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Synchronization of Stochastic Modeling with Square Fuzzy Transition Probability Relational Matrix

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Abstract*: There are number of fields like agriculture, industry, insurance, tourism and others get affected directly or indirectly by a most common phenomenon known as the Rainfall. Till now we are not able to predict rainfall and it is one of the unsolved problems. This paper aims to synchronize stochastic modeling and square fuzzy transition probability relational matrix and it is obtained from the lexical terms along with the help of fuzzy value which in turn calculated form the lexical value. Fuzzy matrices are necessary in modeling uncertain situations in various fields. Here the lexical values are specially assigned heptagonal fuzzy numbers which in turn converted into fuzzy value to measure relation mappings. Along with this, synchronization is done for stochastic models with transition probability matrix and also with square fuzzy transition probability relation matrix to get a clear picture of the result. In this work, the changes in annual rainfall of Tamil Nadu depending on markov chain models are monitored. Statistical technique like Markov chain is applied at metrological stations in order to predict short term precipitation. The annual rainfall from 1901 to 2000 is derived and the frequency distribution table is formed. The class intervals are denoted as states and the transition probability matrix is formed due to the variations in annual rainfall. The uniform random states are formed by generating random number. The available stochastic climate models presently be adapted to form new climatic conditions if the forthcoming conditions is identified with necessary accuracy. Finally, prediction of rainfall based on two categories such as climatic factors and through states is analyzed in this study.

Keywords: Lexical Value, Transition Probability Relational Matrix, Markov chain, frequency distribution, random number.

I. INTRODUCTION

Rainfall plays a major scenario in Agricultural systems. It is one of the most important factors that affect yield in agriculture. The information about rainfall patterns and variability [1] is provided for the analysis if the rainfall records for long periods. In hydrology, Rain plays a major role to find the greatest applications in the design and operations of water resources, engineering works as well as agricultural system [2]. The Four techniques of rainfall predictions are as follows: Stochastic Method, Statistical Method, Artificial Neural Network and Numerical weather prediction. Three primary methods of weather forecasting such as Synoptic, Statistical and Numerical Methods. In Forecasting, Rainmann model and Lorentz model are used. [3]. This paper describes and analyses the annual rainfall pattern at Tamil Nadu as an application of stochastic process and a model based on first order Markov chain was developed. Most of the previous models fail to consider year by year variation in the model parameters. Only within-year seasonal variations were analyzed and the assumptions are made to be constant for year by year. Hidden state Markov model [4] developed recently for accounting the long-term persistence in annual rainfall. The annual rainfall for the period 1901 to 2000 is considered and the frequency distribution table is determined. The class intervals are denoted as states and the ambiguity under different states are replaced with the construction of transition probability matrix.

The famous theory that deals with ambiguity, vagueness and uncertainty is known as the Fuzzy Theory, is introduced [22] and it has a broad range of applications in multiple fields. Fuzzy Matrices are represented as reference function and the idea of determinant of the square fuzzy matrix is given [23]. Further the same study is extended to produce the determinant and the adjoint of square fuzzy matrix in the same year. In this article, the unit fuzzy interval which is also represented as fuzzy value interval, [0, 1]. Fuzzy Matrices and their convergence criteria of the powers are discussed [7]. Fuzzy matrix theory and its extension of Boolean matrices are developed by Kim and Roush.

The uncertainty of transition probability relation matrix is measured numerically and it is expressed linguistically. If this model is stimulated by a technical expert, instead of providing approximate probability values, linguistic relations are better choice. In order to give out solution with uncertainty in transition probability and to develop the method to handle uncertainty in linguistic decisions, the square fuzzy transition probability relation matrix is used. The major scope of this work is to focus how linguistic relations and their fuzzy numbers are used to form transition probability matrix. The fuzzy numbers are taken as the entries of fuzzy transition matrix and heptagonal fuzzy represented as 7-tuples. Thus in order to connect uncertainties with transition probabilities, fuzzy methods is carried out for calculations. Numerous works has been carried out and connected to this view in both theoretical and application part [8-11].

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The transition matrix with fuzzy numbers are considered [12-14] (Lazaro R et.al; 2001, Srikanthan, R. and McMahon, T. A. 2005a, De Michele, C. and Bernardara, P. 2005, Dore, MHI. 2005, Buckley, J.J. and Eslami, E., 2008).

One if the main interesting thing is to note that all classic probability theory can be fuzzified in this way. The main objective is to manage with unsettled information in stochastic processes, when less or no data are available for the dynamic system conceived. Moreover, the data is an unsupervised one and there is uncertainty in the concepts and it is analyzed by using fuzzy tool. The analysis of rainfall in two different cases is given using different factors and by using different states. The Hepta Value is calculated along with the percentage. The content of the research work is formulated as: Preliminaries as first part, fundamentals are defined in part 2. Idea of Square Fuzzy Matrix and Synchronization are given in Section 3. Finally, the section 4 covers conclusion based on our study and future directions.

II. METHODOLOGIES

A. *Definition: 2.1 Fuzzy Matrix (Fu_M)*

The $p\times q$ order Fuzzy Matrix (Fu_M) "Fu_M (A)" is defined as $A=\left[\langle a_{ij},a_{ij\mu}\rangle\right]_{p\times q}$ where $a_{ij},a_{ij\mu}$ represents a membership measure for an element a_{ij} in A. For simplicity, we write $A=a_{ij\mu}$.

B. *Definition: 2.2 Square Fuzzy Matrix (SFu_M)*

The Square Fuzzy Matrix (SFu_M) is defined as a fuzzy matrix having its elements between [0, 1]. A square fuzzy matrix of order $p\times q$ is a fuzzy matrix (where p = q) with elements belonging to [0, 1].

C. *Definition: 2.3 Square Fuzzy Transition Probability Relation Matrix (SFu_PRM)*

A Square Fuzzy Transition Probability Relational Matrix (SFu_PRM) 'A' is a $n\times n$ square fuzzy transition matrix. The domain of row i for a given membership degree $\alpha\in[0,1]$ is the set

$$Dom_i(\alpha)=\left(\overset{n}{\underset{i=1}{\times}}\tilde{P}_{ij}[\alpha]\right)\bigcap\Delta_n=$$

$$\{(p_{i1},\dots,p_{in})\in\left[\tilde{P}_{ij_\alpha}{}^{L}\ \tilde{P}_{ij_\alpha}{}^{U}\right]\wedge\sum_j p_{ij}=1\}.$$

The domain of the whole matrix for a given α is $Dom(\alpha)=\overset{n}{\underset{i=1}{\times}}Dom_i(\alpha)$. Here the elements of $Dom(\alpha)$ are matrices of dimensions $n\times n$ and the number of rows and columns are equal [5].

D. *Definition: 2.4 Membership Function for Heptagonal Fuzzy Number*

The heptagonal fuzzy number represented by its membership function $\tilde{H}=(hp_1,hp_2,hp_3,hp_4,hp_5,hp_6,hp_7;k,\omega)$ is defined as follows

$$\mu_{\tilde{H}}(x)=\begin{cases}0 & for \quad x<hp_1\\ k\left(\dfrac{x-hp_1}{hp_2-hp_1}\right) & for \quad hp_1\le x\le hp_2\\ k & for \quad hp_2\le x\le hp_3\\ k+(\omega-k)\left(\dfrac{x-hp_3}{hp_4-hp_3}\right) & for \quad hp_3\le x\le hp_4\\ k+(\omega-k)\left(\dfrac{hp_5-x}{hp_5-hp_4}\right) & for \quad hp_4\le x\le hp_5\\ k & for \quad hp_5\le x\le hp_6\\ k\left(\dfrac{x-hp_1}{hp_2-hp_1}\right) & for \quad hp_6\le x\le hp_7\\ 0 & for \quad x\ge hp_7\end{cases}$$

The heptagonal fuzzy number defined above is said to be normal if $\omega=1$. If k=0, the heptagonal fuzzy number decreases to triangular fuzzy number and if k=1, it reduces to trapezoidal fuzzy number. The Graphical representation is as follows:

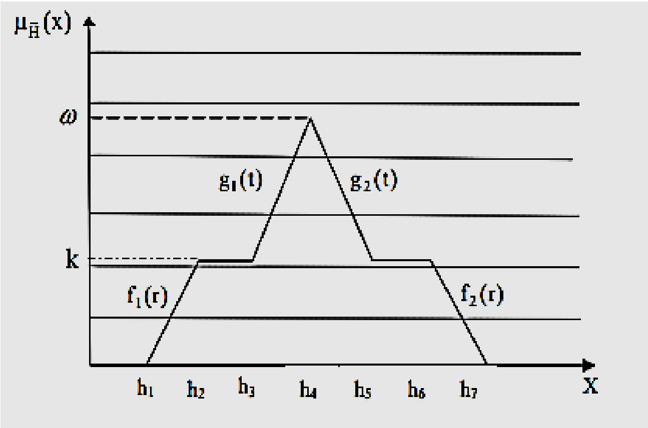

Fig 1. Graphical Representation of Heptagonal Fuzzy Numbers

E. *Definition: 2.5 Hepta Value*

The Hepta Value of a square Fuzzy Transition Probability Relational Matrix is given by

$$Hepta\ Value=\frac{1}{7}\left[\sum_{i,j=1}^{n}(R_iC_j)^{1/7}\right],$$ R_i denotes the row of the matrix and C_j denotes the column of the matrix [17-20].

III. SQUARE FUZZY MATRIX AND SYNCHRONIZATION

A. *Methodology*

Tamil Nadu approximately lies within 8° 04' N latitude and the 78° 0' E longitude. The average annual rainfall stays between 25 and 75 inches (635 and 1,905 mm) per year. The data were obtained from the Chennai Regional Meteorological Department for the year 1901 to 2000. Fuzzy value is calculated by using the heptagonal fuzzy numbers which are taken as lexical values and the lexical terms are assigned to each class intervals as states. The specific state of rainfall are denoted by the alphabets A1, A2, A3, A4, A5, A6 and A7 and the corresponding frequencies are determined from the respective class intervals. The following table indicate the states lexical terms the corresponding class interval and the frequency level along with the fuzzy value:

Table- I: Name of the Table that justify the values

Table- I: Frequency distribution of annual rainfall using Fuzzy Value

Lexical Terms	class	Lexical Value	states	Frequency	Fuzzy Value
Extreme Low (EL)	680.1-779.9	(0,0,0,0,0,0,0)	A1	6	0
Low (LW)	779.9 - 879.9	(0.1,0.15,0.2, 0.25,0.3,0.35, 0.45)	A2	17	0.26
Fairly Low (FL)	879.9 - 979.9	(0.2,0.25,0.3,0.35,0.4,0.45,0.5)	A3	23	0.42
Fair (F)	979.9 - 1079.9	(0.3,0.4,0.5,0.6, 0.7,0.8,0.9)	A4	27	0.58
Fairly High (FH)	1079.9 - 1179.9	(0.4,0.5,0.6,0.7, 0.8,0.9,1)	A5	13	0.74
High (H)	1179.9 - 1279.9	(0.6,0.7,0.8,0.9, 1,1,1)	A6	14	0.91
Very High (VH)	1279.9 - 1379.9	(1,1,1,1,1,1,1)	A7	5	1

The following figure indicates the class interval and the frequency level of the states for the annual rainfall:

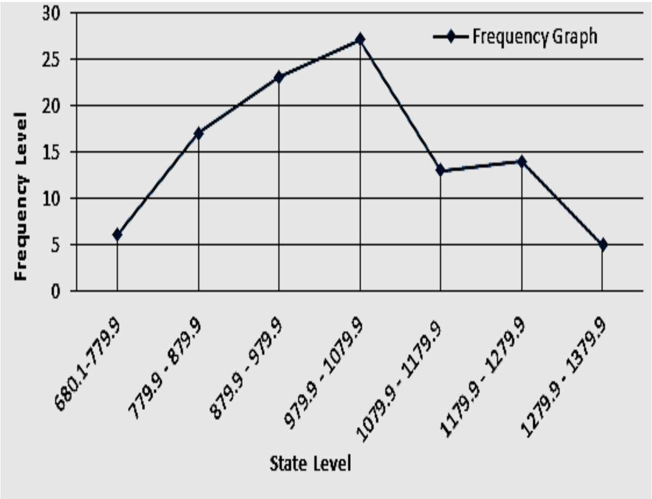

Fig 2. Frequency level for annual rainfall

B. *Stochastic Modeling*

The Markov chain is a stochastic process $\{X_n, n = 0,1,2,...\}$ which is named after a Russian Mathematician, Andrey Markov who introduced the concept of a process from which the sequence or chain with discrete states in time for the probability of transition from one state to another [15]. Later by using a two state first-order Markov chain, the occurrence of rain is analyzed [16]. Then, for generating daily rainfall depths, a multi-state Markov chain models are used [6]. Stochastic modeling is not as much easy task for exhibiting rainfall as a strong variability in time and space [10, 21]. The two advantages of Markov chain model is: (i) The forecasts are available immediately after the observations are done since the predictors use the local information on the weather, (ii) After the processing of climatological data, they need minimal computation. By identifying one variable like fog, frost, wind, cloudiness, precipitation amount and temperature at a time, is enough to forecast in future and is represented by the concept called first-order Markov Chain. The estimation of transition probability for the requirement K (K-1) in a 'K state' Markov chain and rest of tp_{ij} can be evaluated by using the relation $\sum_{j=1}^{k} tp_{ij} = 1$. The stochastic matrix TP is determined by calculating K^2 transition probabilities given as,

$$TP = \begin{bmatrix} tp_{11} & tp_{12} & \cdots & tp_{1k} \\ tp_{21} & tp_{22} & \cdots & tp_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ tp_{k1} & tp_{k2} & \cdots & tp_{kk} \end{bmatrix}$$

The determination of the probabilistic behavior of the Markov Chain is the initial state of the chain when TP is known. Let $tp_j^{(n)}$ denotes the probability that the chain is in state j at step or time n. The $1 \times K$ vector $tp^{(n)}$ has elements $tp_j^{(n)}$.

Thus $tp_j^{(n)} = \left(tp_1^{(n)}, tp_2^{(n)}, ..., tp_k^{(n)}\right)$ and $tp^{(1)} = tp^{(0)} tp$, where $tp^{(0)}$ is the initial probability vector. In general, $tp^{(n+k)} = tp^{(k)} tp^{(n)}$ where, $tp^{(n)}$ is the n[th] power of tp.

C. *Estimation of Parameters*

The transition matrix TP is formed with the parameters of tp_{ij} .The evaluation for tp_{ij} is given by $tp_{ij} = \frac{n_{ij}}{\sum_{j=1}^{k} n_{ij}}$ where

n_{ij} is number of times the observed data moves from i state to j state. Initially, the changes in rainfall region is divided into many states for the calculation of Markov chain transitional probabilities according to frequency distribution.

D. *Probability Transition Matrix*

The population probability transition matrix is obtained for the annual rainfall representing seven states as follows:

$$TP_{ij} = \begin{bmatrix} \frac{0}{6} & \frac{1}{6} & \frac{0}{6} & \frac{3}{6} & \frac{2}{6} & \frac{0}{6} & \frac{0}{6} \\ \frac{1}{17} & \frac{3}{17} & \frac{7}{17} & \frac{4}{17} & \frac{0}{17} & \frac{2}{17} & \frac{1}{17} \\ \frac{2}{23} & \frac{5}{23} & \frac{7}{23} & \frac{5}{23} & \frac{3}{23} & \frac{1}{23} & \frac{2}{23} \\ \frac{1}{27} & \frac{3}{27} & \frac{6}{27} & \frac{7}{27} & \frac{4}{27} & \frac{4}{27} & \frac{1}{27} \\ \frac{1}{13} & \frac{2}{13} & \frac{2}{13} & \frac{2}{13} & \frac{1}{13} & \frac{3}{13} & \frac{1}{13} \\ \frac{1}{14} & \frac{3}{14} & \frac{1}{14} & \frac{3}{14} & \frac{2}{14} & \frac{3}{14} & \frac{0}{14} \\ \frac{0}{5} & \frac{0}{5} & \frac{1}{5} & \frac{2}{5} & \frac{1}{5} & \frac{1}{5} & \frac{0}{5} \end{bmatrix}$$

$$TP_{ij} = \begin{bmatrix} 0..000 & 0.167 & 0.000 & 0.500 & 0.333 & 0.000 & 0.000 \\ 0.059 & 0.176 & 0.412 & 0.235 & 0.000 & 0.118 & 0.059 \\ 0.087 & 0.217 & 0.304 & 0.217 & 0.130 & 0.043 & 0.087 \\ 0.038 & 0.111 & 0.222 & 0.259 & 0.148 & 0.148 & 0.037 \\ 0.077 & 0.154 & 0.154 & 0.154 & 0.077 & 0.231 & 0.077 \\ 0.071 & 0.214 & 0.071 & 0.214 & 0.143 & 0.214 & 0.000 \\ 0.000 & 0.000 & 0.200 & 0.400 & 0.200 & 0.200 & 0.000 \end{bmatrix}$$

The basis of future rainfall state generation is provided by the matrix and the subsequent steps are carried out:

Step 1: The cumulative summation probability transition matrix is calculated within each row which leads to the following matrix:

$$TP_{Cum} = \begin{bmatrix} 0.000 & 0.167 & 0.167 & 0.667 & 1.000 & 1.000 & 1.000 \\ 0.059 & 0.235 & 0.647 & 0.882 & 0.882 & 1.000 & 1.000 \\ 0.087 & 0.304 & 0.608 & 0.825 & 0.955 & 0.998 & 1.000 \\ 0.038 & 0.148 & 0.370 & 0.629 & 0.777 & 0.925 & 1.000 \\ 0.077 & 0.231 & 0.385 & 0.539 & 0.616 & 0.847 & 1.000 \\ 0.071 & 0.285 & 0.356 & 0.570 & 0.713 & 1.000 & 1.000 \\ 0.000 & 0.000 & 0.200 & 0.600 & 0.800 & 1.000 & 1.000 \end{bmatrix}$$

Step 2: A random number is generated between the number 0 and 1 by using the method of uniform random number generator.

Step 3: The consecutive states are calculated when the random number is higher than the cumulative probability of the before state but less than or equal to the cumulative probability of the next state. This method produces any number of rainfall states. The states A1, A2, A3, A4, A5, A6, and A7 are classified in the class interval as presented in Table 1, the rainfall states are obtained according to the following rule:

$$State = \begin{cases} A1 & if & & & u & < & 0.038 \\ A2 & if & 0.038 & \leq & u & \leq & 0.148 \\ A3 & if & 0.148 & \leq & u & \leq & 0.370 \\ A4 & if & 0.370 & \leq & u & \leq & 0.629 \\ A5 & if & 0.629 & \leq & u & \leq & 0.777 \\ A6 & if & 0.777 & \leq & u & \leq & 0.925 \\ A7 & if & 0.925 & \leq & u & \leq & 1.000 \end{cases}$$

Here u is the generated uniform random number.

E. *Lexical Matrix and Fuzzy Value*

The factors which are responsible for the variations of rainfall such as maximum temperature (max temp), minimum temperature (min temp), humidity, wind speed, pressure, cloud cover and intensity. The lexical matrix [24] based on the cumulative probability transition matrix is formed which is entirely depends on the state levels. The lexical terms of each state is replaced by the corresponding fuzzy value as follows:

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$$\text{Lexical Matrix} = \begin{array}{c} * \\ A1 \\ A2 \\ A3 \\ A4 \\ A5 \\ A6 \\ A7 \end{array}\begin{bmatrix} \max temp & \min temp & humidity & windspeed & pressure & cloud\ cover & int\ ensity \\ FH & H & EL & VH & EL & LW & H \\ F & FH & LW & H & LW & FL & FH \\ FL & F & FL & FH & FL & F & F \\ LW & FL & F & F & F & FH & FL \\ EL & LW & FH & FL & FH & H & LW \\ EL & EL & H & LW & H & VH & EL \\ EL & EL & VH & EL & VH & VH & EL \end{bmatrix}$$

$$\text{Lexical Matrix}_{fuzz} = \begin{array}{c} * \\ A1 \\ A2 \\ A3 \\ A4 \\ A5 \\ A6 \\ A7 \end{array}\begin{bmatrix} \max temp & \min temp & humidity & windspeed & pressure & cloud\ cover & int\ ensity \\ 0.74 & 0.91 & 0 & 1 & 0 & 0.26 & 0.91 \\ 0.58 & 0.74 & 0.26 & 0.91 & 0.26 & 0.42 & 0.74 \\ 0.42 & 0.58 & 0.42 & 0.74 & 0.42 & 0.58 & 0.58 \\ 0.26 & 0.42 & 0.58 & 0.58 & 0.58 & 0.74 & 0.42 \\ 0 & 0.26 & 0.74 & 0.42 & 0.74 & 0.91 & 0.26 \\ 0 & 0 & 0.91 & 0.26 & 0.91 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

IV. RESULTS AND DISCUSSION

The Prediction of rainfall is done under two categories: (i) prediction based on the climatic factors and (ii) prediction through states as follows:

Category: 1

The Hepta value is calculated for each climatic factor and the percentage is calculated below:

Table- II: Calculation of Hepta Value

Factors	Max Temp	Min Temp	Humidity	Wind Speed	Pressure	Cloud Cover	Intensity
Hepta Value	0.513	0.653	0.796	0.796	0.796	0.939	0.653
%	51.3	65.3	79.6	79.6	79.6	93.9	65.3

It is observed that the prediction of rainfall due to the climatic factor "Cloud Cover" is more and it is obtained as 93.9% whereas the same is predicted as 51.3% due to the climatic factor "Maximum Temperature". The graphical representation of this prediction is given as follows:

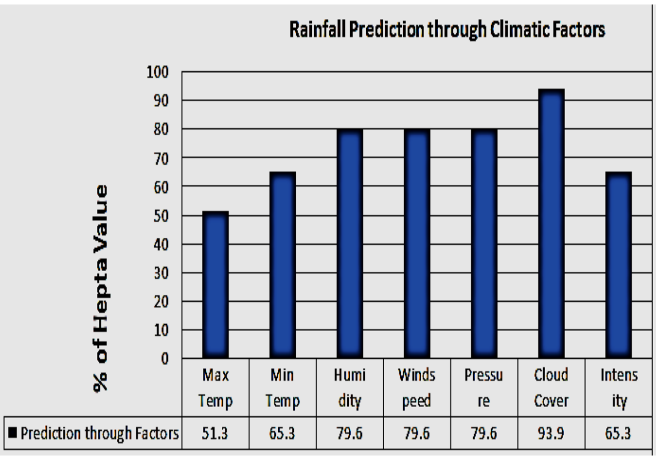

Fig 3. Prediction through Climatic Factors

Category: 2

The Hepta value is calculated for each States and the percentage is calculated below:

Table- III: Percentage of Hepta Value in rainfall prediction using States

States	A1	A2	A3	A4	A5	A6	A7
Hepta Value	0.679	0.909	0.912	0.903	0.776	0.543	0.428
%	67.9	90.9	**91.2**	90.3	77.6	54.3	**42.8**

It is observed that the prediction of rainfall due to the State "A3" which is predicted under the class interval 879.9 - 979.9 is more and it is obtained as 91.2% whereas the same is predicted as 42.8% due to the State "A7" which is predicted under the class interval 1279.9 - 1379.9.

The graphical representation of this prediction is given as follows:

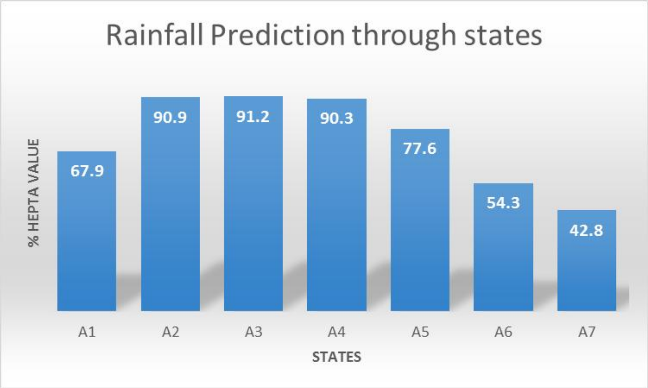

Fig 4. Prediction through States

The following table indicates the prediction of rain fall for the year 2001 to 2010. Similarly this can be predicted for another 20 years starting from 2011 to 2020.

Table- IV: Generated uniform random number and the corresponding future states for rainfall

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Generated Uniform Random Number (GURN)	0.37198	0.08041	0.718332	0.618173	0.057674	0.314373	0.766773	0.577483	0.31145	0.682455
Random States	A2	A1	A4	A4	A1	A2	A6	A4	A2	A4

V. CONCLUSION

In this study, the rainfall prediction is carried out by synchronizing transition probability matrix in stochastic modeling with square fuzzy transition probability matrix. Here the heptagonal fuzzy numbers are taken as the linguistic values in which the fuzzy values are calculated using it. The future trend is estimated by using transition probability matrix. In this study the prediction of rainfall in two categories such as climatic factor method and through states are analyzed with the help of Hepta value. The prediction of rainfall using the climatic factor is found to be high for the "Cloud Cover" which is given by 93.9% and it is less for the "Maximum Temperature" which is given by 51.3%. Also, the prediction of rainfall through the states is found to be more in the state "A3" which is given by 91.2% and it is found to be less in the state "A7" which is given by 42.8%. The prediction of long range forecasting does not show much satisfactory results. Although this model focuses only on the future trend, and it can be improved to forecast with higher accuracy. This model can be used as an alternative for modeling future variations in rainfall. They can be either in the form of too much rain or too little rain which leads to flooding or drought. Thus this Markov model is considered as one of the best available forecasting methods that can be used for the assessment of rainfall.

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