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The New Method of Analyzing the Continuous Curved Flat Bar Subject To Space Load

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***Abstract*: The report presents a new method for linear analysis of continuous curved flat bar, subject to any load in space. This method is a combination of improving the expression of the load and displacement at the two ends of the curved bar element of the Transfer Matrix Method and Finite Element Method (TMMFEM), called the Matrix Method transfer improvements. The research results are to build math problems and programming with Matlab, verify with the results according to SAP2000 oftware and « Strength of materials » documents.**

***Keywords*: Curved flat bar, Space load, Curved elemen,; Improved transfer matrix method.**

I. INTRODUCTION

The curved flat bar has a curve that is located in the plane of the bar axis. Due to the requirements of use and construction, it is common to have a continuous bar structure form mounted on the supports, bearing the load of space. The calculation of this bar system is quite complicated, the structural mechanical materials have not mentioned much. It is necessary to develop an innovative transfer matrix method to calculate a continuous flat bar system, subject to any load in space, such as arch structures; girder; balcony beams; belt etc. if you know the equation for the bar axis curve. Accordingly, build a program to calculate for a particular case, a continuous circular shape of a curved shape

II. THEORY

A. Making the transfer matrix method to improve general curved bar analysis

The method of transfer matrix in general structural analysis of curved bar is mentioned by the author (N.Tram, 1982) [1], with the assumption of elastic deformation material, ignoring the effect of sliding deformation, buckling and curvature of the bar. The space curve element of the coordinate system Oxyz is shown in Figure 1. At the conventional pinned, stress and displacement are positive when in the same direction as the separate coordinate system and vectors at the ends 1 and 2 of the element are $\{P_1\} = \{P_1 \quad M_1\}^T$, $\{U_1\} = \{U_1 \quad \Omega_1\}^T$, $\{P_2\} = \{P_2 \quad M_2\}^T$, $\{U_2\} = \{U_2 \quad \Omega_2\}^T$, with symbols *{P}*, *{M}*, *{U}*, *{Ω}* is the stress, moments, straight and rotating deformation at the pinned. The basic expression of the transfer matrix method is the relationship between the load and displacement at the ends 1 and 2 of the bar element "m" [1].

$$\begin{Bmatrix} U_2 \\ P_2 \end{Bmatrix} = \begin{bmatrix} [A_{12}^U] & [A_2^T]^{-1}\int_{s1}^{s2}[B]ds[A_1]^{-1} \\ O_3 & -[A_{12}^P] \end{bmatrix} \begin{Bmatrix} U_1 \\ P_1 \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} U_1 \\ P_1 \end{Bmatrix} = [T] \begin{Bmatrix} U_1 \\ P_1 \end{Bmatrix} \quad (1)$$

Matrix transfer $[T]$ is the set of geometric and mechanical characteristics of the element. Expression (1) is an algebraic linear system of equations that is located at both sides of the equation. The solution to this system needs to be transformed into a form *[A]{x} = {b}* and increase the calculation time. Overcoming the disadvantages mentioned above, improvements (1) in the direction taken on the same side unknown factor, we have:

$$\begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} = \begin{bmatrix} -T_{12}^{-1}T_{11} & T_{12}^{-1} \\ T_{21} - T_{22}T_{12}^{-1}T_{11} & T_{22}T_{12}^{-1} \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = [k_c]_m \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} \quad (2)$$

Expression (2) is the basic form of the equation involving the force and displacement of pinned by the TMMFEM method, where $[k_c]_m$ is the stiffness matrix of the bar element "m" in the general coordinate system. Applying the TMMFEM method, applying the calculation diagram characteristics of the bar is linear, non-branching, making solutions according to the improved method of transfer matrix as follows:

Step 1. Mark the number of pinned, it usually is the load set point, support or point to calculate displacements, internal forces. If the system has n pinned and from the initial data, we get the stiffness matrix of (n-1) elements.

Step 2. Assembling the element stiffness matrix into the overall stiffness matrix in the general coordinate system, $[K_c]$; with $[L]_m$ - positioning matrix of element m:

$$[K_c] = \sum_{m=1}^{n-1} [L]_m^T [k_c]_m [L]_m \quad (3)$$

Step 3. Making the displacement vector in the general coordinate system, $\{d_c\}$, it is the experiment vector to find; and load vector in the general coordinate system $\{P_c\} = \{\{P_1\} \ \dots \ \{P_s\} \ \dots \ \{P_n\}\}^T$,

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with $s = \overline{1, n}$, including the load set at the pinned and the load within the element to be assigned to the pinned.

Step 4. Making the balance equation in whole system in the general coordinate system:

$$\{P_c\} = [K_c]\{d_c\} \tag{4}$$

Equation (4) has solutions when determinant $K_c \neq 0$, It is necessary to handle boundary conditions on the principle that when pinned s has a connection that prevents straight deformation or rotated direction, the corresponding row value in that pinned in $\{d_c\}$ and $[P_c]$ equal "0". Remove rows and columns in $[K_c]$ corresponding to the row with the value "0" in $\{d_c\}$ and $[P_c]$, we get $[K_b]$. Remove rows with the value "0" in $\{d_c\}$ and $[P_c]$ we get $\{d_b\}$ and $[P_b]$. Then the balance equation for the whole structure is in the form:

$$\{P_b\} = [K_b]\{d_b\} \tag{5}$$

Step 5. Solve the system of equations (5) to find equation $\{d_b\}$, then identified $\{d_c\}$.

Step 6. Calculating pinned stress:

$$\{UL_c\}_m = \{UL_1 \quad UL_2\}_m^T = [k_c]_m\{d_c\}_m ; \{UL_r\}_m = [H\ ]_m^{-1}\{UL_c\}_m \tag{6}$$

With $\{UL_c\}_m, \{UL_r\}_m, [H\ ]_m$ In turn is the inner pinner stress vector in the general coordinate system, the private coordinate system and transfer coordinates matrix, the dimensions (12×12), when change from the private coordinate system to the general coordinate system.

B. Making the stiffness matrix of curved flat elements

Element m, assume the running point *S(x, y, z)*, as Figure 2, indicates the position of the considering section with the private coordinate system *Sx'y'z'*. the curved flat bar has z'-axis parallel to the z-axis and perpendicular to the private coordinate system *Oxy*, *z = z' = 0*. Space load impacts the load vector $\{P\ \}$ and general displacement vector $\{U\ \}$ has fully 6 components. [6,8,9]

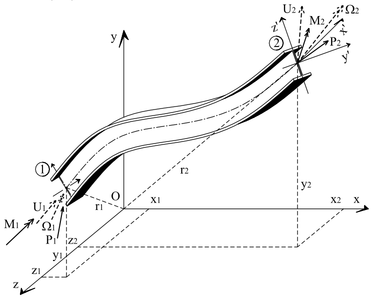

Figure 1. Curved element 1-2

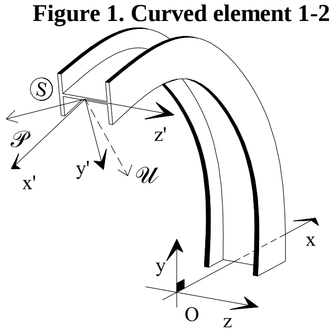

Figure 2. Curved element

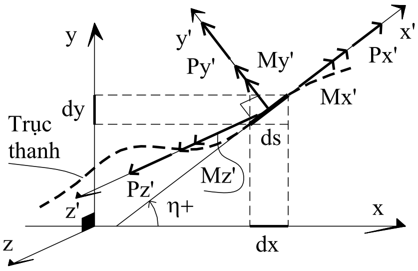

Figure 3. Components *ds* at *S*

$$\{P\ \} = \{P_{x'} \ P_{y'} \ P_{z'} \ M_{x'} \ M_{y'} \ M_{z'}\}^T ; \{U\ \} = \{U_{x'} \ U_{y'} \ U_{z'} \ \Omega_{x'} \ \Omega_{y'} \ \Omega_{z'}\}^T \tag{7}$$

Positioning matrix takes the form:

$$[A_i] = \begin{bmatrix} I_3 & O_3 \\ A_i & I_3 \end{bmatrix} \ (i = 1 \div 2); \ [A_i] = \begin{bmatrix} 0 & 0 & -y_i \\ 0 & 0 & x_i \\ y_i & -x_i & 0 \end{bmatrix}$$

$$[A_{12}^{P}] = \begin{bmatrix} & I_3 & & O_3 \\ 0 & 0 & -(y_2 - y_1) & \\ 0 & 0 & (x_2 - x_1) & I_3 \\ (y_2 - y_1) & -(x_2 - x_1) & 0 & \end{bmatrix}; [A_{12}^{U}] = \begin{bmatrix} & 0 & 0 & (y_1 - y_2) \\ I_3 & 0 & 0 & -(x_1 - x_2) \\ & -(y_1 - y_2) & (x_1 - x_2) & 0 \\ O_3 & & I_3 & \end{bmatrix} \tag{8}$$

Determine the coordinate transfer matrix at the point position *S(x, y)*. Because z and z' axis perpendicular to Oxy, so :

$$\cos z'z = 1; \cos z'x = \cos z'y = \cos x'z = \cos y'z = 0,$$

the coordinate transition matrices take the form:

$$[H_P] = [H_U] = [H_M] = [H_\Omega] = \begin{bmatrix} \cos x'x & \cos y'x & 0 \\ \cos x'y & \cos y'y & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

In Figure 3, show the equation *ds*, from the trigonometric relationship between two coordinate systems, we see:

$$\begin{aligned} &\cos x'x = \cos y'y = \cos\eta = \frac{dx}{ds} = x_s';\\ &\cos x'y = \sin\eta = \frac{dy}{ds} = y_s';\\ &\cos y'x = -\sin\eta = -\frac{dy}{ds} = -y_s' \end{aligned} \tag{10}$$

So we get:

$$[H\ ]_s = \begin{bmatrix} H_P & 0 \\ 0 & H_M \end{bmatrix} = \begin{bmatrix} H_U & 0 \\ 0 & H_\Omega \end{bmatrix} = \begin{bmatrix} x_s' & -y_s' & 0 & \\ y_s' & x_s' & 0 & O_3 \\ 0 & 0 & 1 & \\ & O_3 & & ĐX \end{bmatrix} \tag{11}$$

The elastic matrix is shaped:

$$[M_P] = \begin{bmatrix} 1/EF & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \ [M_M] = \begin{bmatrix} 1/GI_x & 0 & 0 \\ 0 & 1/EI_y & 0 \\ 0 & 0 & 1/EI_z \end{bmatrix} \tag{12}$$

Accordingly, calculate the component matrices of the block matrix $[B\ ]$ in the expression (1):

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$$[M_M]=\begin{bmatrix} \frac{x_s'^2}{GI_x}+\frac{y_s'^2}{EI_y} & \frac{x_s'y_s'}{GI_x}-\frac{x_s'y_s'}{EI_y} & 0 \\ \frac{x_s'y_s'}{GI_x}-\frac{x_s'y_s'}{EI_y} & \frac{y_s'^2}{GI_x}+\frac{x_s'^2}{EI_y} & 0 \\ 0 & 0 & \frac{1}{EI_z} \end{bmatrix};$$

$$[M_M][A]=\begin{bmatrix} 0 & 0 & -y\left(\frac{x_s'^2}{GI_x}+\frac{y_s'^2}{EI_y}\right)+x\left(\frac{x_s'y_s'}{GI_x}-\frac{x_s'y_s'}{EI_y}\right) \\ 0 & 0 & -y\left(\frac{x_s'y_s'}{GI_x}-\frac{x_s'y_s'}{EI_y}\right)+x\left(\frac{y_s'^2}{GI_x}+\frac{x_s'^2}{EI_y}\right) \\ \frac{y}{EI_z} & -\frac{x}{EI_z} & 0 \end{bmatrix}$$

$$[M_P]=\begin{bmatrix} \frac{x_s'^2}{EF} & \frac{x_s'y_s'}{EF} & 0 \\ \frac{x_s'y_s'}{EF} & \frac{y_s'^2}{EF} & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$[A]^T[M_M]=\begin{bmatrix} 0 & 0 & -y\left(\frac{x_s'^2}{GI_x}+\frac{y_s'^2}{EI_y}\right)+x\left(\frac{x_s'y_s'}{GI_x}-\frac{x_s'y_s'}{EI_y}\right) \\ 0 & 0 & -y\left(\frac{x_s'y_s'}{GI_x}-\frac{x_s'y_s'}{EI_y}\right)+x\left(\frac{y_s'^2}{GI_x}+\frac{x_s'^2}{EI_y}\right) \\ \frac{y}{EI_z} & -\frac{x}{EI_z} & 0 \end{bmatrix}^T$$

$$[M_P]+[A]^T[M_M][A]=$$

$$=\begin{bmatrix} \frac{x_s'^2}{EF}+\frac{y^2}{EI_z} & \frac{x_s'y_s'}{EF}-\frac{xy}{EI_z} & 0 \\ \frac{x_s'y_s'}{EF}-\frac{xy}{EI_z} & \frac{y_s'^2}{EF}+\frac{x^2}{EI_z} & 0 \\ 0 & 0 & x^2\left(\frac{y_s'^2}{GI_x}+\frac{x_s'^2}{EI_y}\right)+y^2\left(\frac{x_s'^2}{GI_x}+\frac{y_s'^2}{EI_y}\right)-2xy\left(\frac{x_s'y_s'}{GI_x}-\frac{x_s'y_s'}{EI_y}\right) \end{bmatrix}$$

Set the quantities:

$$C_1=\frac{x_s'^2}{GI_x}+\frac{y_s'^2}{EI_y};\ C_2=\frac{y_s'^2}{GI_x}+\frac{x_s'^2}{EI_y};\ C_3=\frac{1}{EI_z};\ C_4=\frac{x_s'^2}{EF}+\frac{y^2}{EI_z}$$

$$C_5=\frac{y_s'^2}{EF}+\frac{x^2}{EI_z};\ C_6=\frac{x_s'y_s'}{GI_x}-\frac{x_s'y_s'}{EI_y};\ C_7=\frac{x_s'y_s'}{EF}-\frac{xy}{EI_z} \quad (13)$$

We have,

$$[B]=\begin{bmatrix} C_4 & C_7 & 0 & 0 & 0 & yC_3 \\ C_7 & C_5 & 0 & 0 & 0 & -xC_3 \\ 0 & 0 & x^2C_2+y^2C_1-2xyC_6 & xC_6-yC_1 & xC_2-yC_6 & 0 \\ 0 & 0 & xC_6-yC_1 & C_1 & C_6 & 0 \\ 0 & 0 & xC_2-yC_6 & C_6 & C_2 & 0 \\ yC_3 & -xC_3 & 0 & 0 & 0 & C_3 \end{bmatrix}$$

(14)

Thus, we have all the components of the transfer matrix $[T]$ For curved flat bar bearing the space load [5].

C. Making the stiffness matrix of circular curved element

The equation of the circular rod axis with radius R, in the form $x_\varphi = R\cos\varphi; y_\varphi = R\sin\varphi$ (Figure 4). [10-12]

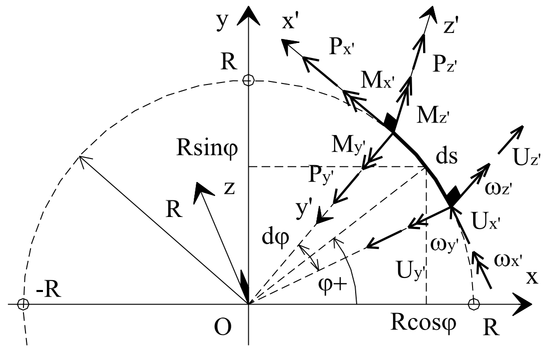

Figure 4. Flat round dome element is subject to space load

Consider the extremely small bar corresponding to the angle and arc differential $d\varphi$, ds, asume the bar to be straight, we have:

$$x_\varphi'=\frac{dx}{d\varphi}-R\sin\varphi \rightarrow dx=-R\sin\varphi d\varphi; dy=R\cos\varphi d\varphi;\ ds=\sqrt{dx^2+dy^2}=Rd\varphi;\ x_s'=\frac{dx}{ds}=-\sin\varphi;$$

$y_s'=\frac{dy}{ds}=\cos\varphi;\ x_s'y_s'=-\sin\varphi\cos\varphi.$ Calculate the terms in expression (14) as follows:

$$C_1=\frac{\sin^2\varphi}{GI_x}+\frac{\cos^2\varphi}{EI_y};C_2=\frac{\cos^2\varphi}{GI_x}+\frac{\sin^2\varphi}{EI_y};C_4=\frac{\sin^2\varphi}{EF}+\frac{R^2\sin^2\varphi}{EI_z};C_5=\frac{\cos^2\varphi}{EF}+\frac{R^2\cos^2\varphi}{EI_z};$$

$$xC_6-yC_1=-\frac{R}{GI_x}\left(\sin\varphi\cos^2\varphi+\sin^3\varphi\right);C_6=-\frac{\sin\varphi\cos\varphi}{GI_x}+\frac{\sin\varphi\cos\varphi}{EI_y};C_7=-\frac{\sin\varphi\cos\varphi}{EF}-\frac{R^2\sin\varphi\cos\varphi}{EI_z};$$

$$x^2C_2+y^2C_1-2xyC_6=\frac{R^2}{GI_x};yC_3=\frac{R\sin\varphi}{EI_z};-xC_3=-\frac{R\cos\varphi}{EI_z};xC_2-yC_6=\frac{R}{GI_x}\left(\cos^3\varphi+\sin^2\varphi\cos\varphi\right).$$

Calculating integral in $\int [B]ds$:

$$T_1 = \int_{\varphi_1}^{\varphi_2} \sin\varphi ds;\ T_2 = \int_{\varphi_1}^{\varphi_2} \sin^2\varphi ds;\ T_3 = \int_{\varphi_1}^{\varphi_2} \sin^3\varphi ds;\ T_4 = \int_{\varphi_1}^{\varphi_2} \sin\varphi\cos\varphi ds;\ T_5 = \int_{\varphi_1}^{\varphi_2} \sin\varphi\cos^2\varphi ds;$$

$$T_6 = \int_{\varphi_1}^{\varphi_2} \sin^2\varphi\cos\varphi ds;\ T_7 = \int_{\varphi_1}^{\varphi_2} \cos\varphi ds;\ T_8 = \int_{\varphi_1}^{\varphi_2} \cos^2\varphi ds;\ T_9 = \int_{\varphi_1}^{\varphi_2} \cos^3\varphi ds;\ T_{10} = \int_{\varphi_1}^{\varphi_2} ds.$$

Replace integrals T_i $(i = 1 \div 10)$ into (14), We get the expression (15):

$$\int_{\varphi_1}^{\varphi_2} [B] ds =$$

$$\begin{bmatrix}
\left(\frac{1}{EF}+\frac{R^2}{EI_z}\right)T_2 & \left(-\frac{1}{EF}-\frac{R^2}{EI_z}\right)T_4 & 0 & 0 & 0 & \frac{R}{EI_z}T_1 \\
\left(-\frac{1}{EF}-\frac{R^2}{EI_z}\right)T_4 & \left(\frac{1}{EF}+\frac{R^2}{EI_z}\right)T_8 & 0 & 0 & 0 & -\frac{R}{EI_z}T_7 \\
0 & 0 & \frac{R^2}{GI_x}T_{10} & -\frac{R}{GI_x}(T_3+T_5) & \frac{R}{GI_x}(T_6+T_9) & 0 \\
0 & 0 & -\frac{R}{GI_x}(T_3+T_5) & \frac{T_2}{GI_x}+\frac{T_8}{EI_y} & \left(-\frac{1}{GI_x}+\frac{1}{EI_y}\right)T_4 & 0 \\
0 & 0 & \frac{R}{GI_x}(T_6+T_9) & \left(-\frac{1}{GI_x}+\frac{1}{EI_y}\right)T_4 & \frac{T_8}{GI_x}+\frac{T_2}{EI_y} & 0 \\
\frac{R}{EI_z}T_1 & -\frac{R}{EI_z}T_7 & 0 & 0 & 0 & \frac{1}{EI_z}T_{10}
\end{bmatrix}$$

Along with the expression (8), replace the expression (2) with the stiffness matrix of the continuous curved element, subjected to space load $[k_c]_m$ with two ends defined by angle φ_1 and φ_2

III. PROGRAMMING FOR ANALYZING CIRCULAR CURVED BAR AND VERIFIABLE CALCULATIONS

A. *Circular curved bar analysis program*

Analysis program name: PCA - V1 *(Analysis Planar Circle Arc - Version 1);* Programming language: Matlab 2010a; PCA program capability - V1: Linear analysis of cigular-flat-curves-continuous bar subject to bearing the space load, with spatial connection in any position.

B. *Example 1: Symmetry verification of PCA-V1 analysis results*

Calculate and verify the displacement symmetry of points A, B, C, D and E of the circular dome of the radius of the rod axis R = 20,0 m as shown in Figure 5a. Steel materials have E = 2,1.10^8 kN/m^2; G = 0,808.10^8 kN/m^2. bearing the centripetal concentrated force, P = 15 kN, placed at the top of the dome and in the arch plane; and P_1 = 10 kN place at the top of the dome in a downward direction and perpendicular to the arch plane. Arch section I1500×400×20×10 mm, has geometric features I_x = 2,551.10^{-06} m^4; I_y = 0,0114 m^4, I_z = 2,135.10^{-04} m^4; F = 0,0306 m^2. Connection at points A and E are fixed in all directions. Use PCA-V1 program to analyze bar structure, divide the system into 4 curved elements, 5 pinned, 30 degrees of freedom as shown in Figure 5b. Button displacement in the general coordinate system is shown in Table 1. Through the analysis results show that the pinned displacements are symmetrical and consistent with the calculation diagram.

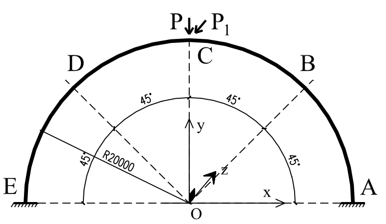

Figure 5a. Diagram of circular arched symmetry calculation

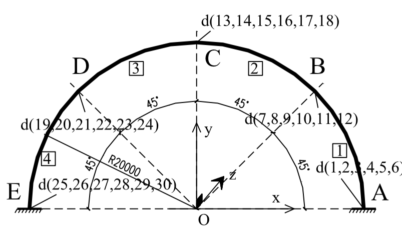

Figure 5b. Divide and marking element symbols

Table 1. Displacement of pinned in the general coordinate system, calculated according to PCA - V1 program

Pinned	A	B	C	D	E	Pinned	A	B	C	D	E
U_x [m]	0	-0.0152	-9.3E-16	0.0152	0	Ω_x [rad]	0	0.0874	1.0071	0.0874	0
U_y [m]	0	-0.0043	0.0313	-0.0043	0	Ω_y [rad]	0	0.4656	-5.5E-15	-0.4656	0
U_z [m]	0	2.8723	7.3625	2.8723	0	Ω_z [rad]	0	-0.0011	-1.2E-16	0.0011	0

A. *Example 2: Verify the results of analysis according to PCA-V1 compared with the results of material strength*

Verify the results of PCA-V1 compared with the results in Example 9-1, according to the author (B.T.Luu., N.V.Vuong, 1998) [3]. Bar calculation diagram and chart of internal force, shown in Figure 6.

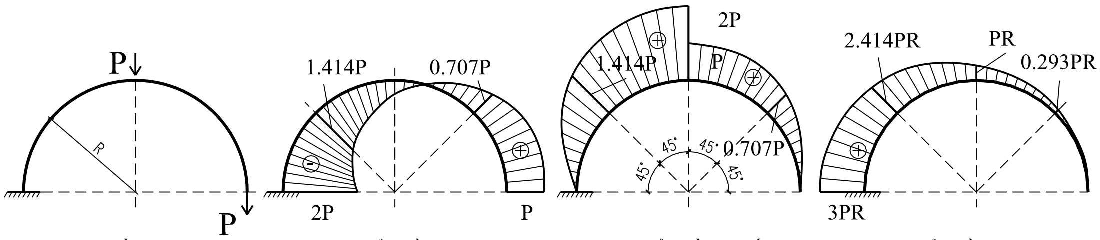

Figure 6. Calculation model and internal force diagram of curved bar [3]

For a numerical solution, for radius R = 10.0 m, P = 5.0 kN. Using PCA-V1, divide the bar into 4 curved elements with the angel 45^0, element notation, degrees of freedom as Example 1. Analysis results in Table 2. Underlined results show when calculated according to PCA- V1 coincides with the material Strength of materials and in other locations of the chart also completely match the calculated value.

Table 2. Pinned stress in the specific coordinate system, calculated according to PCA-V1 program

	Element	1	2	3	4		Elements	1	2	3	4
ends 1	$P_{x'}$ [kN]	-5.0	-3.54	0.0	7.1	ends 2	$P_{x'}$ [kN]	3.54	0.0	-7.1	<u>-10.0</u>
	$P_{y'}$ [kN]	0.0	3.54	<u>10.0</u>	7.1		$P_{y'}$ [kN]	-3.54	<u>-5.0</u>	-7.1	0.0
	$P_{z'}$ [kN]	0.0	0.0	0.0	0.0		$P_{z'}$ [kN]	0.0	0.0	0.0	0.0
	$M_{x'}$ kNm]	0.0	0.0	0.0	0.0		$M_{x'}$ [kNm]	0.0	0.0	0.0	0.0
	$M_{y'}$ [kNm]	0.0	0.0	0.0	0.0		$M_{y'}$ [kNm]	0.0	0.0	0.0	0.0
	$M_{z'}$ [kNm]	0.0	-14.6	-50.0	<u>-120.7</u>		$M_{z'}$ [kNm]	14.6	50.0	120.7	<u>150.0</u>

B. Example 3: Verify analysis results according to PCA-V1 compared to SAP2000 software results

Analysis of flat bar bearing general load as shown in Figure 7a. Validation results calculated compared to SAP2000 software. The parameters of material and bar section are as shown in Example 1. The effect load and the link point are shown in Table 3 and Table 4. With symbols: TD - free (the connection has no fixed position with the axis being considered) and CĐ - fixed (the connection with displacement is fixed with the axis being considered).

Table 3. Loads of pinneds in the general coordinate system

P	Pinned				
	B	C	D	G	H
P_x [kN]	-10.0	5.0	5.0	-18.0	8.0
P_y [kN]	12.0	6.0	-5.0	-15.0	7.0
P_z [kN]	-18.0	0.0	6.0	-10.0	-9.0
M_x [kNm]	12.0	-7.0	-4.0	10.0	-9.0
M_y [kNm]	-5.0	18.0	6.0	-8.0	6.0
M_z [kNm]	6.0	15.0	-8.0	9.0	10.0

Table 4. Support connection in the general coordinate system

U	Pinned				
	A	C	E	F	K
U_x	CĐ	TD	CĐ	TD	CĐ
U_y	CĐ	TD	CĐ	TD	CĐ
U_z	CĐ	CĐ	CĐ	CĐ	CĐ
Ω_x	TD	TD	TD	TD	CĐ
Ω_y	TD	TD	TD	TD	CĐ
Ω_z	TD	TD	TD	TD	CĐ

Use PCA-V1 program for analysis, calculation diagram of the bar as shown in Figure 7a, dividing the bar into 8 curved elements, 9 pinneds, 54 degrees of freedom as shown in Figure 7b. Results calculated in Table 5 and Table 6, Results calculated according to SAP2000 in Table 6 corresponding to bar division making 210 straight elements.

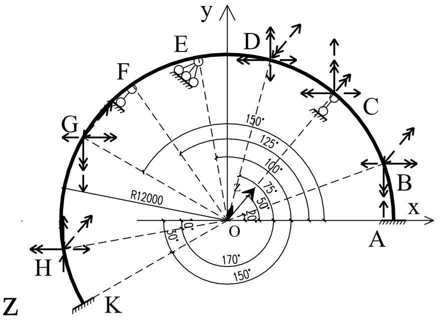

Figure 7a. Diagram of circular arched symmetry calculation

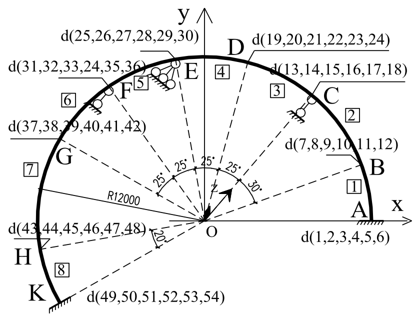

Figure 7b. Divide and marking element symbols

Table 5. Displacement of pinned in the general coordinate system, calculated according to PCA - V1 program

Pinned	A	B	C	D	E	F	G	H	K
U_x [m]	0	-0.0014	0.0005	0.0000	0	0.0004	-0.0012	0.0003	0
U_y [m]	0	-0.0002	0.0011	0.0002	0	-0.0013	0.0001	0.0001	0
U_z [m]	0	-0.0007	0	0.0102	0	0	-0.0839	-0.0241	0
Ω_x [rad]	-0.0034	0.0070	-0.0320	0.0050	-0.0212	0.0544	-0.0168	-0.0238	0
Ω_y [rad]	0.0223	-0.0162	0.0311	-0.0041	-0.0022	0.0278	-0.0578	0.0518	0
Ω_z [rad]	0.0006	-0.0001	0.0002	0.0000	0.0003	-0.0001	-0.0002	0.0002	0

Table 6. Displacement of pinned in the general coordinate system, calculated according to SAP2000 program

Pinned	A	B	C	D	E	F	G	H	K
U_1~U_x [m]	0	-0.0014	0.0005	0.0000	0	0.0004	-0.0012	0.0003	0
U_2~U_y [m]	0	-0.0002	0.0011	0.0002	0	-0.0013	0.0001	0.0002	0
U_3~U_z [m]	0	-0.0008	0	0.0104	0	0	-0.0841	-0.0241	0
R_1~Ω_x [rad]	-0.0034	0.0069	-0.0319	0.0051	-0.0213	0.0544	-0.0169	-0.0238	0
R_2~Ω_y [rad]	0.0224	-0.0161	0.0310	-0.0041	-0.0022	0.0278	-0.0579	0.0519	0
R_3~Ω_z [rad]	0.0006	-0.0001	0.0002	0.0000	0.0003	-0.0001	-0.0002	0.0002	0

Table 7. Pinned stress in the general coordinate system, calculated according to PCA - V1 program

Element		1	2	3	4	5	6	7	8
ends i	P_x [kN]	5.15	-4.85	0.15	5.15	15.78	15.78	-2.22	5.78
	P_y [kN]	-15.13	-3.13	2.87	-2.13	10.34	10.34	-4.66	2.34
	P_z [kN]	8.13	-9.87	4.82	10.82	-12.40	8.42	-1.58	-10.58
	M_x [kNm]	0.00	-21.36	21.87	6.30	3.85	-20.81	21.43	-0.34
	M_y [kNm]	0.00	-10.88	42.29	26.07	-30.09	29.44	-8.11	0.14
	M_z [kNm]	0.00	16.19	-4.63	0.98	-8.89	9.38	-5.76	15.53
ends j	P_x [kN]	-5.15	4.85	-0.15	-5.15	-15.78	-15.78	2.22	-5.78
	P_y [kN]	15.13	3.13	-2.87	2.13	-10.34	-10.34	4.66	-2.34
	P_z [kN]	-8.13	9.87	-4.82	-10.82	12.40	-8.42	1.58	10.58
	M_x [kNm]	33.36	-28.87	-10.30	-3.85	20.81	-11.43	-8.66	41.78
	M_y [kNm]	5.88	-24.29	-20.07	30.09	-29.44	0.11	5.86	14.94
	M_z [kNm]	-10.19	19.63	-8.98	8.89	-9.38	14.76	-5.53	10.46

Bảng 8. Pinned stress in the general coordinate system, calculated according to SAP2000 program

Element		1	2	3	4	5	6	7	8
ends i	F_1~P_x [kN]	5.15	-4.85	0.15	5.15	15.78	15.78	-2.22	5.78
	F_2~P_y [kN]	-15.12	-3.12	2.88	-2.12	10.33	10.33	-4.67	2.33
	F_3~P_z [kN]	8.13	-9.87	4.82	10.82	-12.40	8.42	-1.58	-10.58
	M_1~M_x [kNm]	0.00	-21.36	21.86	6.30	3.85	-20.80	21.44	-0.36
	M_2~M_y [kNm]	0.00	-10.88	42.28	26.08	-30.07	29.42	-8.11	0.14

	M_3~M_z [kNm]	0.00	16.18	-4.63	0.99	-8.85	9.38	-5.78	15.55
ends j	F_1~P_x [kN]	-5.15	4.85	-0.15	-5.15	-15.78	-15.78	2.22	-5.78
	F_2~P_y [kN]	15.12	3.12	-2.88	2.12	-10.33	-10.33	4.67	-2.33
	F_3~P_z [kN]	-8.13	9.87	-4.82	-10.82	12.40	-8.42	1.58	10.58
	M_1~M_x [kNm]	33.36	-28.86	-10.30	-3.85	20.80	-11.44	-8.64	41.81
	M_2~M_y [kNm]	5.88	-24.28	-20.08	30.07	-29.42	0.11	5.86	14.94
	M_3~M_z [kNm]	-10.18	19.63	-8.99	8.85	-9.38	14.78	-5.55	10.40

Compare the results in Table 5 and Table 6, finding that coincidence, some values, though there are differences but not more than 2%. The results of the nodal stress in general electric system by PCA-V1 are shown in Table 7, and according to SAP2000, see Table 8. Comparison of results also found that it almost matched

IV. CONCLUSION

Improved transfer matrix method, built on the basis of transfer matrix method and TMMFEM method, results are reliable and effective enough in linear curve analysis, with continuity and tolerance diagram load space. The research results have developed the PCA - V1 calculation program, using Matlab programming language. At the same time, the analysis results were compared with the results calculated in the material strength and SAP2000 Software; can use software to analyze arch structure, curved beams in construction works.

In addition, the next development is needed: to improve the above method to access advanced analytical problems such as geometrical nonlinearity, connection, materials, or using new materials.

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