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Solution Analysis of A 3rd Order Initial Value Problem

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Abstract: In this article, we have used DT Method (Differential Transform Method) and Numerical Differentiation Method to solve a third order initial value problem $Y''' = f(x, y)$. ***We observed that the solutions by these methods are very close to the exact solution. The methods have been demonstrated and hence the superiority of the ND Method is observed. A numerical example is illustrated by using DT and ND methods and the results were compared with exact solution.***

Index Terms : DT Method, Absolute stability, Multi-Step method, Numerical Differentiation

I. INTRODUCTION

Greater order differential methods consistently occur in bunches of divisions of making use of arithmetics, clinical research research study and also design. Numerous scientists have actually checked out particular along with furthermore multistep techniques for the First worth problems in addition to the info of the subject matter have really been actually provided through considerable amounts of authors [3]To attend to special second order differential solutions, Henrici [5] taken advantage of One-of-a-kind multistep strategies located upon algebraic mix. Techniques of reduction of purchase for fixing however troubled two-point limitation worth issues was actually thought about in [1], [7] Solutions of very first worth issue through making use of algebraic accolade have really been actually analyzed in [2], [9] The approaches discussed through Henrici [5] encouraged our company to consume the work carried out in this particular quick write-up. Special multistep methods have in fact been actually gotten through transforming y(x) left wing palm edge of through a putting polynomial as well as likewise identifying it 3 opportunities. Our experts have really discovered a training course of method as well as likewise the strategy secured within this article possesses a purchase of (k - 2) Our experts have really acquired the places of straight-out safety and security for these methods. For taking care of differential formulations, the differential adjustment approach important which is actually an analytical strategy.

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Zhou [12], has in fact shown the concept of differential adjustment approach in 1986. In this particular strategy, the solution stays in the type of a polynomial which is actually a variety of coming from Taylor compilation strategy. The DT strategy is actually a substitute method for obtaining analytical Taylor compilation alternative of the differential methods ([10], [11].

II. THIRD-ORDER DIFFERENTIAL EQUATIONS GENERAL MULTISTEP METHODS

Let the 3rd order differential equation

$$F(x,\ y,\ y',\ y'',\ y''')=0 \tag{1}$$

The mathematical answer of formula (1) through standard multistep approach of kth measure is actually

$$y_{n+1} = \sum_{j=1}^{k} a_j y_{n+1-j} + h^3 \sum_{j=0}^{k} b_j y_{n+1-j} \tag{2}$$

Here 'h' is the step length and a_j, b_j are constants.
Taking the polynomials

$$\rho(\xi) = \xi^k - \sum_{j=1}^{k} a_j \xi^{k-1} \text{ and } \sigma(\xi) = \sum_{j=1}^{k} b_j \xi^{k-1} \tag{3}$$

We can write the equation (2) as

$$\rho(E)\, y_{n-k+1} - h^3 \sigma(E) y'_{n-k+1} = 0, \text{ where } E(y_n) = y_{n+1} \tag{4}$$

$$\rho(\xi) - \bar{h}\sigma(\xi) = 0, \quad \text{where } \bar{h} = \lambda h^3 \tag{5}$$

Typically the origins of the certain formula (5) as well as likewise are detailed and also the area of the complex - airplane is defined as the area of straight-out protection such that the beginnings of the formula (5) are within device circle whenever rests on the inside the area. By representing the location of outright safety as R as well as its boundary by R, after that the locus of R will absolutely be

$$\bar{h}(\theta) = \rho(e^{i\theta})/\sigma(e^{i\theta}), \quad 0 \le \theta \le 2\pi \tag{6}$$

III. DERIVATION OF THE METHODS

Now p(x) can be written as

$$p(x) = \sum_{m=0}^{k} (-1)^m \binom{-s}{m} \nabla^m y_{n+1}, \qquad s = \frac{(x - x_{n+1})}{h} \tag{7}$$

In equation (1), taking $x = x_{n+1-r}$ i.e. $s = -r$ and replacing $y'''(x)$ by $p'''(x)$, we obtain

$$\sum_{m=0}^{k} \delta_{r,\,m} \nabla^m y_{n+1} = h^3 f_{n+1-r} \tag{8}$$

where $$\delta_{r,m} = \frac{d^3}{ds^3}\left[(-1)^m \binom{-s}{m}\right] \tag{9}$$

IV. GENERATING FUNCTION OF $\boldsymbol{\delta_{r,m}}$

We define $$D_{r,t} = \sum_{m=0}^{\infty} \delta_{r,m} t^m \tag{10}$$

Putting $\delta_{r,m}$ from (9) in (10), we get

$$D_{r,t} = \sum_{m=0}^{\infty} \delta_{r,m} t^m = -(1-t)^{-s}[\log(1-t)]^3$$

$$\therefore \sum_{m=0}^{\infty} \delta_{r,m} t^m = (1-t)^r [\log(1-t)]^3 \ at\ s = -r \tag{11}$$

Taking r = $\frac{1}{2}$ in (8), we get a class of method given by

$$\sum_{m=0}^{k} \delta_{\frac{1}{2},\,m} \nabla^m y_{n+1} = h^3 f_{n+\frac{1}{2}} \tag{12}$$

After simplifying, the equation (12) reduces to

$$\sum_{j=0}^{k} a_j y_{n+1-j} = h^3 f_{n+\frac{1}{2}} \tag{13}$$

Coefficients a_j are given up table 2.
Neighborhood truncation error of the formula (13) is provided by

$$LTE = \delta_{0,k+1} h^{k+1} y^{k+1}(\eta) \tag{14}$$

Table 1

m	0	1	2	3	4	5	6	7	8	9
$\delta_{\frac{1}{2},m}$	0	0	0	1	1	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{1237}{1920}$	$\frac{357}{640}$	$\frac{471539}{967680}$

Table 2
Coefficients of α_j ; $j = 0\ (1) k,\ k = 3\ (1)\ 9$

K	J								
	0	1	2	3	4	5	6	7	8
3	1	-3	3	-1					
4	2	-7	9	-5	1				
5	$\frac{23}{8}$	$\frac{-91}{8}$	$\frac{142}{8}$	$\frac{-110}{8}$	$\frac{43}{8}$	$\frac{-7}{8}$			
6	$\frac{29}{8}$	$\frac{-127}{8}$	$\frac{232}{8}$	$\frac{-230}{8}$	$\frac{133}{8}$	$\frac{-43}{8}$	$\frac{6}{8}$		
7	$\frac{8197}{1920}$	$\frac{-39139}{1920}$	$\frac{81657}{1920}$	$\frac{-98495}{1920}$	$\frac{75215}{1920}$	$\frac{-36297}{1920}$	$\frac{10099}{1920}$	$\frac{-1237}{1920}$	
8	$\frac{9268}{1920}$	$\frac{-47707}{1920}$	$\frac{111645}{1920}$	$\frac{-158471}{1920}$	$\frac{150185}{1920}$	$\frac{-96273}{1920}$	$\frac{40087}{1920}$	$\frac{-9805}{1920}$	$\frac{1071}{1920}$

It is observed the k-step strategy (14) possesses the purchase k-2 as well as it is actually positively stable.For the system, our experts possess

$$\rho(\xi) = \sum_{j=0}^{k} a_j \xi^{k-j} \ and\ \sigma(\xi) = \xi^k. \tag{15}$$

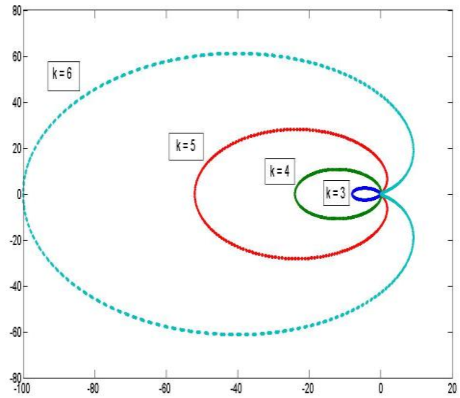

Figure 1. r = 1/2 and k = 3, 4, 5 and 6

Figure 1: Stability region(13) for k = 3, 4, 5

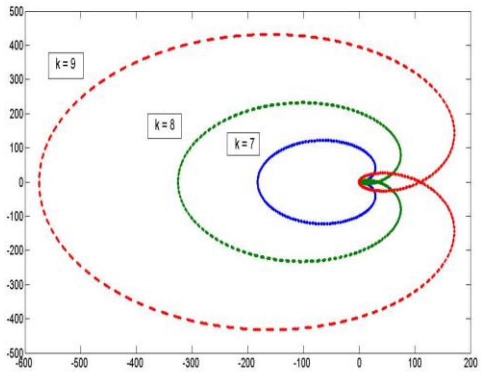

Figure 2. r = 1/2 and k = 7, 8 and 9

Figure 2: Absolute stability region of (13) for k = 7, 8 and 9

V. DIFFERENTIAL TRANSFORMATION METHOD

Differential transformation of function $y(x)$ is defined as follows

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$$Y(k)=\frac{1}{k!}\left[\frac{d^k y(x)}{dx^k}\right]_{x=0} \qquad (16)$$

In (16), Y(k) is the changed function of the original function y(x).
Differential inverse transform of is specified as adheres to

$$y(x)=\sum_{k=0}^{\infty} x^k Y(k) \qquad (17)$$

From (16) and (17), we obtain

$$y(x)=\sum_{k=0}^{\infty}\frac{x^k}{k!}\left[\frac{d^k y(x)}{dx^k}\right] \qquad (18)$$

Eq. (18) indicates that the concept of differential makeover is originated from the Taylor collection advancement. From the definitions (16) as well as (17), it is extremely simple to obtain the adhering to mathematical procedures:

Original Function	Transform Function
$y(x)=g(x)\pm h(x)$	$Y(k)=G(k)\pm H(k)$
$y(x)=cg(x)$	$Y(k)=cG(k)$
$y(x)=e^{ax}$	$Y(k)=\frac{a^k}{k!}$
$y(x)=\frac{d^n g(x)}{dx^n}$	$Y(k)=\frac{(k+n)!}{k!}G(k+n)$
$y(x)=\sin(ax+b)$	$Y(k)=\frac{a^k}{k!}\sin\left(\frac{k\pi}{2}+b\right)$

VI. NUMERICAL EXAMPLE

As an example, consider initial value problem

$$y''=2e^x-y,\quad y(0)=0,\ y'(0)=2,\ y'(0)=0 \qquad (19)$$

$$y_{n+1}=\frac{7}{2}y_n-\frac{9}{2}y_{n-1}+\frac{5}{2}y_{n-2}-\frac{1}{2}y_{n-3}+\frac{1}{2}h^3 f_{n+\frac{1}{2}} \qquad (20)$$

(2) The fourth order ND method (13) discussed with k = 5 is

$$y_{n+1}=\frac{91}{23}y_n-\frac{142}{23}y_{n-1}+\frac{110}{23}y_{n-2}-\frac{43}{23}y_{n-3}+\frac{7}{23}y_{n-4}+\frac{8}{23}h^3 f_{n+\frac{1}{2}} \qquad (21)$$

Presently using the DT approach to the similar differential formula (19),
making use of the differential improvement on both sides of (19), we obtain

$$Y(k+3)=\frac{k!}{(k+3)!}\left[\frac{2}{k!}-Y(K)\right] \qquad (22)$$

$$\text{And}\qquad Y(0)=0,\ Y(1)=2,\ Y(2)=0 \qquad (24)$$

Replacing formula (24) in formula (23) in addition to taking k = 0, 1, 2, 3, ..., we get

$$Y(3)=\frac{1}{3}\quad Y(4)=0\quad Y(5)=\frac{1}{60}$$

$$Y(6)=0\quad Y(7)=\frac{1}{2520}\quad Y(8)=0$$

$$Y(9)=\frac{1}{181440}\quad Y(10)=0\quad Y(11)=\frac{1}{19958400}$$

$$Y(12)=0\quad Y(13)=\frac{1}{3113510400}\quad \ldots\ldots$$

By DT method, the solution can be expressed as

$$y(x)=\sum_{k=0}^{\infty}x^kY(k)$$

$$y(x)=\sum_{k=0}^{\infty}x^kY(k)$$

$$=2x+\frac{1}{3}x^3+\frac{1}{60}x^5+\frac{1}{2520}x^7+\frac{1}{181440}x^9+\frac{1}{19958400}x^{11}+\frac{1}{3113510400}x^{13}+\ldots\ldots$$

Table 3 : ND technique towards k = 5 and h = 0.01

X	Y(Exact Solution)	ND Method	Absolute Error	DT Method	Absolute Error
0	0.0000000000E+00	-1.0939058455E-14	1.0939058455E-14	0.0000000000E+00	0.0000000000E+00
0.1	2.0033350004E-01	2.0033350004E-01	4.0550895974E-14	2.0033350004E-01	3.1194491434E-13
0.2	4.0267200508E-01	4.0267200508E-01	9.2925667161E-14	4.0267200508E-01	2.1879720258E-12
0.3	6.0904058689E-01	6.0904058689E-01	1.4677148386E-13	6.0904058689E-01	4.2851278081E-12
0.4	8.2150465161E-01	8.2150465161E-01	2.0350388041E-13	8.2150465161E-01	4.3689496465E-12
0.5	1.0421906110E+00	1.0421906110E+00	2.5734969711E-13	1.0421906110E+00	1.2505330105E-11
0.6	1.2733071643E+00	1.2733071643E+00	3.1841196346E-13	1.2733071643E+00	3.5182967650E-12
0.7	1.5171674037E+00	1.5171674037E+00	3.7925218521E-13	1.5171674037E+00	2.0940138512E-11
0.8	1.7762119644E+00	1.7762119644E+00	4.4919623576E-13	1.7762119644E+00	2.4807711441E-11
0.9	2.0530334514E+00	2.0530334514E+00	5.1869619710E-13	2.0530334514E+00	1.6035173189E-11
1	2.3504023873E+00	2.3504023873E+00	5.9641180883E-13	2.3504023873E+00	1.3931966691E-11
1.1	2.6712949402E+00	2.6712949402E+00	6.7901240186E-13	2.6712949402E+00	4.1936232265E-11
1.2	3.0189227108E+00	3.0189227108E+00	7.6383344094E-13	3.0189227108E+00	6.5636385216E-13
1.3	3.3967648746E+00	3.3967648746E+00	8.6020079948E-13	3.3967648745E+00	9.3542062984E-11
1.4	3.8086030029E+00	3.8086030029E+00	9.7077901273E-13	3.8086030027E+00	2.3658808246E-10
1.5	4.2585589102E+00	4.2585589102E+00	1.0800249584E-12	4.2585589095E+00	6.8567018729E-10
1.6	4.7511359064E+00	4.7511359064E+00	1.2132517213E-12	4.7511359046E+00	1.7795782625E-09
1.7	5.2912638677E+00	5.2912638677E+00	1.3473666627E-12	5.2912638632E+00	4.4503281060E-09
1.8	5.8843485762E+00	5.8843485762E+00	1.5010215293E-12	5.8843485657E+00	1.0451258525E-08
1.9	6.5363258231E+00	6.5363258231E+00	1.6635581801E-12	6.5363257995E+00	2.3573319297E-08

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Table 4 : ND method with k = 5 and h = 0.02

X	Y	NDMethod	Absolute Error	DT Method	Absolute Error
0	0.0000000000E+00	-1.4301991335E-12	1.4301991335E-12	0.0000000000E+00	0.0000000000E+00
0.1	2.0033350004E-01	2.0033350004E-01	1.9108048477E-12	2.0033350004E-01	3.1194491434E-13
0.2	4.0267200508E-01	4.0267200509E-01	5.2701731867E-12	4.0267200508E-01	2.1879720258E-12
0.3	6.0904058689E-01	6.0904058690E-01	8.6829432533E-12	6.0904058689E-01	4.2851278081E-12
0.4	8.2150465161E-01	8.2150465162E-01	1.2182810316E-11	8.2150465161E-01	4.3689496465E-12
0.5	1.0421906110E+00	1.0421906110E+00	1.5803802711E-11	1.0421906110E+00	1.2505330105E-11
0.6	1.2733071643E+00	1.2733071643E+00	1.9582779842E-11	1.2733071643E+00	3.5182967650E-12
0.7	1.5171674037E+00	1.5171674037E+00	2.3558932583E-11	1.5171674037E+00	2.0940138512E-11
0.8	1.7762119644E+00	1.7762119644E+00	2.7771118738E-11	1.7762119644E+00	2.4807711441E-11
0.9	2.0530334514E+00	2.0530334514E+00	3.2261748828E-11	2.0530334514E+00	1.6035173189E-11
1	2.3504023873E+00	2.3504023873E+00	3.7070346792E-11	2.3504023873E+00	1.3931966691E-11
1.1	2.6712949402E+00	2.6712949403E+00	4.2258196942E-11	2.6712949402E+00	4.1936232265E-11
1.2	3.0189227108E+00	3.0189227109E+00	4.7860826413E-11	3.0189227108E+00	6.5636385216E-13
1.3	3.3967648746E+00	3.3967648746E+00	5.3943516320E-11	3.3967648745E+00	9.3542062984E-11
1.4	3.8086030029E+00	3.8086030030E+00	6.0571547777E-11	3.8086030027E+00	2.3658808246E-10
1.5	4.2585589102E+00	4.2585589103E+00	6.7799987846E-11	4.2585589095E+00	6.8567018729E-10
1.6	4.7511359064E+00	4.7511359065E+00	7.5712769387E-11	4.7511359046E+00	1.7795782625E-09
1.7	5.2912638677E+00	5.2912638678E+00	8.4384055299E-11	5.2912638632E+00	4.4503281060E-09
1.8	5.8843485762E+00	5.8843485763E+00	9.3894669817E-11	5.8843485657E+00	1.0451258525E-08
1.9	6.5363258231E+00	6.5363258232E+00	1.0434320075E-10	6.5363257995E+00	2.3573319297E-08

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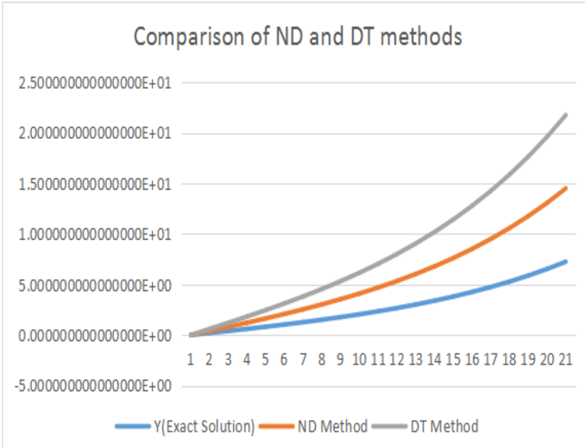

Figure 3: Comparison of ND method with h = 0.01 and DT method

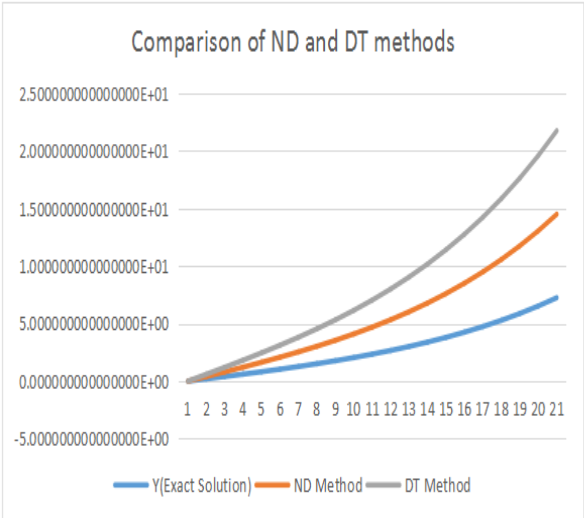

Figure 4: Comparison of ND method with h = 0.02 and DT method

VII. DISCUSSION AND CONCLUSION

The strategies based upon mathematical distinction are observed to be most definitely safe and secure outside some shut boundaries. We have in fact gotten the alternative by ND in addition to DT methods which are obtained in this short write-up in addition to are extremely nearer to the first treatment. The absolute mistakes are remarkably little. From the tables 3 and also 4 and the charts 3 as well as 4, it is evident that the outcomes acquired by ND method transcend to those acquired by DT method.

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