

Performance of ZF and CB Precoding Techniques for Large-Scale MU-MIMO System

Jagtar Singh, Deepak Kedia



Abstract: Large Scale Multi User-MIMO (MU-MIMO) is a key technology with reference to 5G to achieve higher spectrum as well as energy efficiency. The new technology refers to the use of a large number of antennas at the base station serving many user terminals in the same time and frequency resource allowing the channel vectors nearly orthogonal as a result, there is a reduction in inter-user interference and users may be served with the significant data rate. The linear precoding techniques play a vital role in the reduction of interference among users and cells. In this paper, we have derived, analyzed and compared two important precoding techniques i.e. Zero-forcing (ZF) and Conjugate beamforming (CB) for large-scale multiuser-MIMO. We analyze these precoding techniques with respect to spectral efficiency and downlink power with imperfect channel state information (CSI) as well as with perfect CSI. It is shown that ZF performs better as compared to CB precoding for achieving higher spectral efficiency and requires lower downlink power. CB outperforms the ZF in terms of downlink transmit power when there is a requirement to achieve low spectral efficiency and also for cell-edge users, hence energy efficient in these cases. It is shown from simulation results that ZF precoding is the better choice for attaining higher spectral and energy efficiency for a large scale multiuser-MIMO communication system.

Index Terms: MU-MIMO, Large Scale MU-MIMO, Precoding, Spectral efficiency, Energy efficiency, downlink power, Conjugate Beamforming, Zero Forcing

I. INTRODUCTION

Huge data throughput is needed in next-generation wireless communication systems as the demand for higher data rate is rapidly growing more in the near future [1-3]. In past years, the wireless data traffic has been growing fast and is expected to become 200 to 1,000 times up to 2020 [1]. In future also, the demand for higher data rates will be even more [2, 3]. To meet this future demand for higher throughput, new technologies are required. The new technology known as large-scale MU-MIMO will cater the needs without increasing bandwidth and power. As there is always a scarcity of bandwidth, the new technology should improve the spectral efficiency without increasing bandwidth. The widely known

method to enhance spectral efficiency by deploying a couple of antennas on the transmitter as well as on user terminals known as MIMO technology. MIMO technique is a conventional known method to increase the spatial multiplexing gain and reliability of a wireless communication system. To enhance gain due to spatial multiplexing there is a shift from MIMO to MU-MIMO [4]. MU-MIMO technology has earned a lot of attention from the last many years and now has been integrated into wireless broadband standards [5]. Currently, this new research field known as large-scale MU-MIMO (a.k.a Hyper MIMO, Massive MIMO) attracted a lot of research attention [6-12]. This new emerging technology will fulfill the demands of the future wireless communication system. Large Scale MU-MIMO is based on the use of more antennas as compared to conventional MU-MIMO technology, especially at the transmitter side. The use of large number of base station (BS) antennas as well as on the user terminal side will significantly enhance the energy as well as spectral efficiency [13]. Interference is the main limiting factor in a large-scale multiuser-MIMO communication system which diminishes throughput. Precoding plays an important role to diminish the interference in the multiuser-MIMO system and studied in the last few years [13-17]. In [15], the performance for uplink large MIMO using maximum ratio combining (MRC), zero-forcing (ZF), and minimum mean square error (MMSE) filters were investigated. It was presented that the transmitted energy can be scaled inversely proportional to the antennas (M) with perfect CSI and $1/\sqrt{M}$ with imperfect CSI, where M represents the antennas at the base station [15]. In [16-19], multicell processing is considered for the reduction of pilot contamination effect in large MU-MIMO. In [20], the performance of MISO broadcast channels is studied with precoding techniques as Regularized-ZF, maximal ratio transmission (MRT), and ZF in the case of a single cell. In [21], the precoding techniques are studied as ZF and MRT and downlink rates are shown in the case of multicell large MIMO systems. From the previous research works, it is observed that no deep analysis was performed on spectral efficiency for ZF and CB precoding especially for exact mathematical expression and that is challenging for a large-scale MU-MIMO communication system.

The specific contributions of the paper are:

- In this work, the performance analysis of linear precoders such as Conjugate beamforming (CB), zero-forcing (ZF) is performed. It is considered that equal power is distributed between the users.

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The user terminals are considered to be fixed. We have derived detailed and exact expressions for spectral efficiency and achievable rate for a large scale MU-MIMO system with CB and ZF precoding techniques.

- Our simulation results show that as antennas at BS increases, the spectral efficiency in the case of ZF precoding improves as compared to CB precoding scheme. It is also shown that as the downlink power decreases the spectral efficiency for ZF as well as CB precoding decreases. Simulation results also show that ZF precoding is the better choice to obtain higher spectral and energy efficiency as compared to CB precoding technique.

This paper is arranged into six sections as follows. In section II system model for a large MIMO is presented. Section III provides detailed mathematical derivations for ZF and CB precoding techniques. In section IV, signal to interference plus noise (SINR) is derived for ZF and CB precoding. We also analyze expressions for achievable data rate and spectral efficiency. Section V, discusses the simulation results. Section VI provides the conclusion of the paper.

II. SYSTEM MODEL

A downlink of the large-scale multiuser-MIMO communication system with a base station having M transmit antennas with K number of single-antenna user terminals is considered here. Figure 1 shows the system model. The base station is serving simultaneously K number of user terminals. Here it is assumed that K user terminals share the same time and frequency resources and the base station is considered to have perfect CSI. The channel knowledge is acquired at the BS during the training duration. The particular training schemes depend on time-division duplex (TDD) and frequency-division duplex (FDD) protocols. When employing FDD at the base station, it becomes challenging to obtain CSI since the amount of downlink resources required for training pilots becomes proportional to antennas at the base station.

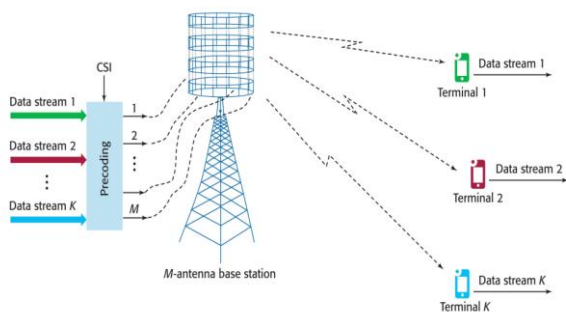


Figure 1. A downlink large scale MU-MIMO system model

On the other hand, when employing TDD operation, the base station can acquire CSI from uplink training phase due to the use of channel reciprocity. Therefore, the overhead due to pilots becomes proportional to the users and TDD operation is preferred in most of the cases in large-scale MU-MIMO system [8, 13, 16].

Consider H be the channel matrix among base station antennas and users terminals is represented by

$$H = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1M} \\ h_{21} & h_{22} & \dots & h_{2M} \\ \dots & \dots & \dots & \dots \\ h_{K1} & h_{K2} & \dots & h_{KM} \end{bmatrix} \quad (1)$$

Assume that $H \in K \times M$, here k^{th} column of channel matrix H is represented by h_k , denotes the channel vector $M \times 1$ among k^{th} user terminal and base station. Generally, the propagation channel is designed by assuming large-scale and small-scale fading. Here, we have considered small-scale fading, assuming that components of matrix H are independently and identically distributed (i.i.d.) Gaussian distribution with unit variance and zero mean.

The K users will receive their corresponding message signal from the vector $K \times 1$, so the signal received y is

$$y = Hx + n \quad (2)$$

Here x represents precoded signal for input vector v and n represents the additive white Gaussian noise (AWGN) having i.i.d. elements $n_k \in CN(0; N_0)$.

The energy of the i^{th} component in v is

$$E\{v_i^* v_i\} = 1, \quad i=1, 2, \dots, K \quad (3)$$

The base station utilizes CSI for precoding of the symbols. Consider W is the complex value $M \times K$ precoding matrix having $\|W\|=1$ and P represents the transmitted power from the base station. Signal transmitted from BS is given by

$$x = \sqrt{P}Wv \quad (4)$$

The transmitted signal on the i^{th} antenna is written as

$$x_i = \sqrt{P} \sum_{j=1}^K w_{ij} v_j \quad (5)$$

The k^{th} user terminal received signal can be

$$y_k = \sqrt{P} \sum_{m=1}^M h_{k,m} x_m + n_k$$

$$= \sqrt{P} \sum_{j=1}^K \sum_{m=1}^M h_{k,m} w_{m,j} v_j + n_k \quad (6)$$

By looking for the desired signal part when $j=k$, the Eq. (6) can be written by

$$y_k = \sqrt{P} \sum_{m=1}^M h_{k,m} w_{m,k} v_k + \sqrt{P} \sum_{j \neq k} \sum_{m=1}^M h_{k,m} w_{m,j} v_j + n_k \quad (7)$$

Here the first part of above equation is a useful signal part. For k^{th} user terminal, the signal to interference plus noise (SINR) is written from Eq. (7) as

$$SINR = \frac{E \left(\sqrt{p} \sum_{m=1}^M h_{k,m} w_{m,k} v_k \right)}{E \left(\sqrt{p} \sum_{j \neq k} \sum_{m=1}^M h_{k,m} w_{m,j} v_j + n_k \right)} \quad (8)$$

From Eq. (7) it is observed that first part is the desired signal, the second part is the interference known as multi-user interference and third part represents additive white Gaussian noise (AWGN) for each user having mean zero, variance σ^2 i.e CN(0, σ^2) for all users. Here W is precoding matrix and v is transmitted signal vector for K users.

III. PRECODING SCHEMES

Precoding techniques are designed to separate data streams and reduce the inter-user interference in a MU-MIMO system. Base station pre-filters the signal for desired users in downlink and base station post-filters received signal from all users. The CSI is needed at the base station to perform these processings. In downlink [22], the optimum precoding is Dirty paper coding (DPC) in the sense that it attains the sum-rate capacity. Successive interference cancellation detection achieves capacity in the uplink [23]. These schemes are the non-linear schemes that are more complex in practice. Linear schemes are low complexity and have suboptimal performance in conventional MU-MIMO system [24-25]. As antennas at transmitter increases, the linear schemes become near-optimal. So we carry out the detailed mathematical derivations of the linear precoding schemes in case of large MIMO system and performance evaluation is carried out with respect to spectral efficiency and required downlink transmit power.

A. Zero forcing Precoding

ZF is a linear transmission approach permitting a multiuser transmission without producing multiuser interference. This type of beamforming is feasible when the antennas M at base station satisfy $M \geq K$ and user terminals having single antenna. The zero forcing precoding is a prominent linear precoding technique. By using ZF precoding the MUI can be made to zero at each user. Consider the system of equations as

$$y_{K \times 1} = H_{K \times M} x_{M \times 1} \quad (9)$$

$$e = y - Hx \quad (10)$$

A natural estimate of x would be that will minimizes $\|e\|^2$
 $\min \|e\|^2 = \min \|y - Hx\|^2 \quad (11)$

The estimate of x which minimizes $\|y - Hx\|^2$ is named as least square solution.

$$\hat{x} = \arg \min_x \|y - Hx\|^2 \quad (12)$$

To find minimum value differentiate with respect to x and put equal to zero. Least square cost function can be written in a simplified manner as

$$\|y - Hx\|^2 = (y - Hx)^T (y - Hx) \quad (13)$$

$$= (y^T - x^T H^T)(y - Hx)$$

$$= y^T y - y^T Hx - x^T H^T y + x^T H^T Hx$$

$$= y^T y - 2x^T H^T y + x^T H^T Hx \quad (14)$$

The derivative of the least square cost functions is given as

$$\frac{d}{dx} \|y - Hx\|^2 = \frac{d}{dx} (y^T y - 2x^T H^T y + x^T H^T Hx) \quad (15)$$

$$= -2H^T y + 2H^T Hx \quad (16)$$

To obtain minimum value, the derivative of the least squares function is set to be zero. So setting Eq. (16) to zero we get

$$\frac{d}{dx} \|y - Hx\|^2 = -2H^T y + 2H^T Hx = 0$$

Therefore the estimate of x is given by

$$\hat{x} = (H^T H)^{-1} H^T y \quad (17)$$

The zero forcing scheme for a complex channel matrix H is

$$\hat{x} = (H^H H)^{-1} H^H y \quad (18)$$

The quantity $(HH^H)^{-1}H^H$ is known as pseudo inverse for complex channel matrix H. This quantity is known as zero forcing precoding matrix.

$$W_{ZF} = (H^H H)^{-1} H^H \quad (19)$$

With the increase of BS antennas, the calculation of inverse of matrix $(HH^H)^{-1}$ becomes challenging and sometimes the matrix $(HH^H)^{-1}$ does not yield a stable inverse. In ZF precoding scheme the matrix HH^H is invertible if the condition $M \geq K$ is satisfied.

B. Conjugate Beamforming Precoding

The Minimum Mean Square Error (MMSE) algorithm is intended for minimizing error among the transmitted and received symbol caused by interference as well as from noise distortions. Consider the linear estimate of x is $\hat{x} = c^T y$ which minimizes the MSE. Consider y, x be the zero mean. The Mean Square Error (MSE) is

$$MSE = E \left\{ (x - \hat{x})^2 \right\} = E \left\{ (c^T y - x)^2 \right\} \quad (20)$$

The Eq. (20) can be

$$MSE = E \left\{ (c^T y - x)(c^T y - x)^T \right\} \quad (21)$$

$$= E \left\{ (c^T y - x)(y^T c - x) \right\}$$

$$= E \left\{ c^T y y^T c - 2c^T y x + x^2 \right\}$$

$$= c^T E \left\{ y y^T \right\} c - 2c^T E \left\{ y x \right\} + E \left\{ x^2 \right\}$$

$$= c^T R_{yy} c - 2c^T r_{yx} + p_d \quad (22)$$

To determine the optimum combiner c which minimizes the MSE, differentiate with respect to c and put equal to zero.

$$\frac{d}{dc} MSE = \frac{d}{dc} (c^T R_{yy} c - 2c^T r_{yx} + p_d) \\ = 2R_{yy} c - 2r_{yx}$$

Therefore the optimal combiner c is

$$\frac{d}{dc} MSE = 0$$

$$\Rightarrow 2R_{yy} c - 2r_{yx} = 0$$

$$c = R_{yy}^{-1} r_{yx}$$

$$(23)$$

Finally, the MMSE estimate is

$$x = c^T y = \left(R_{yy}^{-1} r_{xy} \right)^T y = r_{yx}^T R_{yy}^{-1} y \quad (24)$$

$$x = r_{xy} R_{yy}^{-T} y \quad (25)$$

Where R_{yy} is covariance of y and R_{xy} is the cross-covariance of x, y . Now, since the symbols are i.i.d having power P_d with

noise variance is σ^2 , $E\{xx^H\} = P_d I_M$ and $E\{nn^H\} = \sigma^2 I_K$ hence

$$\begin{aligned} R_{yy} &= E\{yy^H\} \\ &= E\{(Hx+n)(Hx+n)^H\} \\ &= E\{(Hx+n)(x^H H^H + n^H)\} \\ &= E\{(Hxx^H H^H + nx^H H^H n^H + nn^H)\} \\ &= HE\{xx^H\}H^H + E\{nx^H\}H^H + HE\{nn^H\} \end{aligned} \quad (26)$$

Assuming independent channel symbols and noise, we have

$$E\{nx^H\} = 0 \quad \text{and} \quad E\{xn^H\} = 0$$

$$R_{yy} = HE\{xx^H\}H^H + E\{nn^H\} \quad (27)$$

$$= P_d HH^H + \sigma^2 I \quad (28)$$

Also,

$$R_{xy} = E\{xy^H\} = E\{x(Hx+n)^H\}$$

$$R_{xy} = E\{x(x^H H^H + n^H)\}$$

$$R_{xy} = E\{xx^H H^H + xn^H\} = P_d H^H$$

Hence the least MMSE estimate is

$$x = R_{xy} R_{yy}^{-1} y = P_d H^H \left(P_d HH^H + \sigma^2 I \right)^{-1} y \quad (29)$$

Consider now the following:

$$\begin{aligned} P_d H^H HH^H + \sigma^2 H^H &= P_d H^H HH^H + \sigma^2 H^H \\ \Rightarrow H^H \left(P_d HH^H + \sigma^2 I \right) &= \left(P_d HH^H + \sigma^2 I \right) H^H \\ \Rightarrow \left(P_d HH^H + \sigma^2 I \right)^{-1} H^H &= H^H \left(P_d HH^H + \sigma^2 I \right)^{-1} \\ \Rightarrow P_d \left(P_d HH^H + \sigma^2 I \right)^{-1} H^H &= P_d H^H \left(P_d HH^H + \sigma^2 I \right)^{-1} \end{aligned}$$

Hence the Least MMSE estimate is

$$\begin{aligned} x &= R_{xy} R_{yy}^{-1} y \\ &= P_d H^H \left(P_d HH^H + \sigma^2 I \right)^{-1} y \\ &= P_d \left(P_d HH^H + \sigma^2 I \right)^{-1} H^H y \\ &= \left(H^H H + \frac{\sigma^2}{P_d} I \right)^{-1} H^H y \end{aligned}$$

$$\begin{aligned} x &= \left(H^H H + \frac{\sigma^2}{P_d} I \right)^{-1} H^H y \\ x &= \left(H^H H + \frac{1}{SNR} I \right)^{-1} H^H y \end{aligned} \quad (30)$$

For the low values of SNR in Eq. (30), the second term dominates and the first term can be neglected. Therefore the Eq. (30) is written as

$$x = H^H * SNR * y \quad (31)$$

The quantity H^H is known as conjugate beamforming precoding matrix. So conjugate beamforming precoding is

$$W_{CB} = H^H \quad (32)$$

Eq. (32) is known as conjugate beamforming precoding matrix. CB is a linear precoding technique that maximizes the signal gain at the desired users. This equation is also called matched filtering in connection with the uplink transmission. The advantage of the beamforming precoding is that it does not require the inverse of matrix, as a result, complexity reduces in case of CB precoding as compared to ZF precoding. With the increase in antennas, the size of channel matrix H increases. As a result, the inverse calculation of matrix HH^H becomes challenging in the case of ZF precoding. So with the increase in antennas, the conjugate beamforming precoding becomes easier to implement with low complexity.

IV. SPECTRAL EFFICIENCY

A. Spectral Efficiency for CB precoding Technique

Consider h_k be the channel vector between antennas and k^{th} user. The components of the matrix H are complex Gaussian variables i.i.d. having mean zero and unit variance $CN(0, 1)$. The received vector is given by

$$y = \sqrt{P_d} Hx + n \quad (33)$$

In matrix form

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_K \end{bmatrix} = \sqrt{P_d} \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1M} \\ h_{21} & h_{22} & \dots & h_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ h_{K1} & h_{K2} & \dots & h_{KM} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_K \end{bmatrix} \quad (34)$$

The input precoded signal is written as

$$x = \sqrt{P} Wv \quad (35)$$

The Eq. (34) in compact form is written as

$$\bar{y} = \sqrt{P_d} \left[\bar{h}_1 \quad \bar{h}_2 \quad \dots \quad \bar{h}_M \right] \bar{x} + \bar{n} \quad (36)$$

Now consider the application of the conjugate beamforming

$$\bar{r} = H^H \bar{y} \quad (37)$$

Substituting Eq. (36) in Eq. (37) and writing in matrix form

$$\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_M \end{bmatrix} = \sqrt{P_d} \begin{bmatrix} \bar{h}_1^{-H} \\ \bar{h}_2^{-H} \\ \vdots \\ \bar{h}_M^{-H} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix} + \begin{bmatrix} \bar{h}_1^{-H} \\ \bar{h}_2^{-H} \\ \vdots \\ \bar{h}_M^{-H} \end{bmatrix} n$$

Where r_1, r_2, \dots, r_M are corresponding received signals from the user terminals. Consider r_1 corresponding to user 1.

$$r_1 = \bar{h}_1^{-H} \begin{bmatrix} \bar{h}_1 & \bar{h}_2 & \dots & \bar{h}_M \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix} + \bar{h}_1^{-H} n \quad (38)$$

Eq. (38) can be written as

$$r_1 = \sqrt{P_d} \|\bar{h}_1\|^2 x_1 + \sqrt{P_d} \bar{h}_1^{-H} \bar{h}_2 x_2 + \dots + \sqrt{P_d} \bar{h}_1^{-H} \bar{h}_M x_M + \bar{h}_1^{-H} n \quad (39)$$

The first term of Eq. (39) is the intended signal for the user 1 and power for the desired signal can be written as

$$E \left\{ \left(\sqrt{P_d} \|\bar{h}_1\|^2 x_1 \right)^2 \right\} = P_d \|\bar{h}_1\|^4 \quad (40)$$

The second term of Eq. (39) is the interference and known as multiuser interference (MUI). MUI power can be

$$E \left\{ \left(\sqrt{P_d} \bar{h}_1^{-H} \bar{h}_2 x_2 + \dots + \sqrt{P_d} \bar{h}_1^{-H} \bar{h}_M x_M \right)^2 \right\} = P_d \sum_{i=2}^M \|\bar{h}_i\|^2 \quad (41)$$

The third term of Eq. (39) is the noise; therefore the noise power is given by

$$E \left\{ \left(\bar{h}_1^{-H} n \right)^2 \right\} = \sigma^2 \|\bar{h}_1\|^2 \quad (42)$$

For the kth user, signal to interference plus noise (SINR) ratio from Eqs. (40), (41) and (42) is

$$\text{SINR} = \frac{\text{Desired Signal Power}}{\text{MUI Power} + \text{Noise Power}}$$

$$\text{SINR}_k = \frac{P_d \|\bar{h}_1\|^4}{P_d \sum_{i=2}^M \|\bar{h}_i\|^2 + \sigma^2 \|\bar{h}_1\|^2} \quad (43)$$

Achievable data rate for a particular user in case of downlink large MU-MIMO system is

$$R_k = B \log_2 (1 + \text{SINR}_k) \text{ bits/s} \quad (44)$$

The achievable rate for conjugate beamforming precoding scheme is given by

$$R_k = B \log_2 \left(1 + \frac{P_d \|\bar{h}_1\|^4}{P_d \sum_{i=2}^M \|\bar{h}_i\|^2 + \sigma^2 \|\bar{h}_1\|^2} \right) \quad (45)$$

The spectral efficiency can be defined as achievable data rate divided by channel bandwidth, so

$$\text{Spectral efficiency } \eta = \frac{R_k}{B} = \log_2 (1 + \text{SINR}) \text{ bits/s/Hz} \quad (46)$$

So the spectral efficiency (η) for the Conjugate Beamforming can be written using Eq. (42) as

$$\eta_{CB} = \log_2 \left(1 + \frac{P_d \|\bar{h}_1\|^4}{P_d \sum_{i=2}^M \|\bar{h}_i\|^2 + \sigma^2 \|\bar{h}_1\|^2} \right) \text{ bits/s/Hz} \quad (47)$$

B. Spectral Efficiency for ZF precoding Technique

The equation for system model is

$$y = \sqrt{P_d} Hx + n$$

The ZF precoding matrix is as derived in equation (19) is

$$W_{ZF} = (H^H H)^{-1} H^H$$

Here W denotes precoding matrix and satisfies the condition $W.H=I$ (48)

In this case, W is known as the left inverse of channel matrix H .

Then by applying the zero-forcing precoding technique

$$r = W.y = W \sqrt{P_d} (Hx + n) = \sqrt{P_d} (WHx + Wn)$$

$$r = \sqrt{P_d} (x + Wn)$$

We are able to suppress MUI with ZF precoding. Noise co-variance can be written as

$$E \{ nn^H \} = \sigma^2 I \quad (49)$$

Let $Wn = n$, Now the co-variance of n is

$$\begin{aligned} E \{ nn^H \} &= E \{ (Wn)(Wn)^H \} \\ &= E \{ (Wn)(n^H W^H) \} \\ &= \sigma^2 WW^H \end{aligned} \quad (50)$$

Put the value of W_{ZF} from Eq. (19) we get

$$E \{ nn^H \} = \sigma^2 (H^H H)^{-1} \quad (51)$$

The value of SINR for ZF precoding using equation (48) can be

$$\text{SINR} = \frac{P_d}{\sigma^2 (H^H H)^{-1}} \quad (52)$$

Now, achievable data rate with the use of ZF precoding is given by

$$R_k = B \log_2 \left(1 + \frac{P_d}{\sigma^2 (H^H H)^{-1}} \right) \quad (53)$$

Spectral efficiency using ZF precoding can be written as

$$\eta_{CB} = l \log_2 \left(1 + \frac{P_d}{\sigma^2 (H^H H)^{-1}} \right) \text{ bits/s/Hz} \quad (54)$$

V. SIMULATION RESULTS

This section discusses the performance of a large scale MIMO system with precoding techniques such as Zero forcing (ZF), Conjugate beamforming (CB) for a downlink single-cell scenario. The results are analyzed and compared in reference to spectral efficiency, downlink transmit power versus antennas at base stations with perfect as well as imperfect CSI, considering different antenna arrangements and different precoding techniques.

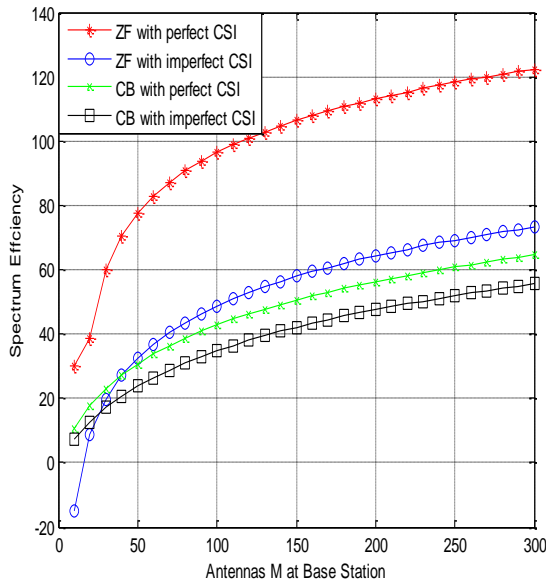


Figure 2. Spectral efficiency vs antennas M at base station for ZF, CB precoding at transmitter considering perfect and imperfect CSI. In this case users terminals K=15 simultaneously served and downlink power is P_d=15 dB. Figure 2 depicts spectral efficiency (SE) for downlink large MU-MIMO versus number of antennas M. The ZF and CB precoding techniques are compared and analyzed considering perfect as well as imperfect CSI. Here we choose the value of downlink transmitted power of 15 dB and considering the number of users to be served are 15. Assuming that each user having single antenna. With the increase in BS antennas, the spectral efficiency tends to a higher value when base stations having perfect CSI, but when having imperfect CSI spectral efficiency approaches to the lower value as compared to the perfect CSI. With the increase in antennas (M) from 50 to 100, the spectral efficiency increases approximately 15 bits/s/Hz. Performance of large MU-MIMO is better with ZF precoding as compared to CB precoding. As antennas at BS increases the spectral efficiency will approach to ZF in the case of CB precoding technique. The advantage of using CB

precoding is the reduced complexity, hence easier implementation.

In Figure 3, assuming a similar simulation setting as in the case of Figure. 2 but considering the downlink transmit power P_d=10 dB. It is depicted from Figure 3, the spectral efficiency decreases in the case of both precoding techniques.

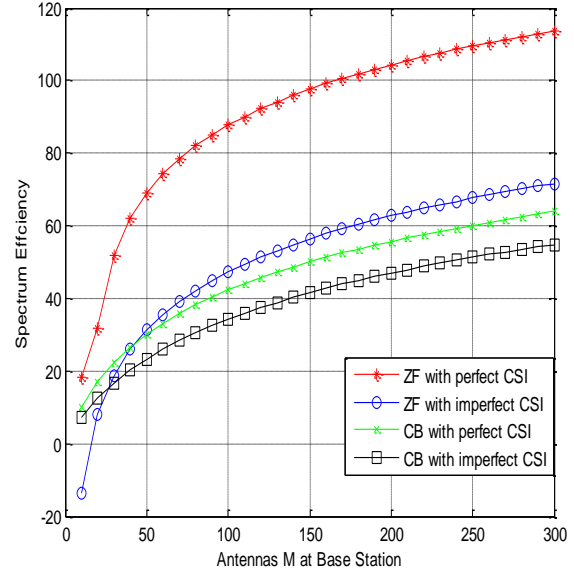


Figure 3. Spectral efficiency vs antennas M at base station for ZF, CB precoding at transmitter considering perfect and imperfect CSI.

In this case, users terminals K=15 simultaneously served and downlink power is P_d=10 dB.

Next, we show that downlink transmit power required for achieving fixed spectrum efficiency of value 15 bits/s/Hz. Figure 4 depicts the required downlink power for achieving 1 bit/s/Hz per terminal versus M antennas at the transmitter. As the antennas M at transmitter increases there is a decrease in the downlink power. As depicted from the Figure 4 that both the precoding schemes require the same downlink power for achieving 1 bit/s/Hz. Higher downlink transmit power is required in case of imperfect CSI.

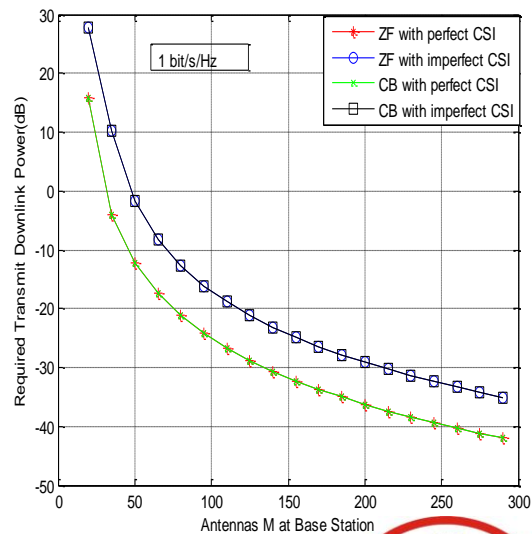


Figure 4. Required downlink power vs antennas M at base



station for achieving 1 bit/s/Hz/user for CB, ZF precoding with perfect as well as with imperfect CSI.

The user terminals are fixed to $K = 15$, and targeted SE of 15 bits/s/Hz.

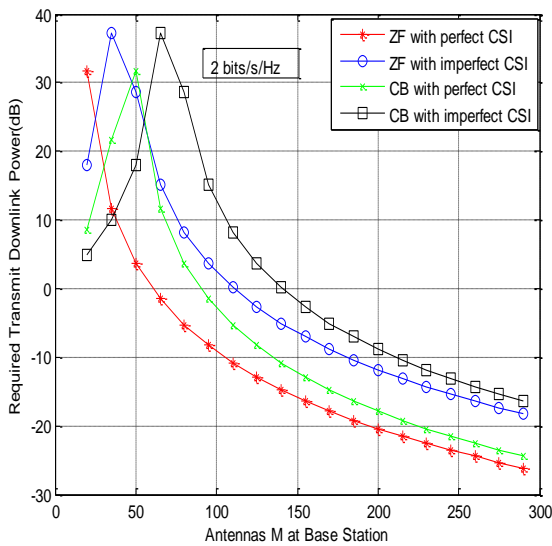


Figure 5. Required downlink power vs antennas M at base station for achieving 2 bit/s/Hz/user for CB, ZF precoding with perfect as well as with imperfect CSI. The user terminals are fixed to $K = 15$, and targeted SE of 30 bits/s/Hz. Figure 5 depicts the downlink power required for achieving 2 bit/s/Hz. It is seen from the Figure 4 and Figure 5, as spectral efficiency increases from the value 15 bits/s/Hz to 30 bits/s/Hz i.e. to double the required SE, the required downlink power approximately increases to double. Figure 5 shows that the ZF precoding performs better as compared to CB precoding in terms of downlink transmit power. Therefore, to get higher SE the ZF precoding is the better choice and energy efficiency is also better as compared to CB precoding.

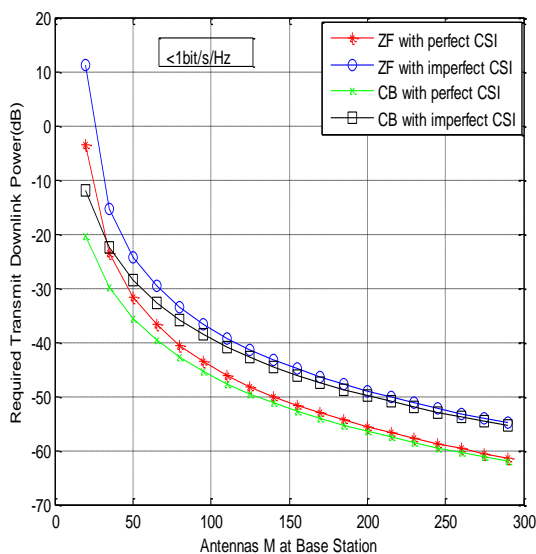


Figure 6. Required downlink power vs antennas M at base station for achieving less than 1 bit/s/Hz/user for CB, ZF precoding with perfect as well as with imperfect CSI. The user terminals are fixed to $K = 15$, and targeted SE of 5 bits/s/Hz.

Figure 6 shows the downlink transmit power versus antennas M at base stations to achieve less than 1 bit/s/Hz per user and targeted spectral efficiency is 5 bits/s/Hz for perfect as well as imperfect CSI. It is seen that the CB technique perform better as compared to ZF precoding in terms of downlink transmit power. So CB precoding is energy efficient to achieve lower spectral efficiency.

VI. CONCLUSION

The new large scale MU-MIMO technique provides the possibility to increase spectral efficiency and reduction in required downlink power when antennas M at transmitter increases when compared with a conventional MU-MIMO system such as LTE, LTE-A. In this paper, the spectral efficiency and required downlink power at the base station for large-scale MU-MIMO are investigated for perfect and imperfect CSI. Detailed derivations and analysis for ZF, CB precoding schemes are provided in a simplified manner. We analyzed two linear precoding techniques, zero-forcing and conjugate beamforming for the large MU-MIMO system. Simulations depict that the ZF precoding technique outperforms the CB precoding technique in terms of downlink transmit power when there is a requirement to attain higher spectral efficiency. Therefore, the use of ZF precoding in the large-scale MU-MIMO system provides high energy efficiency as compared to CB precoding. CB precoding is energy efficient when required to attain low spectral efficiency as compared to ZF precoding. We conclude that ZF precoding performs well to achieve the high spectral and energy efficiency in a large MU-MIMO system that is required for future generation wireless communication system.

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