula:

$$Q_i^k=\begin{cases}\left[(1,1,1,1,1;1),(1,1,1,1,1;1)\right] & if\ k=0,\\ \bigoplus_{j=1}^{n_k}A_{ij}^k & if\ k=1,2,...,\eta-1\end{cases} \qquad (10)$$

$\oplus$ is defined as the following co-norm operator:

$$a\oplus b=(a+b)-a\times b \qquad (11)$$

For $k=1,2,...\eta$ and $i=1,2,...m$; $Q_i^k=\left[Q_i^{kL},Q_i^{kU}\right]$

$$=\left[\left(q_{1i}^{kL},q_{2i}^{kL},q_{3i}^{kL},q_{4i}^{kL},q_{5i}^{kL};h_{qi}^{kL}\right),\left(\left(q_{1i}^{kU},q_{2i}^{kU},q_{3i}^{kU},q_{4i}^{kU},q_{5i}^{kU};h_{qi}^{kU}\right)\right)\right] \qquad (12)$$

2. The Priority-based weight W_i^k of k[th] class for an alternative $a_i\in A$ is calculated using the following formula:

$$W_i^k=\left[\begin{array}{l}\left(\prod_{\varphi=1}^{k}q_{1i}^{\varphi-1,L},\prod_{\varphi=1}^{k}q_{2i}^{\varphi-1,L},\prod_{\varphi=1}^{k}q_{3i}^{\varphi-1,L},\prod_{\varphi=1}^{k}q_{4i}^{\varphi-1,L},\prod_{\varphi=1}^{k}q_{5i}^{\varphi-1,L};\min_{\varphi=1}^{k}h_{qi}^{\varphi-1,L}\right),\\ \left(\prod_{\varphi=1}^{k}q_{1i}^{\varphi-1,U},\prod_{\varphi=1}^{k}q_{2i}^{\varphi-1,U},\prod_{\varphi=1}^{k}q_{3i}^{\varphi-1,U},\prod_{\varphi=1}^{k}q_{4i}^{\varphi-1,U},\prod_{\varphi=1}^{k}q_{5i}^{\varphi-1,U};\min_{\varphi=1}^{k}h_{qi}^{\varphi-1,U}\right)\end{array}\right] \qquad (13)$$

where $k=1,2,...\eta$; $i=1,2,...m$.

For $k=1,2,...\eta$ and $i=1,2,...m$; $W_i^k=\left[W_i^{kL},W_i^{kU}\right]$

$$=\left[\left(w_{1i}^{kL},w_{2i}^{kL},w_{3i}^{kL},w_{4i}^{kL},w_{5i}^{kL};h_{wi}^{kL}\right),\left(\left(w_{1i}^{kU},w_{2i}^{kU},w_{3i}^{kU},w_{4i}^{kU},w_{5i}^{kU};h_{wi}^{kU}\right)\right)\right] \qquad (14)$$

3. The normalized priority-based weight W_i^k of k[th] class for an alternative $a_i\in A$ is calculated using the following formula:

$$W_i^{'k}=W_i^k\varnothing\left(\sum_{\gamma=1}^{\eta}W_i^{\gamma}\right);\ \text{for}\ k=1,2,...\eta;\ i=1,2,...m. \qquad (15)$$

$$\sum_{\gamma=1}^{\eta}W_i^{\gamma}=\left[\begin{array}{l}\left(\sum_{\gamma=1}^{\eta}w_{1i}^{\gamma L},\sum_{\gamma=1}^{\eta}w_{2i}^{\gamma L},\sum_{\gamma=1}^{\eta}w_{3i}^{\gamma L},\sum_{\gamma=1}^{\eta}w_{4i}^{\gamma L},\sum_{\gamma=1}^{\eta}w_{5i}^{\gamma L};\min_{\gamma=1}^{\eta}h_{wi}^{\gamma L}\right)\\ \left(\sum_{\gamma=1}^{\eta}w_{1i}^{\gamma U},\sum_{\gamma=1}^{\eta}w_{2i}^{\gamma U},\sum_{\gamma=1}^{\eta}w_{3i}^{\gamma U},\sum_{\gamma=1}^{\eta}w_{4i}^{\gamma U},\sum_{\gamma=1}^{\eta}w_{5i}^{\gamma L};\min_{\gamma=1}^{\eta}h_{wi}^{\gamma U}\right)\end{array}\right] \qquad (16)$$

$\xi\in\{1,2,3,4,5\}$;

$$w_{\xi i}^{'kL}=w_{\xi i}^{kL}\varnothing\sum_{\gamma=1}^{\eta}\left(w_{(6-\xi)i}^{\gamma L}\right)\ \text{and}\ w_{\xi i}^{'kU}=w_{\xi i}^{kU}\varnothing\sum_{\gamma=1}^{\eta}\left(w_{(6-\xi)i}^{\gamma U}\right);$$

$$h_{wi}^{\gamma L}=\min\left\{h_{wi}^{kL},\min_{\gamma=1}^{\eta}h_{wi}^{\gamma L}\right\}\ \text{and}\ h_{wi}^{\gamma U}=\min\left\{h_{wi}^{kU},\min_{\gamma=1}^{\eta}h_{wi}^{\gamma U}\right\}$$

For $k=1$, 2η, and $i=1,2,...m$; $W_i^{'k}=\left[W_i^{'kL},W_i^{'kU}\right]$

$$=\left[\left(w_{1i}^{'kL},w_{2i}^{'kL},w_{3i}^{'kL},w_{4i}^{'kL};h_{wi}^{kL}\right),\left(\left(w_{1i}^{'kU},w_{2i}^{'kU},w_{3i}^{'kU},w_{4i}^{'kU};h_{wi}^{kU}\right)\right)\right] \qquad (17)$$

4. The prioritized interval type-2 pentagonal fuzzy aggregation (PIT2PFA) operator is a function $PIT2PFA:\Omega^n\rightarrow\Omega$ defined as:

$$PIT2FA\left(A_{i1},A_{i1},...A_{in}\right)=\bigoplus_{k=1}^{\eta}\left(\bigoplus_{j=1}^{n_k}\left(W_i^{'k}\otimes A_{ij}^k\right)\right) \qquad (18)$$

A_{ij}^k denotes an evaluative rating of an alternative $a_i\in A$ with respect to criterion $x_i\in X$ for $i=1,2,...m$, $j=1,2,...n$.

$$PIT2PFA\left(A_{i1},A_{i1},...A_{in}\right)=\left[P_i^L,P_i^U\right]$$

$$=\left[\left(p_{1i}^L,p_{2i}^L,p_{3i}^L,p_{4i}^L,p_{5i}^L;h_{pi}^L\right),\left(p_{1i}^U,p_{2i}^U,p_{3i}^U,p_{4i}^U,p_{5i}^U;h_{pi}^U\right)\right] \qquad (19)$$

where $p_{\xi i}^L=\sum_{k=1}^{\eta}\sum_{j=1}^{n_k}w_{\xi i}^{'kL}.a_{\xi ij}^{kL}$; $p_{\xi i}^U=\sum_{k=1}^{\eta}\sum_{j=1}^{n_k}w_{\xi i}^{'kU}.a_{\xi ij}^{kU}$ for $\xi\in\{1,2,3,4,5\}$ and

$$h_{pi}^L=\min_{k-1}^{\eta}\min_{j=1}^{n_k}\left(\min\left\{h_{w'i}^{kL},h_{ij}^{kL}\right\}\right);$$

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$h_{pi}^{U}=\min_{k-1}^{\eta}\min_{j=1}^{n_k}\left(\min\left\{h_{wi}^{kU},h_{ij}^{kU}\right\}\right)$ for each ξ.

5. The pro-value and con-value obtained as interval type-2 pentagonal fuzzy numbers in the previous step are de-fuzzified using an existing ranking method.

6. The de-fuzzified values for pro-value and con-value are then used to obtain final ranking score using the newly proposed formula in (7) and the alternatives are ranked.

H. Confirmation with the tentative decision

In this stage, we take a cognitive approach by taking into account the interaction between decision conditions and subjective utility values. We use the significant principles of Fuzzy Trace Theory (FTT) [4], [6] to confirm our tentative decision. A decision maker reviews the whole process of decision making with reference to the gist representation of the issue concerned. In this model, gist representation is basically expression of the values and the goal (primary and secondary) of the decision maker. The decision maker seeks confirmation by checking whether the final mathematical result corroborates the gist representations of decision options. We propose the two following steps at this stage:

a) Formation of gist representation for decision options based on three principles:

(i) What good/value is it to decision maker?

(ii) What good/value is it to the immediate context (family, relatives, work place)?

(iii) What good/value is it to the society at large (common good)?

b) Evoking gist representations via meaningful cues/guides in decision context. The strength of gist representation depends on vividness of representation in decision maker's mind. The vivid cues could be subjective feelings or emotions (positive or negative) towards alternatives available to be opted for.

Consider a decision maker has k gist representations namely $g_1, g_2, \ldots g_k$. Let's call this a gist set $G = \{g_1, g_2, \ldots g_k\}$. An alternative-gist matrix is formed using gist set G and alternative set $A = \{a_1, a_2 \ldots a_n\}$ to represent the relation between the purpose/goal of a decision and the respective alternatives.

Table- III: Alternative-gist matrix

	g_1	g_2	g_3	…	g_k	min (a_{ij})
a_1	a_{11}	a_{12}	a_{13}	…	a_{1k}	min (a_{1j})
a_2	a_{21}	a_{22}	a_{23}	…	a_{2k}	min (a_{2j})
.	.	.	.	.	.	.
a_n	a_{n1}	a_{n2}	a_{n3}	…	a_{nk}	min (a_{4j})

Each element a_{ij} of the matrix represents the degree of satisfaction of j^{th} gist with respect to i^{th} alternative. The final consolation score is defined as follows:

$$\text{Consolation Score}=\max_{i=1}^{n}\left(\min_{j=1}^{k}\left(a_{ij}\right)\right) \qquad (20)$$

If the alternative with the consolation score aligns with tentative decision the decision maker arrives at final decision. If not, the decision maker goes back to the first stage and checks with preference structures or preference order and makes changes according to his preference order. He consults the Experts for better clarity and repeats the whole procedure. Once decision maker is confirmed with the decision, the appropriate decision is taken. Tertiary observation at the last step continually orients and motivates the decision maker to reach the goal of the decision. Thus, the model suggests a putting continuous effort to be goal-focused.

IV. A CASE STUDY: AN APPLICATION

We illustrate the newly proposed method with a case study to clarify and verify the method. Consider a situation of religious community of three persons. The community plans to buy either 2-wheeler or 4-wheeler. Decision has to be made for the better alternative. The following are the decision constraints:

1. Context and the field reality
2. Need felt by the three persons in the community
3. The objects of decision themselves i.e. 2-wheeler (a_1) and 4-wheeler (a_2)
4. The concerns of the higher authorities

A. Step 1

The alternative set $A=\{a_1,a_2\}$ is formed; here a_1 is 2-wheeler and a_2 is 4-wheeler.

B. Step 2

All the arguments/opinions are collected using all the above four decision constraints.

Table- IV: List of Pros and Cons for the alternatives

	2-wheeler	
	Pros	Cons
1	Easy to maintain (no need of driver)	Cannot help sick people whenever required
2	Cheaper/affordable	Cannot be used to carry things
3	Handy to use for travel to visit people	Tiresome in summer
4	In emergency any person can use	Not comfortable in winter
5	Single or double comfortable	Not useful in rainy season
6	……………	Cannot be used for many persons
	4-wheeler	
1	Can carry many people/sick persons	Costly/not affordable
2	Can be used for shopping and marketing	Maintenance cost high/Need a driver
3	Comfortable	Cannot use to travel all the interior places
4	Safety	Cannot be used by every person
5	Can be used in any season if roads are good	Not useful in rainy season if roads are bad

C. Step 3

All the opinions are filtered, repetition and redundancy are removed and global criteria C set is formed as $C=\left\{c_i^{+1},c_j^{-1},c_k^{+2},c_l^{-2},c_m^{+3},c_n^{-3}\right\}$, for $i,j,k,l,m,n=1,2,3\ldots$

Table- V: Criteria sets for the alternative a_1

Prioritized categorized criteria (C_i)	Global criteria sets with pros and cons for corresponding categorized criteria set	
	Pros	Cons
Usefulness (C_1)	{3,4}/ $\left\{c_1^{+1}, c_2^{+1}\right\}$	{1, 2, 6}/ $\left\{c_1^{-1}, c_2^{-1}, c_3^{-1}\right\}$
Comfort and Safety (C_2)	{5}/ $\left\{c_1^{+2}\right\}$	{3, 4, 5}/ $\left\{c_1^{-2}, c_2^{-2}, c_3^{-2}\right\}$
Maintenance (C_3)	{1, 2}/ $\left\{c_1^{+3}, c_2^{+3}\right\}$	……..

Table- VI: Criteria sets for the alternative a_2

Prioritized categorized criteria (C_i)	Global criteria sets with pros and cons for corresponding categorized criteria set	
	Pros	Cons
Usefulness (C_1)	{1, 2, 5}/ $\left\{c_1^{+1}, c_2^{+1}, c_3^{+1}\right\}$	{3, 4, 5}/ $\left\{c_1^{-1}, c_2^{-1}, c_3^{-1}\right\}$
Comfort and Safety (C_2)	{3, 4}/ $\left\{c_1^{+2}, c_2^{+2}\right\}$	………….
Maintenance (C_3)	……………	{1, 2}/ $\left\{c_1^{-3}, c_2^{-3}\right\}$

D. Step 4

All the criteria in global criteria set are categorized and then categorized criteria sets are prioritized. Thereafter, criteria-wise pros-cons matrix is formed. The entries in the matrix are linguistic values assigned by the decision maker. The table below shows the linguistic values assigned by the decision maker to the arguments as in the table 5.

Table- VII: Criteria-wise pro-con matrix with truth values

	2-wheeler		4-wheeler	
Criteria	**Pros**	**Cons**	**Pros**	**Cons**
C_1	VT	VT	VT	VT
	T	T	T	N
	…..	T	T	T
C_2	T	N	T	…..
	…..	N	T	…..
		T	…..	…..
C_3	T	…..	…..	T
	T	…..	…..	N
	…..	…..	…..	…..

E. Step 5

For each truth value in table 8, the corresponding IT2PFNs is assigned using the interval type-2 pentagonal fuzzy scale given in the following table:

Table- VIII: Linguistic Variables with corresponding IT2PFNs

Linguistic Variables	Interval type-2 pentagonal fuzzy numbers
Extremely false (EF)	[(0, 0, 0, 0, 0; 1), (0, 0, 0, 0, 0; 1)]
Very false (VF)	[(0, 0.015, 0.05, 0.085, 0.12; 1), (0, 0.005, 0.05, 0.095, 0.14; 1)]
False (F)	[(0.13, 0.165, 0.2, 0.235, 0.27; 1), (0.11, 0.155, 0.2, 0.245, 0.29; 1)]
Medium false (MF)	[(0.28, 0.315, 0.35, 0.385, 0.42; 1), (0.26, 0.305, 0.35, 0.345, 0.44; 1)]
Neutral (N)	[(0.43, 0.465, 0.5, 0.535, 0.57; 1), (0.41, 0.455, 0.5, 0.545, 059; 1)]
Medium true (MT)	[(0.58, 0.615, 0.65, 0.685, 0.72; 1), (0.56, 0.605, 0.65, 0.695, 0.74; 1)]
True (T)	[(0.73, 0.765, 0.8, 0.835, 0.87; 1), (0.71, 0.755, 0.8, 0.845, 0.89; 1)]
Very true (VT)	[(0.88, 0.915, 0.95, 0.985, 1; 1), (0.86, 0.905, 0.95, 0.995, 1; 1)]
Extremely true (ET)	[(1, 1, 1, 1, 1; 1), (1, 1, 1, 1, 1; 1)]

The synthesized value Q_i^k is computed by using the newly suggested formula in (10).The result thus obtained is as in the table below:

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Table- IX: Synthesized values of Pros and Cons for alternatives

A_i	K	The synthesized value Q_i^{+k} pros	The synthesized value Q_i^{-k} for cons
A_1	0	[(1, 1, 1, 1, 1; 1), (1, 1, 1, 1, 1; 1)]	[(1, 1, 1, 1, 1; 1), (1, 1, 1, 1, 1; 1)]
	1	[(0.9676, 0.98, 0.99, 0.9975, 1, 1; 1), (0.9594, 0.9767, 0.99, 0.9992, 1, 1; 1)]	[(0.9913, 0.9953, 0.998, 0.9996, 1; 1), (0.9882, 0.9943, 0.998, 0.9999, 1; 1)]
	2	[(0.73, 0.765, 0.8, 0.835, 0.87; 1), (0.71, 0.755, 0.8, 0.845, 0.89; 1)]	[(0.9123, 0.9328, 0.95, 0.9643, 0.976; 1), (0.8991, 0.9237, 0.95, 0.9679, 0.9815; 1)]
A_2	0	[(1, 1, 1, 1, 1; 1), (1, 1, 1, 1, 1; 1)]	[(1, 1, 1, 1, 1; 1), (1, 1, 1, 1, 1; 1)]
	1	[(0.9913, 0.9953, 0.998, 0.9996, 1; 1), (0.9882, 0.9943, 0.998, 0.9999, 1; 1)]	[(0.9815, 0.9893, 0.995, 0.9989, 1; 1), (0.9761, 0.9873, 0.995, 0.9997, 1; 1)]
	2	[(0.9271, 0.9448, 0.96, 0.9728, 0.9831; 1), (0.9159, 0.94, 0.96, 0.976, 0.9879; 1)]	…..
	3	…..	[(0.8461, 0.8743, 0.9, 0.9233, 0.9441; 1), (8289, 0.8665, 0.9, 0.9295, 0.9549; 1)]

F. Step 6

The Priority-based weight W_i^k of k[th] class for an alternative is calculated using the formula in (13) and the result obtained is as follows:

Table- X: The Priority-based Weight W_i^k

A_i	K	W_i^{+k} (pros)	W_i^{-k} (cons)
A_1	1	[(1, 1, 1, 1, 1; 1), (1, 1, 1, 1, 1; 1)]	[(1, 1, 1, 1, 1; 1), (1, 1, 1, 1, 1; 1)]
	2	[(0.9676, 0.98, 0.99, 0.9975, 1, 1; 1), (0.9594, 0.9767, 0.99, 0.9992, 1, 1; 1)]	[(0.9913, 0.9953, 0.998, 0.9996, 1; 1), (0.9882, 0.9943, 0.998, 0.9999, 1; 1)]
	3	[(0.7064, 0.7497, 0.792, 0.8329, 0.87; 1), (0.6812, 0.7374, 0.792, 0.8443, 0.89; 1)]	[(0.9044, 0.9284, 0.9481, 0.9639, 0.976; 1), (0.8885, 0.9184, 0.9481, 0.9678, 0.9815; 1)
A_2	1	[(1, 1, 1, 1, 1; 1), (1, 1, 1, 1, 1; 1)]	[(1, 1, 1, 1, 1; 1), (1, 1, 1, 1, 1; 1)]
	2	[(0.9913, 0.9953, 0.998, 0.9996, 1; 1), (0.9882, 0.9943, 0.998, 0.9999, 1; 1)]	[(0.9815, 0.9893, 0.995, 0.9989, 1; 1), (0.9761, 0.9873, 0.995, 0.9997, 1; 1)]
	3	[(0.9190, 0.9404, 0.9581, 0.9724, 0.9831; 1), (0.9051, 0.9346, 0.9581, 0.9759, 0.9879; 1)]	[(0.8305, 0.865, 0.9223, 0.9441; 1), (0.8091, 0.8555, 0.8955, 0.9292, 0.9549; 1)]

G. Step 7

The normalized priority-based weight $W_i^{'k}$ of k[th] class for an alternative is calculated using the formulas in (15) and (16) and the result is obtained as follows:

Table- XI: The Normalized Priority-based Weight $W_i^{'k}$

A_i	K	$W_i^{'+k}$ (pros)	$W_i^{'-k}$ (cons)
A_1	1	[(0.3484, 0.3533, 0.3595, 0.3663, 0.374; 1), (0.346, 0.3517, 0.3595, 0.3685, 0.3787; 1),]	[(0.336, 0.3374, 0.3394, 0.342, 0.3453; 1), (0.3354, 0.337, 0.3394, 0.3433, 0.3476; 1)]
	2	[(0.3371, 0.3462, 0.3559, 0.3654, 0.374; 1), (0.332, 0.3435, 0.3559, 0.3682, 0.3787; 1)	[(0.3331, 0.3358, 0.3388, 0.3419, 0.3453; 1), (0.3314, 0.335, 0.3388, 0.3422, 0.3476; 1)]
	3	[(0.2461, 0.2649, 0.2847, 0.3051, 0.3254; 1), (0.2357, 0.2593, 0.2847, 0.3111, 0.3371; 1)]	[(0.3039, 0.3133, 0.3218, 0.3297, 0.3371; 1), (0.298, 0.3095, 0.3218, 0.3323, 0.3412; 1)]
A_2	1	[(0.3352, 0.3365, 0.3383, 0.3406, 0.3436; 1), (0.3347, 0.336, 0.3383, 0.3413, 0.3456; 1)]	[(0.3397, 0.3423, 0.346, 0.3504, 0.3556; 1), (0.3384, 0.3414, 0.346, 0.3518, 0.359; 1)]
	2	[(0.3323, 0.3349, 0.3376, 0.3405, 0.3436; 1), (0.3307, 0.3341, 0.3376, 0.3414, 0.3456; 1)	[(0.3334, 0.3387, 0.3442, 0.35, 0.3556; 1), (0.3303, 0.3371, 0.3442, 0.3517, 0.359)]
	3	[(0.3081, 0.3164, 0.3241, 0.3312, 0.3378; 1), (0.3029, 0.3141, 0.3241, 0.3332, 0.3415; 1)]	[(0.2821, 0.2843, 0.3098, 0.3231, 0.3358; 1), (0.2738, 0.2921, 3098, 0.3269, 0.3429; 1)]

H. Step 8

Using the PIT2F aggregation operator in (18) we get the following aggregated value for pros value and cons value in terms of interval type-2 pentagonal fuzzy numbers:

$\tilde{A}_1^+ = [(1.1664, 1.264, 1.369, 1.481, 1.591; 1),$
$(1.114, 1.235, 1.369, 1.515, 1.653; 1)]$

$\tilde{A}_1^- = [(1.3159, 1.3943, 1.4753, 1.5594, 1.6404; 1),$
$(1.2718, 1.3716, 1.4753, 1.5861, 1.686; 1)]$

$\tilde{A}_2^+ = [(1.2696, 1.335, 1.403, 1.473, 1.539; 1),$
$(1.233, 1.316, 1.403, 1.494, 1.576; 1)]$

$\tilde{A}_2^- = [(1.0201, 1.084, 1.181, 1.268, 1.351; 1),$
$(0.977, 1.076, 1.181, 1.293, 1.398; 1)]$

I. Step 9

Using the ranking based on centroid point and spread [5] we convert the above four IT2PFNs into generalized standardized type-1 pentagonal fuzzy numbers and then using direct method [5] based on modal value, spread value and normal value we obtain ranking score for the obtained standardized type-1 pentagonal fuzzy numbers as follows:

$R\left(\tilde{A}_1^+\right) = 18.8975$; $R\left(\tilde{A}_1^-\right) = 35.482$; $R\left(\tilde{A}_2^+\right) = 45.7026$; $R\left(\tilde{A}_2^-\right) = 21.903$

J. Step 10

Using the formula in equation (7) we get the following ranking score for the alternatives:

$R\left(\tilde{A}_1\right) = -8.2913$ and $R\left(\tilde{A}_2\right) = 11.8998$

As $R\left(\tilde{A}_1\right) > R\left(\tilde{A}_2\right)$, we have $A_2 \succ A_1$ i.e. A_2 is preferred to A_1.

K. Step 11

In the given decision problem, we have two following gist representations or values of the decision:

g_1= For the better and greater usefulness

g_2 = Life is important than money.

Table- XIII: Name of the Table that justify the values

	g_1	g_2	min(g_1, g_2)
a_1	0.5	0.4	0.4
a_2	0.9	0.9	0.9

Consolation score = max (min (0.5, 0.4), min (0.9, 0.9))
= 0.9

Clearly the score 0.9 indicates that the alternative a_2 is preferred. This confirms the decision arrived at in stage 2.

V. CONCLUSION

The proposed interval type-2 pentagonal fuzzy number based multi-criteria decision making method combines pros and cons of an issue. After reaching the tentative decision in the second stage; we also suggest to confirm the tentative decision proposing the concept of consolation score. We have introduced necessary mathematical formulas for computations. This decision method is an integral and comprehensive approach to decision making problem concerning life-issues.

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REFERENCES

1. D. Bouyssou, D. Dubois, H. Prade, and M. Pirlot (Eds.), *Decision Making Process: Concepts and methods*. John Wiley & Sons, 2013, pp. 475-502.
2. G.E. Ganss, *The Spiritual Exercises of Saint Ignatius (4th Edition)*. Gujarat Sahitya Prakash Press, India, 2011, pp. 21-89 and 121-132.
3. H.A. Simon, "Motivational and Emotional Controls of Cognition", *Psychological Review*, vol. 74 (1), 1957, pp. 29-39.
4. J.C. Brainerd and F.V. Reyna, "Fuzzy-trace theory: Dual process in memory, reasoning and cognitive neurosciences," Vol. 28, 2001, pp. 41-100.
5. M. Ajay and T. Pathinathan, "Interval Type-2 Pentagonal Fuzzy Numbers," *Journal of Applied Science and Computations*, vol. 6 (5), 2019, pp. 218-230.
6. S. Rahimi-Golkhandan, D.M.N. Garavito, B.B. Reyna-Brainerd and V.F. Reyna, "A Fuzzy-Trace Theory of Risk and Time Preferences," *in Decision Making: Integrating Cognition and Motivation", In Impulsivity, Springer*, 2017, pp. 115-144.
7. T. Pathinathan and K. Ponnivalavan, "Pentagonal Fuzzy Numbers," *International Journal of Computing Algorithm*, vol. 3, 2014, pp. 1003-1005.
8. T. Pathinathan and M. Ajay, "Fuzzy Multi-Criteria Decision-making Based on Ignatian Discernment Process," *International Journal of Information, Control and Computer Sciences*, vol. 12 (5), pp. 355-364, 2018.
9. T.Y. Chen, "Multiple Criteria Decision Making Analysis using Prioritised Interval Type-2 Fuzzy Aggregation Operators and Its Application to Site Selection," *Technological and Economic Development of Economy*, vol. 23(1), 2017, pp. 1-21.
10. W. Pedrycz, P. Ekel, R. Parreiras, *Fuzzy Multi-Criteria Decision Making*. John Wiley & Sons, Ltd., 2011, 155-199.

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