Building the Stiffness Matrix and Load Vector of the Taper Elements with the I-Section That Considering the Effect of Shear Force and Stiffness of the Connection

Hong Son Nguyen, Trung Thanh Pham, Quang Hung Nguyen

Abstract: This paper shows how to build stiffness matrix and load vector by energy method for taper, I section elements includes the effects of shear and semi-rigid connection, use for structural analysis problem. Mathematical model of the section is exponential. Thereby, programming the structural analysis program for taper elements and verify results stiffness matrix, load vector as well as assessing the effects of shear and semi-rigid connection to force and displacement of this element.

Keywords: Taper elements, Stiffness matrix, Semi-rigid connection.

I. INTRODUCTION

In the structure of constructions, bar components (with the height of the section that changing follow to the lengths) are used more widely in construction, sections meet the architectural requirements, more suitTab. for bearing capacity in the structures, and the making of changing sections is not difficult for manufacturers.

For steel frame structure, the type of beveled cross-section is used quite a lot, usually the beam and column components with I cross-section have the wing plate size and the thickness of the belly plate is constant, the section height (the height of vell plate) changes. The rate of change in the height of the section depends on the distribution of internal force in the section according to the length, the conditions of connection at the two ends of the component.

Documents on structural mechanics and numerical methods have presented how to construct the stiffness matrix and load vector for a constant section bar element, with different connection conditions at either end or applied. With different load cases, the results have built a sample element Tab. Recently, a number of domestic and foreign documents mentioned the establishment of variable section element stiffness matrices, but mainly rectangular sections with linear changing heights or based on the shape function of the regular section bar element and the analytical results are approximate, SAP 2000 commercial software also mentioned the problem of structural analysis with beveled structure, the structure is consistent with the section linear changes, or changes in parabola form with cross sectional characteristics are quadratic and tertiary exponential functions. However, in order to have the matrix and load vector of the taper element and I cross-section, we also need further studies, to achieve the results of the structural structural problem with elegant more accurate beveled face, and to approach more general problems of steel frame structural analysis that SAP 2000 software has not mentioned.

In addition, when analyzing the stress and deformation state of bending structures such as beams, frames, plates ... the effect of shear force and sliding deformation is often overlooked. However, for bar elements such as beams with a thin bell plate, the impact of the shear force needs to be considered. Especially for the I-shaped taper element, with semi-rigid connection at both ends, the effect of shear force should be considered clearly. In addition, structural steel structures such as beams and columns are linked together to form a bearing structure. Many research results show that the connection has certain elasticity, also known as soft connection or semi-rigid connections. The problem of analyzing the steel frame structure with consideration of the linkage hardness has been interested recently by many domestic and foreign authors, however, for the taper element, taking into account the influence of the connection stiffness, there are almost no in-depth studies.

II. THEORY

A. Model of taper element, I cross-section

1) Geometric characteristics of taper

Due to beam and column with taper components, the cross section I is widely used as a bearing element in steel frame, the height of the section often changes. To determine the geometrical characteristics of sections at each position in length, including area, inertia moment, torsion moment, these cross sectional features are mathematically performed. The relational form of quantities of geometric dimensions in the section is quite complicated.

In fact, to reduce numerical integration after each calculation and still give accurate results, the representation of the geometric characteristics of the section through the law of the exponential exponential function, or by exponential function with radix is the ratio of taper, also mentioned by some documents.
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Accordingly, with the taper element with the I section, in the coordinate system Oyz, there are cross section characteristics defined:

Area section bar, \( A(z) \)

\[ A(z) = A_1 (1 + rz/L)^n \]  

(1)

\[ \ell(z) = \ell_1 (1 + rz/L)^m \]  

(2)

Where: \( A(z) \) – section area at position “z”; \( \ell(z) \) – moment of inertia at position “z”; \( A_1, \ell_1, n \) and \( m \) – respectively the area of cross section, moment of inertia, height of beam section at position \( z = L, \) \( r = h_2/h_1 - 1 \)

Notice that at the ends of the two elements, the section-specific values are correct, but in the middle position there is a certain error. Below we examine some sections.

2) Shape coefficient of some sections

Considering section 1, hollow box and solid box shape, for a constant width, thickness, and height, the shape of coefficients "n" and "m" are shown in Tab. 1.

<table>
<thead>
<tr>
<th>No</th>
<th>Section</th>
<th>Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>rectangular</td>
<td>n 2.1 + 2.6</td>
</tr>
<tr>
<td>2</td>
<td>hollow box</td>
<td>n 2.1 + 2.6</td>
</tr>
<tr>
<td>3</td>
<td>solid box</td>
<td>n 3.0</td>
</tr>
</tbody>
</table>

3) Error when determining section characteristics through exponential function

As mentioned, the exponential exponential performance has the convenience of calculation, but there is a certain error compared to the exact formula. Accordingly, the survey of this error for some I-beam cross section is popular in practice, the survey position is the mid-beam point, the survey section characteristics include area, inertia moment in the bending plane, the coefficient of taper coefficient changes as follows 0; 1.0; 2.0; 3.0; 4.0. Survey results are shown in Tab. 2.

The following symbol: \( h_b \times h_1 \times h_2 \times t_w \times t_l (\text{mm}) \) is used to denote the geometric parameters of the cross sectional bar I. Where: \( h_b \) – the width of the wing, \( h_1 \) – the height at the ends "1", \( h_2 \) – the height at the ends "2", \( t_w \) – the width of bell plate, \( t_l \) – the width of wing.

<table>
<thead>
<tr>
<th>r</th>
<th>A(z)</th>
<th>I(z)</th>
<th>A(z)</th>
<th>I(z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>1.0</td>
<td>1.44</td>
<td>0.76</td>
<td>1.40</td>
<td>1.00</td>
</tr>
<tr>
<td>2.0</td>
<td>3.53</td>
<td>2.24</td>
<td>3.20</td>
<td>2.77</td>
</tr>
<tr>
<td>3.0</td>
<td>5.39</td>
<td>3.87</td>
<td>4.64</td>
<td>4.54</td>
</tr>
<tr>
<td>4.0</td>
<td>6.93</td>
<td>5.45</td>
<td>5.74</td>
<td>6.13</td>
</tr>
</tbody>
</table>

Tab. 2 shows when the taper ratio \( r \leq 3 \) the biggest error of \( A, I \), is about 5%. The content of this article only takes into account the taper element with \( r \leq 3 \).

B. The making of stiffness matrix and load vector for taper element, I-section, two rigid connection ends

1) Method of making matrix

Using the making method of the stiffness matrix presented in Section 4.4 of the document [4], we have the stiffness matrix of the element built according to the following formula:

\[ [k] = [d]^{-1} [\Phi^t] [\Phi]^{-1} [d]^{-1} [\Phi^t] \]  

(5)

Where: \([d] = [k_1]^{-1}\) – matrix between pinned displacement vector and reaction force. \([\Phi] – matrix between the internal forces of the pinned.

2) Making stiffness matrix:

a) Making stiffness matrix with tension - compression bar element

The taper element has an I section (belly plate thickness and wing size are constant), length \( L \) and cross section characteristics at any position \( z \): \( \ell(z) \) and \( A(z) \)

\[ [k] = [d]^{-1} [\Phi^t] [\Phi]^{-1} [d]^{-1} [\Phi^t] \]  

(5)

Where: \([d] = [k_1]^{-1}\) – matrix between pinned displacement vector and reaction force. \([\Phi] – matrix between the internal forces of the pinned.

b) Making stiffness matrix for flat bending bar

The displacement at point 2 under the effect of force U2 is:

\[ u_2 = \int_0^L \frac{d}{E} \frac{dx}{A(z)} \frac{dU}{E \cdot A(z)} \]  

(6)

Where: \( E \) – Elastic modulus of bar material; \( A(z) \) – bar section area at position \( z \).

From balance equation we have: \( U_1 = -U_2 \) thus \( [\Phi] = -1 \).

So, stiffness matrix of tension - compression bar element is defined as follows:

\[ [k] = [k_1]^{-1} [k_1]^{-1} \]  

(8)

Fig. 3a.b. Internal forces and displacements
Make a connection matrix between displacement vector and reaction force of pinned. To determine \( \mathbf{v}_i \) and \( \mathbf{r}_i \). We fixed the first 2 of the bar as shown in Fig. 3b.

In the plane problem, with the moments, shear force, the potential energy expression of elastic deformation for the general force-bearing linear bar has length L as follows:

\[
\mathcal{E} = \int_0^L \frac{M(z)^2 dz}{E I(z)} + \int_0^L \gamma(z) W(z) dz \frac{dz}{2 GA(z)}
\]

(9)

Where: \( \mathcal{E} \) – elastic deformation potential energy; \( M(z) \) – Expreesion of bending moment at the section with the-z-coordinate; \( W(z) \) – expression of shear force at the section with the-z-coordinate; \( B(z) \) – Anti-bending stiffness of section (Anti-bending stiffness plane \( yz \)) at section with-z-coordinates; \( G \) – Elastic slip modulus; \( GA(z) \) – shear strength of section at section with-z-coordinates;

\( \gamma(z) \) – the coefficient depends on the y-shape of the cross-section at the section with-z-coordinates, reflecting the uneven distribution of the stress; For I section or hollow rectangular \( \gamma(z) = A(z)/A(z) \);

\( A(z) \) – vertical section area at position z.

Coefficient \( \gamma(z) \) can be expressed through the exponential rule with radix is the ratio of taper according to the following formula:

\[
\gamma(z) = \gamma_1(1 + rz/L)^n
\]

(10)

\( \gamma_1, \gamma_2 \) – in turn, the coefficient depends on the sectional shape according to the y at \( z = 0 \) and \( z = L \);

\( p = ln(\gamma_1/\gamma_2)/ln(1/h_i/h_i) \)

(11)

According to Fig. 3b, moment and shear force at the z position of the bar element are determined from the moment balance equation for pinned 1 and the shear balance equation:

\[
M(z) = \mathbf{V}_1(L - z) + \mathbf{M}_1; \quad V(z) = -\mathbf{V}_1
\]  

(12a,b) Replace (12a,b) into (9) we have:

\[
\mathcal{E} = \int_0^L \frac{\mathbf{V}_1(L - z) + \mathbf{M}_1}{E I(z)} \frac{dz}{2} + \int_0^L \frac{\gamma(z) \mathbf{V}_1 dz}{2 GA(z)}
\]

Applying Castigliano’s theorem presented in the document [3], we have:

\[
\mathbf{V}_1 = \frac{\mathcal{E}}{\mathbf{M}(L - z) + \mathbf{M}_1(z)} ; \quad \mathbf{M}_1 = \frac{\mathcal{E}}{\mathbf{E} I(z)}
\]

(13a)

Rewrite (13a) and (13b):

\[
\mathbf{M}_1 = \int_0^L \frac{\mathbf{V}_1(L - z) + \mathbf{M}_1}{E I(z)} \frac{dz}{2} + \int_0^L \frac{\gamma(z) \mathbf{V}_1 dz}{2 GA(z)}
\]

(13c)

\[
\mathbf{M}_1 = \int_0^L \frac{dz}{E I(z)} + \int_0^L \frac{\gamma(z) \mathbf{V}_1 dz}{GA(z)}
\]

(13d)

Where: \( \beta = E/G \); \( \mathbf{M}_1, \mathbf{M}_2 \) – moment at the ends 1 and 2; \( \mathbf{V}_1, \mathbf{V}_2 \) – shear force at the ends 1 and 1; \( \mathbf{r}_1, \mathbf{r}_2 \) – rotation angle at the ends 1 and 2; \( \mathbf{v}_1, \mathbf{v}_2 \) – displacement in the direction perpendicular to the bar axis at the pinned 1 and 2.

Expression (13c,d) write in matrix form:

\[
\begin{bmatrix}
\mathbf{v}_1 \\
\mathbf{v}_2 \\
\mathbf{r}_1 \\
\mathbf{r}_2
\end{bmatrix} = \begin{bmatrix}
\mathbf{a}_1 \\
\mathbf{a}_2 \\
\mathbf{a}_3 \\
\mathbf{a}_4
\end{bmatrix}
\]

(20)

Set basic integrals as follows:

\[
T_1 = \int_0^L \frac{dz}{L_1(z)} \frac{(1 + r)^n}{L_1 - 1 - n}
\]

\[
T_2 = \int_0^L \frac{dz}{L_1(z)} \frac{(1 + (1 + r)^n)}{L_1 - 1 - n}
\]

\[
\mathbf{M}_1 = \int_0^L \frac{dz}{E I(z)} \frac{(2 + (1 + r)^n)}{L_1 - 1 - n}
\]

(22)

(23)

Therefore, the matrix \( \mathbf{[\Phi]} \) as follows:

\[
\mathbf{[\Phi]} = \begin{bmatrix}
-1 & 0 \\
-L & -1
\end{bmatrix}
\]

(24)
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Making the stiffness matrix Applying formula (5) stiffness matrix is defined as follows:

\[
[K_e] = \begin{bmatrix}
K_{e_{11}} & K_{e_{12}} & K_{e_{13}} & K_{e_{14}} \\
K_{e_{21}} & K_{e_{22}} & K_{e_{23}} & K_{e_{24}} \\
K_{e_{31}} & K_{e_{32}} & K_{e_{33}} & K_{e_{34}} \\
K_{e_{41}} & K_{e_{42}} & K_{e_{43}} & K_{e_{44}}
\end{bmatrix}
\]

(25)

with: \([d]^{-1} = [K_e]^{-1} ; [d']^{-1} = [K_e]^{-1} \) ; \([\phi]^{-1} = [K_e]^{-1} \) .

\([\phi]^{-1} = [K_e]^{-1} ; [\phi]^{-1} = [K_e]^{-1} \) ;

We define the elements of the stiffness matrix \( K_i \). Then we rewrite the stiffness matrix in the following order:

\[
[K_e] = \begin{bmatrix}
K_{e_{22}} & K_{e_{23}} & K_{e_{24}} & K_{e_{25}} \\
K_{e_{52}} & K_{e_{53}} & K_{e_{54}} & K_{e_{55}} \\
K_{e_{62}} & K_{e_{63}} & K_{e_{64}} & K_{e_{65}} \\
K_{e_{72}} & K_{e_{73}} & K_{e_{74}} & K_{e_{75}}
\end{bmatrix}
\]

(26)

where:

\[
\begin{align*}
K_{e_{22}} &= \frac{12EL}{L^2} \left[ \frac{1}{2} T_2 + T_2 \right] + \frac{6EL}{L^2} \left[ \frac{1}{2} T_2 + T_2 \right] \\
K_{e_{23}} &= \frac{4EL}{L^4} \left[ \frac{1}{2} T_2 + T_2 \right] + \frac{6EL}{L^4} \left[ \frac{1}{2} T_2 + T_2 \right] \\
K_{e_{24}} &= \frac{12EL}{L^2} \left[ \frac{1}{2} T_2 + T_2 \right] + \frac{6EL}{L^2} \left[ \frac{1}{2} T_2 + T_2 \right] \\
K_{e_{25}} &= \frac{12EL}{L^2} \left[ \frac{1}{2} T_2 + T_2 \right] + \frac{6EL}{L^2} \left[ \frac{1}{2} T_2 + T_2 \right] \\
K_{e_{52}} &= \frac{12EL}{L^2} \left[ \frac{1}{2} T_2 + T_2 \right] + \frac{6EL}{L^2} \left[ \frac{1}{2} T_2 + T_2 \right] \\
K_{e_{53}} &= \frac{12EL}{L^2} \left[ \frac{1}{2} T_2 + T_2 \right] + \frac{6EL}{L^2} \left[ \frac{1}{2} T_2 + T_2 \right] \\
K_{e_{54}} &= \frac{12EL}{L^2} \left[ \frac{1}{2} T_2 + T_2 \right] + \frac{6EL}{L^2} \left[ \frac{1}{2} T_2 + T_2 \right] \\
K_{e_{55}} &= \frac{12EL}{L^2} \left[ \frac{1}{2} T_2 + T_2 \right] + \frac{6EL}{L^2} \left[ \frac{1}{2} T_2 + T_2 \right]
\end{align*}
\]

(27)

The composition of the stiffness matrix for bending resistant bar elements are values \( K_i \) \((i,j = 2,3,5,6)\) obtained above.

The stiffness matrix of the tension-compression-bending element in the plane is the "total" of the tension-compression stiffness matrix with the bending stiffness matrix. "Total" means to sort in order: \( K_i = K_{e_i} + K_{x_i} \)

3) Load vector

\[
\begin{bmatrix}
V_1 \\
M_1 \\
L \\
q_1 \\
V_2
\end{bmatrix}
\]

Fig. 4. Bending element bar, with rigid connection, trapezoidal load

Determine displacement and rotation angle at pinned 1 \(( \theta_v ; \theta_i )\)

The moment balancing equation for pinned 2 and the shear force balancing equation of the bar element is:

\[
\begin{align*}
\theta_{12} + \theta_{13} - V_1 L + (2a + q_1) L^2 / 6 &= 0 ; \quad V_2 V_1 = 0.5(q_1 + q_2) L \\
\theta_{12} &= 0.5(q_1 + q_2) L \\
\end{align*}
\]

(27a,b)

Moments and shear force at the z-position of the bar are determined from the moment balancing of the left part and the shear force balancing equation, we get:

\[
\begin{align*}
\theta(z) &= [2q + Mr_2 - q_2 z^2 / 2 + (q_1 + q_2) z^2 / 6L] z / 2 \\
V(z) &= [2q + Mr_2 - q_2 z^2 / 2 + (q_1 + q_2) z^2 / 6L] z / 2
\end{align*}
\]

(27c,d,e)

Replace (27c,d) into (9) we have:

\[
\begin{align*}
\frac{\partial}{\partial z} \left[ \frac{V_2 - Mr_2 - q_2 z^2 / 2}{2} \right] + \int_{z}^{\gamma} \frac{V_2 - q_2 z^2 / 2 + (q_1 + q_2) z^2 / 6L}{2G(z)} dz \\
\frac{\partial}{\partial z} \left[ \frac{q_2 z^2 / 2}{2} \right] + \int_{z}^{\gamma} \frac{q_2 z^2 / 2 + (q_1 + q_2) z^2 / 6L}{2G(z)} dz
\end{align*}
\]

Applying Castigliano's theorem, we have:

\[
\begin{align*}
\frac{\partial}{\partial z} \left[ \frac{V_2 - Mr_2 - q_2 z^2 / 2}{2} \right] + \int_{z}^{\gamma} \frac{V_2 - q_2 z^2 / 2 + (q_1 + q_2) z^2 / 6L}{2G(z)} dz \\
\frac{\partial}{\partial z} \left[ \frac{q_2 z^2 / 2}{2} \right] + \int_{z}^{\gamma} \frac{q_2 z^2 / 2 + (q_1 + q_2) z^2 / 6L}{2G(z)} dz
\end{align*}
\]

(28)

Compact expression; change basic integrals and apply boundary conditions at pinned 1: linear displacement perpendicular to the bar axis and rotation at the 1st pinned equal to 0 \(( \theta_v = 0 ; \theta_i = 0 )\) obtain the equation as follows:

\[
\begin{align*}
V_1 = & \frac{3Lq_1 [T_1 T_2 - T_2 (T_2 + 2T_2)] \left[ T_1 T_2 - T_2 (T_2 + 3T_2) \right]}{6L \left[ T_2 - T_2 (T_2 + 3T_2) \right]} \\
M_1 = & \frac{3Lq_1 [T_1 T_2 - T_2 (T_2 + 2T_2)] \left[ T_1 T_2 - T_2 (T_2 + 3T_2) \right]}{6L \left[ T_2 - T_2 (T_2 + 3T_2) \right]}
\end{align*}
\]

(31a,b)

Replace (31a,b) into (27a,b) that we obtain \( \theta_{12} \) and \( \theta_{13} \):

\[
\begin{align*}
\theta_{12} &= \theta_{13} = \frac{3L^2 (q_1 + q_2) \left[ T_2 - T_2 (T_2 + 2T_2) \right] + 3Lq_1 \left[ T_2 - T_2 (T_2 + 3T_2) \right]}{6L \left[ T_2 - T_2 (T_2 + 3T_2) \right]}
\end{align*}
\]

(32a,b)

From the above result, the load vector of the bending bar is obtained with uniformly distributed load:

\[
\begin{bmatrix}
F_i \\
M_i \\
V_i \end{bmatrix}
\]

(33)

C. Making the stiffness matrix and load vector for I-shaped taper bar element with semi-rigid connection at both ends

1) Making stiffness matrix of bending bar

The potential energy part of the pinned takes into account the stiffness:

\[
C = 0.5k_i (M_i / k_i)^2 \quad + 0.5k_i (M_i / k_i)^2
\]

(34)

Where: \( k_i ; k_2 ; M_i ; M_i \) respectively the stiffness and bending moment of pinned 1 and 2.
The total elastic energy potential of the taper section element is semi-rigid connection:

$$C = C_e + C_s = \int \left( \frac{1}{2} (M(z)^2 dz + 0.5 \frac{1}{2} (\gamma(z) V(z) dz)^2 + k_i (\frac{M(z)}{k_i})^2 \right)$$

(35)

According to the Fig. 3b, moment and shear force at the z position of the bar determined from the moment balance equation for the left part and the shear balance equation:

$$M(z) = V(z) (L-z) + M_0; V(z) = -\frac{\partial M(z)}{\partial z};$$

(36a,b)

Replace (36a,b) into (35) that we have:

$$C = \int \left( \frac{1}{2} (M(z)^2 dz + 0.5 \frac{1}{2} (\gamma(z) V(z) dz)^2 + \frac{M_0^2}{2k_i} + \frac{M(z) V(z)}{k_i} \right)$$

(37)

Balance equation for moment and shear force follows Fig. 3c is:

$$M(z) + M_0 + Vz - L = 0; V(z) = 0$$

(38a,b)

Replace (38a,b) into (37) that we have:

$$C = \int \left( \frac{1}{2} (M(z)^2 dz + 0.5 \frac{1}{2} (\gamma(z) V(z) dz)^2 + \frac{M_0^2}{2k_i} + \frac{M(z) V(z)}{k_i} \right)$$

(39a,b)

Applying Castigliano’s theorem, we have:

$$\frac{\partial C}{\partial \phi_2} = \int \left( \frac{1}{2} (M(z)^2 dz + 0.5 \frac{1}{2} (\gamma(z) V(z) dz)^2 + \frac{M_0^2}{2k_i} + \frac{M(z) V(z)}{k_i} \right)$$

(40a,b)

Expression (40a,b) written in matrix form:

$$\left[ \frac{\partial C}{\partial \phi_2} \right] = \left[ \frac{\partial C}{\partial \phi_2} \right] \left[ \frac{\partial \phi_2}{\partial \phi_2} \right] \left[ \frac{\partial \phi_2}{\partial \phi_2} \right]$$

where:

$$\frac{\partial C}{\partial \phi_2} = \int \left( \frac{1}{2} (M(z)^2 dz + 0.5 \frac{1}{2} (\gamma(z) V(z) dz)^2 + \frac{M_0^2}{2k_i} + \frac{M(z) V(z)}{k_i} \right)$$

(41a)

$$\frac{\partial \phi_2}{\partial \phi_2} = \int \left( \frac{1}{2} (M(z)^2 dz + 0.5 \frac{1}{2} (\gamma(z) V(z) dz)^2 + \frac{M_0^2}{2k_i} + \frac{M(z) V(z)}{k_i} \right)$$

(41b)

Replace the basic integral values from T1 to T8 into (41a,b,c) that we have:

$$\frac{\partial C}{\partial \phi_2} = \left[ \begin{array}{c} \phi_2 \end{array} \right] = \left[ \begin{array}{cccc} \phi_2 & \phi_2 & \phi_2 & \phi_2 \end{array} \right]$$

(42)

Stiffness matrix:

$$\left[ K_2 \right] = \left[ \begin{array}{cccc} K_{21} & K_{22} & K_{23} & K_{24} \\ K_{22} & K_{22} & K_{23} & K_{24} \\ K_{23} & K_{23} & K_{23} & K_{24} \\ K_{24} & K_{24} & K_{24} & K_{24} \end{array} \right]$$

$$\left[ K_2 \right]$$

$$\left[ MS \right] = \left[ L^2 + 2M(z) - T_2 + \alpha_1 T_1 \right]$$

$$\left[ K_2 \right] = \frac{\partial C}{\partial \phi_2} = \left[ \begin{array}{cccc} K_{21} & K_{22} & K_{23} & K_{24} \end{array} \right]$$

(43)

(44a,b)

Replace (44a,b) into (35) we get:

$$C = \int \left( \frac{1}{2} (M(z)^2 dz + 0.5 \frac{1}{2} (\gamma(z) V(z) dz)^2 + \frac{M_0^2}{2k_i} + \frac{M(z) V(z)}{k_i} \right)$$

(45)

$$V(z) = \left[ \begin{array}{c} \frac{\partial C}{\partial \phi_2} \end{array} \right]$$

(46)

$$\left[ \frac{\partial C}{\partial \phi_2} \right]$$

2) Making the load vector for taper element, semi-rigid connection

Fig. 5. The bar element has a semi-rigid connection, the trapezoidal load

Determine displacement and rotation angle at pinned 2 ($$\phi_2; \phi_2$$)

Moment balance equation with pinned 1 and shear force balance equation:

$$M(z) + M_0 + V(z) - L = 0; V(z) = 0.5(q + q_2)$$

(43)

Moments and shear force at the z position of the bar are determined from the moment balance equation for the left part and the shear balance equation, we get:

$$M(z) = V(z) - L = M_0 - q_2 (2L + z)(L-z)/6 - q_2 (L-z)/6;$$

(44a)

$$V(z) = [q_2 (z - L)/2 - V(z);$$

(44b)

Replace (44a,b) into (35) we get:

$$C = \int \left( \frac{1}{2} (M(z)^2 dz + 0.5 \frac{1}{2} (\gamma(z) V(z) dz)^2 + \frac{M_0^2}{2k_i} + \frac{M(z) V(z)}{k_i} \right)$$

(45)

$$\left[ \frac{\partial C}{\partial \phi_2} \right]$$

2) Making the load vector for taper element, semi-rigid connection

Fig. 5. The bar element has a semi-rigid connection, the trapezoidal load

Determine displacement and rotation angle at pinned 2 ($$\phi_2; \phi_2$$)

Moment balance equation with pinned 1 and shear force balance equation:

$$M(z) + M_0 + V(z) - L = 0; V(z) = 0.5(q + q_2)$$

(43)

Moments and shear force at the z position of the bar are determined from the moment balance equation for the left part and the shear balance equation, we get:

$$M(z) = V(z) - L = M_0 - q_2 (2L + z)(L-z)/6 - q_2 (L-z)/6;$$

(44a)

$$V(z) = [q_2 (z - L)/2 - V(z);$$

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Replace (44a,b) into (35) we get:

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Replace (44a,b) into (35) we get:

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$$M(z) + M_0 + V(z) - L = 0; V(z) = 0.5(q + q_2)$$

(43)

Moments and shear force at the z position of the bar are determined from the moment balance equation for the left part and the shear balance equation, we get:

$$M(z) = V(z) - L = M_0 - q_2 (2L + z)(L-z)/6 - q_2 (L-z)/6;$$

(44a)

$$V(z) = [q_2 (z - L)/2 - V(z);$$

(44b)

Replace (44a,b) into (35) we get:

$$C = \int \left( \frac{1}{2} (M(z)^2 dz + 0.5 \frac{1}{2} (\gamma(z) V(z) dz)^2 + \frac{M_0^2}{2k_i} + \frac{M(z) V(z)}{k_i} \right)$$

(45)
Building the Stiffness Matrix and Load Vector of the Taper Elements with the I-Section That Considering the Effect of Shear Force and Stiffness of the Connection

\( \bar{\delta}_z = \frac{dC}{dM} = \int_0^L \left[ \nabla_z (L-z) + \bar{M}_z \frac{d}{dL} (2L-z)(L-z)^3 - \frac{d(2L-z)}{dL} - \frac{d}{dL} (L-z)^3 \right] \, dz \)

\[ \bar{V}_z = \frac{L^2}{6} (q_2 + 2q_1) + \bar{V}_z = \frac{L^3}{12} \frac{d}{dL} (2L-z) + \frac{d}{dL} (L-z)^3 \]

Determine displacement and rotation angle at pinned 1 \((\eta_1; \gamma_1)\).

The moment balance equation for node 2 and the shear force balance equation of the bending bar element is as above Fig. 5 as: \( \bar{M}_x + \bar{M}_z = \bar{V}_z + (2q_1 + q_2) L^2 / 6 = 0; \bar{V}_z = (q_1 + q_2) L \)

(46)

\[ \bar{M}_z(z) = \nabla_z - \bar{M}_z - q_2 z^2 / 2 + (q_1 - q_2) z^3 / (6L) \]

(47a)

\[ \bar{V}_z(z) = \nabla_z - (q_1 + q_2) / (z/L) \]

(47b)

Replace (47a.b) into (37) we get:

\[ \bar{C} = \int_0^L \left[ \frac{\nabla_z - \bar{M}_z - q_2 z^2 / 2 + (q_1 - q_2) z^3 / (6L) \right] \, dz \]

\[ \bar{V}_z = \frac{L^2}{2G}(q_2 + 2q_1) + \frac{L^3}{12} \frac{d}{dL} (2L-z) + \frac{d}{dL} (L-z)^3 \]

(48)

Applying Castigliano’s theorem, we have:

\[ \frac{\partial C}{\partial V_1} = \int_0^L \left[ \frac{\nabla_z - \bar{M}_z - q_2 z^2 / 2 + (q_1 - q_2) z^3 / (6L) \right] \, dz \]

\[ \bar{M}_z = \int_0^L \left[ \nabla_z - \bar{M}_z - q_2 z^2 / 2 + (q_1 - q_2) z^3 / (6L) \right] \, dz \]

(49)

(50a.1)

Thus, we obtain the load vector for the taper element, section I, two semi-rigid connection ends subject to the trapezoidal distribution load as follows:

\[ \{ F \} = \{ V \} \{ \bar{M} \} \{ \bar{V} \} \]

(53)

III. PROGRAMMING CALCULATIONS AND NUMERICAL EXAMPLES

A. Programming base

On the built-in stiffness matrix and load vector matrix, the author set up a computer program based on 2014 MathLab specialized software named SAC (Static Analysis with Rigid connections) for analyzing the flat bar with I-section with height varies with length.

B. Example 1: Determination of internal force - displacement of taper cross-section steel beams with uniformly distributed load, two rigid connection ends
Known: I-beam cross section: I – (350÷700)×250×6×8

length L = 6 m, q = 10 kN/m, elastic modulus E = 21×10^4 MPa.

Requirements: Determine the bending moment and shear force, rigid connection)

The results are analyzed according to SAC and SAP (Tab. 7 - beam elements considering the impact of shear force, rigid connection); (Tab. 8 - beam elements without consideration of shear force, rigid connection)

C. The results are analyzed according to SAC and SAP and the difference according to the 2 software is recorded in (Tab. 9 - beam elements considering the impact of shear force, rigid connection); (Tab. 10 - beam elements without consideration of shear force, rigid connection)

D. Comparing the calculation results of the case of beam elements consider (PA1) and not consider (PA2) the rigid connection

When consider the shear force effect

### Tab. 7. Results of internal forces, beam displacement

<table>
<thead>
<tr>
<th>Elements</th>
<th>Internal force and displacement</th>
<th>Pinned</th>
<th>Results</th>
<th>Difference (%)</th>
</tr>
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<tbody>
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<td></td>
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<td>SAP</td>
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### Tab. 8. Results of internal forces, beam displacement

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### Tab. 9. Results of internal forces, beam displacement

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### Tab. 10. Results of internal forces, beam displacement

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Building the Stiffness Matrix and Load Vector of the Taper Elements with the I-Section That Considering the Effect of Shear Force and Stiffness of the Connection

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When not consider the shear force effect

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When not consider semi-rigid connection.

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When consider semi-rigid connection.

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<th>Results</th>
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When considering the stiffness of the connection with the assumed stiffness at the two ends of the connection is $2 \times 10^5$ kNm/rad. The result of the largest difference in internal force at pinned 2 has a difference of 43.5%.

E. Comparing the results of calculating the case of beam elements consider and not consider the effect of shear forces

IV. Conclusion

Through the content mentioned above, the author draws some conclusions and recommendations as follows:

- When dividing the bar into 2 elements, the calculation results for internal force and displacement at the pinned according to the programs SAC and SAP2000 have the largest value difference of 0.52% and the smallest is 0.1%. Thereby, it shows that the stiffness matrix and load vector have been set to be accurate, and the SAC calculation program can be trusted.

- When considering the effect of shear force, the internal force results are not much different, but the displacement result at pinned 2 has a difference of 43.5%.

- When considering the stiffness of the connection with the assumed stiffness at the two ends of the connection is $2 \times 10^5$ kNm/rad. The result of the largest difference in internal force at pinned 2 is 41.7%; The difference in displacement results at pinned 2 is 42.5%...

F. Comments on calculation results

- When dividing the bar into 2 elements, the calculation results for internal force and displacement at the pinned according to the programs SAC and SAP2000 have the largest value difference of 0.52% and the smallest is 0.1%. Thereby, it shows that the stiffness matrix and load vector have been set to be accurate, and the SAC calculation program can be trusted.

- When considering the effect of shear force, the internal force results are not much different, but the displacement result at pinned 2 has a difference of 43.5%.

- When considering the stiffness of the connection with the assumed stiffness at the two ends of the connection is $2 \times 10^5$ kNm/rad. The result of the largest difference in internal force at pinned 2 is 41.7%; The difference in displacement results at pinned 2 is 42.5%..
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