International Journal of Recent Technology and Engineering (IJRTE)
ISSN: 2277-3878 (Online), Volume-8 Issue-2, July 2019

Numerical Computing of Frequencies for Rectangular and Square Plates with Transcendental Thickness Variation

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Abstract: ***The frequencies of the thin plate play an important role for the design of the engineering machines, naval structure, etc. Tremendous amount of the literature on the study of vibrations of rectangular and square plate are available with various kinds of the thickness variations. From the study it is observed that transcendental thickness variation has not been studied by the researchers which satisfy the boundary conditions. Therefore, the present work is the attempt in this direction. A well-known Rayleigh-Ritz method has been used for computation of the first three frequencies alongwith mode shapes and by varying the boundary conditions at the side of the plate. The convergence of the computed results has been presented through table and comparison has also been made with the existing result.***

Index Terms**:** ***Rayleigh-Ritz Method, Frequencies, Mode Shape, Convergence and Comparison, Plates.***

I. INTRODUCTION

The study of vibration of plates has significant role in the engineering design. Exhaustive literature is available on the computation of first few frequencies alongwith representation of mode shapes for rectangular and square plates with various combinations of the boundary conditions applied at the side of the plate. The thickness of thin plate is varying, uniformly, linearly, quadratically, etc but it is revealed from the exhaustive literature that transcendental thickness variation is not studied by the researchers and the scientists which satisfy the various boundary conditions applied on the boundary of the plate.

Let us briefly explain related literature available on the study of rectangular and square plates. The monographs of Leissa [1-10] are an excellent source of information on the study of the various kinds of the plates with varying different types of thickness variation. Extensive numerical results are available in these monographs and very helpful for the researchers and scientists for comparing the results. One of the researchers Laura did tremendous research in the field of vibration of plates. Laura and Grossi [11] have obtained transverse vibration of rectangular plates with edges of the plates are elastically restrained against translation and rotation of the plate. There is lots of work on the thin rectangular plate with different boundary conditions. Shuyu [12] has studied the flexural vibration of thin rectangular plates when all the sides of the plate are completely free and performed the experiments to compute the resonance frequencies with excellent agreement between the measured results and the existing one in the literature. Rayleigh-Ritz method, finite element method, superposition method, wavelet method and many more have been applied by scientists and researchers for computation of first frequencies for various shapes of thin plate. Superposition technique has been applied by Gormen [13] for free vibration analysis of rectangular plate, with and without elastic super normal to the boundaries convergence with rapidly convergence of the results alongwith the comparisons of the results. In another paper [14], author computed the eigen-values for the elastically supported plates with various aspect ratios and dimension less elastic support coefficients. Wu et al. [15] used the method based on Bessel's function for obtaining the exact value for the free vibration of rectangular thin plates with three boundary conditions like fully simply-supported, fully clamped, two opposite sides are simply supported and other two sides are clamped. The method provides simple, direct, and highly accurate solutions for these kinds of plates. In the year 2009, Xing and Liu [16] have applied two-eigen function theory by the use of amplitude deflection and the generalized curvature for obtaining the results for free vibration solutions of rectangular Mindlin plate. Dozio [17] applied the Ritz method by the use of trigonometric function and obtained accurate in plate modal properties of the rectangular plate with arbitrary non-uniform elastic edge restraints. Dozio observed that there is a support of free in-plate vibration of plates having triangularly and parabolically varying elastic edge supports. By varying the stiffness of plate, natural frequencies and modal shapes are also computed.

In the year 2011, Liu and Xing [18] were successfully obtained exact solutions with 10 sets of distinct eigen solutions for the free in-plane vibration of isotopic rectangular plates with simply-support boundary conditions at two opposite edges with computation of classical boundary conditions at the two other edges.

Revised Manuscript Received on 30 July 2019.
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The exact solutions are validated and compared with differential quadrature method. In the year 2013, Zhang [19] has studied model and analysis of FGM rectangular plates based on the physical natural surface and high order shear deformation theory in the rectangular plate. Material properties are used with Poisson's ratio depending on the change of temperature and position of the plate and considered as constant. The presented approach has many merits engineering application. Hamiltonian system based benchmark bending solutions for the rectangular thin plates with supported corner point have studied by Li et al. [20] and obtained the comprehensive numerical results with future validation of various, approximate numerical method.

Further, Zhang et al. [21] have obtained orthotropic rectangular plate by the use of two dimensional improved Fourier series method, extensive numerical computations are represented through the reliability and effectiveness of obtained solutions. Banerjee et al. [22] have also studied rectangular plate and solved the bi-harmonic equation to obtain the solution in terms of force displacement relationship of the freely vibrating plate. Wittrick-Williams algorithm is used for ensuing dynamic stiffness matrix to provide the solutions of the problem. Dynamic stiffness method is also used by Danilovic and Petrojjevic [23] for getting the eigenvalues of rectangular plate by converting it as isotropic. The results have been compared with the superposition method used by the Gorman's. The obtained results are high precision accuracy and low memory requirement in comparison of finite element method. In the year 2015, Li et al. [24] reported a benchmark bending solutions of rectangular thin plates with the point-supported at the corner and efficient results were obtained by the author.

Wang and Yuan [25] have applied Discrete Singular Convolution (DSC) and Taylor series expansion method for free vibration analysis of beam and rectangular plates with free boundary conditions and it is observed that the Taylor series expansion method gives accurate results in general and performance proposed by discrete singular decomposition results are excellent and independent of boundary conditions. Recently in the year 2017, Zhou et al. [26] also obtained exact solutions for the free in-plane vibration of rectangular plates with arbitrary boundary conditions, exact natural frequencies and mode shapes can be easily obtained by authors. When two opposite edges of plate have either type of simply-supported boundary conditions and resting plate edges have the arbitrary boundary conditions. Xing et al. [27] have also studied overall assessment of closed form solution methods for vibrations of the rectangular plates. The presented results are giving high accuracy and reliable for different type of rectangular plate.

On the basis of above literature, it is observed that the thickness variation in the closed form of transcendental function has not been studied by the researchers for the various shapes of the plates for which the boundary condition must be satisfied. Hence, the present work is attempt in this direction to perform the exhaustive numerical computations for the rectangular thin plate with transcendental thickness variation. The obtained results have been compared for the different combinations of the boundary conditions with the existing one.

II. BACKGROOUND

Let us briefly explain some of the fundamentals used for computation of the frequencies and mode shapes for the rectangular and square plate with thickness variation.

A. Thin Plate

A thin plate may be defined from the theory of beam which was developed by Kirchoff in the year 1888 by dividing the mid-surface plan which is used to represent three dimensional plate in two dimensional form with following assumptions:

1) Straight line normal to mid surface remain straight after deformation;

2) Straight line normal to mid surface remain normal to mid surface after deformation.

3) The thickness of the plate never changes during deformation.

B. Boundary Condition

The boundary condition is divided into the two classes namely essential or geometric boundary condition and natural or dynamic (or force) boundary condition. Essential (or geometric) boundary conditions are also known as Dirichlet boundary and natural (or dynamic) boundary conditions are also known as Neumann boundary conditions.

Any problem related to the plate may basically use three possible combinations of boundary conditions, one is all essential type, second is all natural type and third is mixed type (some natural and essential type).

In the present work, the three kinds of the boundary conditions have been used one is clamped, second is simply-supported and third is completely-free which are defined below and representation of the boundary condition is depicted in figure [1] on the domain of plate as represented as R.

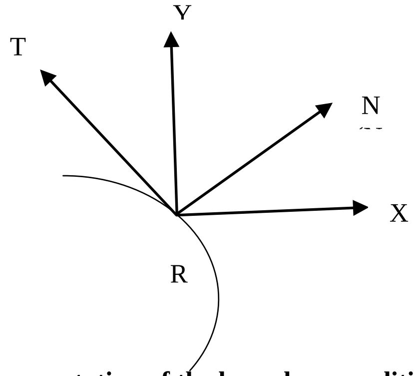

Fig. 1 Representation of the boundary condition

a. Clamped Boundary Condition

In clamped boundary conditions, displacement and slope both are must be zero. In the Mathematical term, it is represented as:

$$\omega = 0 \tag{1}$$

and

$$\frac{\partial \omega}{\partial \xi} = 0 \tag{2}$$

where ω represents the displacement and clamped boundary conditions are the type of essential boundary conditions.

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DOI: 10.35940/ijrte.B3197.078219
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b. Simply-Supported Boundary Condition

In this case, displacement and bending moment must be zero and these boundary conditions written in mathematical term are given below:

$$\omega = 0 \qquad (3)$$

$$M_{\xi} = 0 \qquad (4)$$

Where M shows the bending moment and these boundary conditions are essential and natural both.

c. Completely-Free Boundary Condition

In this case, it is called as free boundary conditions in which, the bending moment and shear force must be zero. In Mathematical form, these may be represented as:

$$M_{\xi} = 0 \qquad (5)$$

$$Q_{\xi} + \frac{\partial M_{\xi\eta}}{\partial \eta} = 0 \qquad (6)$$

This type of boundary condition is of the category of natural boundary condition.

III. NUMARICAL FORMULATION

Rayleigh's-Ritz method is the extension of Rayleigh's method and this method is used for all continuous systems. Let us consider a plate which is defined in the domain R and bounded by x=0, x=a, y=0 and y=b as shown below in figure [2] with h is thickness of the plate. Displacement ω at point (x,y) at time t is given below:

$$w(x,y) = W(x,y)\sin\omega t \qquad (7)$$

Where w(x,y) is the maximum displacement at (x,y).

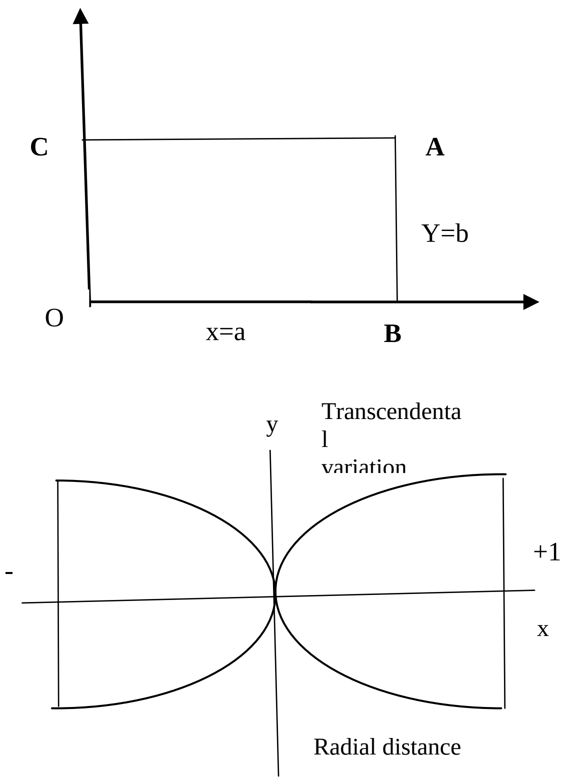

Fig.2 Representation of the thin plate with thickness variation

Now maximum kinetic energy and maximum strain energy are given by

$$T_{\max} = \frac{1}{2}\omega^2 \iint_R \rho h W^2 dxdy \qquad (8)$$

$$U_{\max} = \frac{1}{2}\iint_R D[(\nabla^2 W)^2 + 2(1-\nu)\{W_{xy}{}^2 - W_{xx}W_{yy}\}]dxdy \qquad (9)$$

After solving the maximum kinetic energy and strain energy, one can get the Rayleigh quotient given by

$$\omega^2 = \frac{\iint_R D\left[(\nabla^2 W)^2 + 2(1-\nu)\{W_{xy}{}^2 - W_{xx}W_{yy}\}\right]dxdy}{\iint_R h\rho W^2 dxdy} \qquad (10)$$

where, $D = Eh^3 / (12(1-\nu^2))$, (11)

D is the flexural rigidity and E is young modulus, ρ as mass per unit volume and ν as the passion ratio (0.3) fixed. Let us introduce the following non-dimensional variables and parameters

$$\xi = x/a,\ \eta = y/b,\ \mu = a/b, \qquad (12)$$

Let us consider the N-term approximation with displacement W

$$W = \sum_{j=1}^{N} C_j \phi_j(\xi,\eta), \qquad (13)$$

where C_j are constants and $\phi_j(\xi,\eta)$ are the basis function which satisfying the essential boundary conditions of the plate. By putting equation (13) in (10) and minimizing ω^2 with constants C_1, C_2.........C_N, then one can get the following:

$$\sum_{j=1}^{N}\left(a_{ij} - a^4\omega^2 p b_{ij}\right)c_j = 0 \qquad (14)$$

The above equation can be rewritten in the form

$$\sum_{j=1}^{N}\left(a_{ij} - \lambda^2 b_{ij}\right)c_j = 0 \quad i = 1,2........N \qquad (15)$$

Where $\lambda^2 = a^4\omega^2 p$ and $p = \frac{12\rho(1-\nu^2)}{E}$ then final value of

$$\lambda^2 = \frac{12a^4\rho(1-\nu^2)\omega^2}{Eh_0} \qquad (16)$$

$$a_{ij} = \iint_{R'} f^3\left[\phi_i^{\xi\xi}\phi_j^{\xi\xi} + \mu^2\nu\left(\phi_i^{\xi\xi}\phi_j^{\nu\nu} + \phi_j^{\xi\xi}\phi_i^{\nu\nu}\right) + 2\mu^2(1-\nu)\phi_i^{\xi\nu}\phi_j^{\xi\nu} + \mu^4\phi_i^{\nu\nu}\phi_j^{\nu\nu}\right]d\xi d\eta \qquad (17)$$

$$b_{ij} = \iint_{R'} f\phi_i\phi_j d\xi d\eta \qquad (18)$$

The f is considered as the thickness variation and given by

$$f = 1 + \alpha\sin\xi \qquad (19)$$

where, R' is the new domain in non-dimensional form bounded by $\xi = 0$, $\xi = 1$, $\eta = 0$ and $\eta = 1$, parameter λ represents the frequency parameter.

After solving the above equations for λ^2 with constants values of C_1,

$C_2, \ldots\ldots C_N$, with the basis function ϕ_i satisfying the essential boundary conditions, we get an Eigen value problem. The basis ϕ_i is taken of the form of

$$\phi_i(\xi,\eta) = \xi^p \eta^q (1-\xi)^r (1-\eta)^s \{1, \xi, \eta, \xi^2, \xi\eta, \eta^2 \ldots\ldots\ldots\} \tag{20}$$

Where p=0, 1 or 2, depending on side $\xi = 0$ is free, simply-supported and clamped. Similar representation is given to q, r and s. The parameters, $\eta = 0, \xi = 1, \eta = 1$ are controlling the boundary conditions at other sides. The basis function $\phi_i(\xi,\eta)$ satisfies the all the essential boundary conditions. The complicated integrals involved in the equations are given by.

$$\iint_R \xi^p \eta^q (1-\xi)^r (1-\eta)^s \, d\xi d\eta = \frac{p!q!r!s!}{(p+r+1)!(q+s+1)!} \tag{21}$$

The Eigen-value problem can be solved by the generalized Jacobi method.

IV. NUMARICAL DISCUSSION

For extensive numerical computation on the rectangular and square plates, the following parameters have been selected:

1) The value of Poisson distribution is considered as ν=0.3 for the isotropic plate;
2) The taper parameter α is varying from 0 to 1 with an interval as 0.2;
3) The numeric values of boundary condition are considered as 2,1,0 for the clamped, simply-supported and completely-free plate, respectively and controlled by the values of p, q, r and s;
4) The frequencies are represented as λ_1, λ_2 and λ_3 for the first, second and third frequency, respectively;
5) The aspect ratio a/b is taken as 1 for square plate and 2.0 for the rectangular plate.

The problem has been solved by the generalized Jacobi method which is used for computation of the first three frequencies and corresponding mode shapes. Table [1] represents the first three frequencies for the rectangular plate with different combinations of the boundary conditions. From the table, one can observed that the frequencies are increasing as the value of taper parameter is increasing from 0 to 1. This is because of the stiffness of the plate is increasing.

Table 1 First three frequencies for the rectangular plate (a/b=2.0)

α	0.0	0.2	0.4	0.6	0.8	1.0
FFFF λ_1	0.00000	0.0000	0.0000	0.0000	0.0000	0.0000
λ_2	21.467	23.580	25.731	27.900	30.076	32.254
λ_3	26.585	29.168	31.781	34.390	36.980	39.547
FFFS λ_1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
λ_2	13.469	14.778	16.133	17.513	18.907	20.310
λ_3	34.811	38.140	41.580	45.060	48546	52.024
FFFC λ_1	14.002	15.339	16.738	18.170	19.620	21.082
λ_2	21.460	23.516	25.665	27.864	30.089	32.332
λ_3	40.767	44.653	48.681	52.762	56.853	60.933
FFSF λ_1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
λ_2	13.039	15.145	17.324	19.530	21.743	63.472
λ_3	42.854	46.920	51.071	55.231	59.369	73.580
FFSS λ_1	6.6438	7.6422	8.6819	9.7440	10.819	11.903
λ_2	25.376	27.770	30.234	32.727	35.235	37.750
λ_3	58.772	63.977	68.990	73.622	77.865	81.824
FFSC λ_1	16.140	17.480	18.870	20.288	21.726	23.177
λ_2	31.487	34.331	37.255	40.213	43.186	46.165
λ_3	63.453	69.342	75.357	81.388	87.379	93.300
FFCF λ_1	3.4512	4.1566	4.9138	5.7046	6.5185	7.3507
λ_2	14.818	17.044	19.323	21.616	23.906	26.186
λ_3	21.477	24.216	27.028	29.863	32.699	35.528

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DOI: 10.35940/ijrte.B3197.078219
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FFCS λ_1	8.5156	9.7148	10.950	12.202	13.462	14.727
λ_2	30.983	34.034	37.169	40.332	43.499	46.661
λ_3	64.157	68.243	72.163	75.953	79.648	83.273
FFCC λ_1	17.142	18.545	19.991	21.461	22.945	24.440
λ_2	36.423	39.771	43.199	46.650	50.101	53.545
λ_3	73.516	80.442	87.419	94.285	100.92	107.17
FSFS λ_1	38.945	42.238	45.029	74.512	49.832	52.061
λ_2	46.751	51.477	56.905	62.648	68.487	74.318
λ_3	70.748	77.472	84.465	91.566	98.697	105.82
FSFC λ_1	61.226	65.707	68.918	71.721	74.354	76.899
λ_2	67.279	74.648	83.280	91.987	100.41	108.48
λ_3	87.706	96.102	105.03	114.33	123.93	133.76
FSSS λ_1	41.197	43.583	45.870	48.084	50.251	52.387
λ_2	59.067	64.471	70.006	75.559	81.082	86.559
λ_3	94.506	103.19	112.11	121.09	130.06	138.95
FSSC λ_1	62.922	66.161	69.060	71.766	74.360	76.886
λ_2	77.408	84.479	91.757	99.015	106.17	113.20
λ_3	108.94	118.96	129.31	139.77	150.23	160.64
FSCF λ_1	8.5156	9.7148	10.950	12.202	13.462	14.727
λ_2	30.983	34.034	37.169	40.352	43.499	46.661
λ_3	64.157	68.243	72.162	75.953	79.648	83.273
FSCS λ_1	41.702	43.995	46.218	48.388	50.524	52.636
λ_2	63.016	68.552	74.168	79.772	85.328	90.829
λ_3	103.19	112.58	122.10	131.61	141.06	150.44
FSCC λ_1	63.264	66.345	69.167	71.833	74.402	76.912
λ_2	80.607	87.575	94.667	101.72	108.68	115.54
λ_3	116.70	127.20	137.91	148.65	159.36	170.00
FCFC λ_1	89.151	94.499	98.012	101.14	104.09	106.95
λ_2	93.709	104.82	116.87	127.88	137.82	147.01
λ_3	110.13	120.93	133.08	146.39	160.39	174.62
FCSC λ_1	90.400	94.564	97.990	101.10	104.05	106.90
λ_2	101.65	111.00	120.57	129.90	138.92	147.64
λ_3	128.31	140.12	152.54	165.25	178.03	19075
FCCC λ_1	90.628	94.604	97.984	101.08	104.02	106.87
λ_2	104.17	113.14	122.27	131.22	139.94	148.43
λ_3	135.10	147.18	159.61	172.17	184.77	197.37
SFFS λ_1	6.6438	6.9077	7.1762	7.4435	7.7077	7.9684
λ_2	25.376	27.789	30.274	32.778	35.279	37.766
λ_3	58.772	64.501	70.271	75.989	81.625	87.180
SFFC λ_1	16.140	17.878	19.7121	21.597	23.509	25.437
λ_2	31.487	34.596	37.813	41.065	44.320	47.563
λ_3	63.453	69.537	75.747	81.954	88.108	94.201
SFSF λ_1	9.5125	10.403	11.275	12.123	12.946	13.747
λ_2	27.522	30.057	32.728	35.463	38.227	41.004
λ_3	38.531	42.144	45.787	49.406	52.983	56.514

SFSS	λ_1	16.135	17.637	19.204	20.801	22.412	24.028
	λ_2	46.740	51.116	55.550	59.971	64.350	68.680
	λ_3	75.284	82.239	89.066	95.698	102.13	108.40
SFSC	λ_1	22.815	24.955	27.168	29.414	31.672	33.933
	λ_2	50.751	55.507	60.340	65.169	69.963	74.711
	λ_3	98.890	108.16	117.54	126.03	134.02	141.70
SFCF	λ_1	15.082	16.839	18.632	20.423	22.199	23.955
	λ_2	31.310	34.154	37.110	40.108	43.116	46.122
	λ_3	49.095	54.000	58.979	63.934	68.829	73.656
SFCS	λ_1	20.601	22.659	24.786	26.932	29.076	31.2111
	λ_2	56.318	61.762	67.285	72.782	78.215	83.576
	λ_3	77.332	84.114	90.776	97.264	103.58	109.74
SFCC	λ_1	26.290	28.754	31.284	33.830	36.373	38.906
	λ_2	59.766	65.485	71.289	77.070	82.790	88.437
	λ_3	101.42	110.27	118.78	126.44	134.78	142.34
SSFS	λ_1	41.197	46.429	51.717	56.886	61.877	66.683
	λ_2	59.067	64.841	70.885	77.107	83.457	89.898
	λ_3	94.506	103.63	112.97	122.31	131.59	140.80
SSFC	λ_1	62.922	70.991	78.662	85.651	92.036	97.982
	λ_2	77.408	85.183	93.703	102.86	112.42	122.16
	λ_3	108.94	119.52	130.47	141.57	152.75	163.99
SSSS	λ_1	49.348	53.941	58.492	62.954	67.317	71.589
	λ_2	78.958	86.369	93.926	101.50	109.03	116.50
	λ_3	128.33	140.38	152.66	164.88	176.93	188.78
SSSC	λ_1	69.327	75.733	81.918	87.853	93.562	99.082
	λ_2	94.586	103.52	112.70	121.96	131.21	140.40
	λ_3	140.33	153.52	166.97	180.42	193.75	206.93
SSCS	λ_1	51.674	56.207	60.694	65.095	69.406	73.632
	λ_2	86.135	94.070	102.11	110.12	118.04	125.88
	λ_3	140.91	154.01	167.26	180.44	193.48	206.35
SSCC	λ_1	71.078	77.227	83.188	88.944	94.513	99.921
	λ_2	100.80	110.03	119.43	128.83	138.17	147.41
	λ_3	151.96	166.07	180.38	194.64	208.78	222.74
SCFC	λ_1	90.400	101.65	111.07	119.04	126.27	133.06
	λ_2	101.65	112.33	124.97	138.58	152.25	165.57
	λ_3	128.31	141.00	154.36	168.16	182.37	196.97
SCSC	λ_1	95.263	103.91	111.93	119.39	126.42	133.14
	λ_2	115.80	126.86	138.38	150.07	161.75	173.31
	λ_3	156.37	171.17	186.41	201.73	216.98	232.09
SCCC	λ_1	96.573	104.78	112.50	119.77	126.69	133.33
	λ_2	121.00	132.10	143.49	154.94	166.34	177.62
	λ_3	167.06	182.55	198.33	214.15	229.92	245.55
CFFF	λ_1	3.4512	3.4117	3.3806	3.3551	3.3337	3.3154
	λ_2	14.818	15.302	15.941	16.462	16.957	17.430
	λ_3	21.477	22.775	24.050	25.292	26.497	27.669

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CFFS λ_1	8.5156	8.9253	9.3326	9.7286	10.111	10.481
λ_2	30.983	33.731	36.547	39.371	42.176	44.953
λ_3	64.157	71.870	79.486	86.711	93.525	100.03
CFFC λ_1	17.142	18.998	20.953	22.956	24.980	27.014
λ_2	36.423	39.895	43.471	47.066	50.645	54.195
λ_3	73.516	80.291	87.160	94.003	100.78	107.47
CFSF λ_1	15.082	16.094	17.092	18.065	19.009	19.926
λ_2	31.310	34.214	37.243	40.317	43.401	46.479
λ_3	49.310	53.275	57.476	61.630	65.715	69.726
CFSS λ_1	20.601	22.335	24.122	25.924	27.723	29.512
λ_2	56.318	61.316	66.358	71.358	76.286	81.137
λ_3	77.332	84.882	92.362	99.676	106.81	113.77
CFSC λ_1	26.290	28.711	31.200	33.708	36.214	38.708
λ_2	59.766	65.150	70.594	76.004	81.347	86.614
λ_3	101.42	111.30	120.89	130.13	139.03	147.62
CFCF λ_1	22.076	24.067	26.100	28.125	30.124	32.091
λ_2	36.018	39.307	42.693	46.097	49.487	52.852
λ_3	60.897	66.476	72.118	77.709	83.210	88.616
CFCS λ_1	26.443	28.845	31.305	33.765	36.203	38.611
λ_2	67.249	73.433	79.683	85.876	91.972	97.963
λ_3	79.831	87.240	94.574	101.75	108.76	115.61
CFCC λ_1	31.135	33.992	36.905	39.815	42.701	45.553
λ_2	70.229	76.700	83.243	89.732	96.124	102.41
λ_3	103.42	113.00	122.34	131.36	140.09	148.54
CSFS λ_1	41.702	47.167	52.775	58.340	63.787	69.091
λ_2	63.016	69.330	75.863	82.502	89.201	95.936
λ_3	103.19	113.10	123.21	133.32	148.37	15.332
CSFC λ_1	63.264	71.721	80.070	87.963	95.338	102.26
λ_2	80.607	88.947	97.831	107.18	116.87	126.77
λ_3	116.70	128.12	139.86	151.68	163.51	175.29
CSSS λ_1	51.674	56.759	61.833	66.825	71.715	76.504
λ_2	86.135	94.287	102.57	110.83	119.02	127.11
λ_3	140.91	154.05	167.30	180.46	193.45	206.29
CSSC λ_1	71.078	78.122	85.023	91.711	98.180	104.45
λ_2	100.80	110.49	120.39	130.34	140.23	150.05
λ_3	151.96	166.23	180.70	195.13	209.42	223.56
CSCS λ_1	54.743	59.827	64.882	69.848	74.711	79.473
λ_2	94.585	103.37	112.25	121.06	129.76	138.32
λ_3	154.78	169.16	183.67	198.05	212.20	226.11
CSCC λ_1	73.396	80.222	86.911	93.410	99.720	105.86
λ_2	108.22	118.33	128.57	138.78	148.88	158.85
λ_3	165.03	180.39	195.92	211.34	226.56	241.56
CCFC λ_1	90.628	102.60	113.43	122.88	131.43	139.41
λ_2	104.17	115.36	128.04	141.80	155.93	169.99
λ_3	135.10	148.57	162.261	177.00	191.165	206.50

CCSC	λ_1	96.573	106.09	115.15	123.72	131.85	139.64
	λ_2	121.00	132.83	145.05	157.41	169.76	182.01
	λ_3	167.06	182.90	199.03	215.21	231.35	247.40
CCCC	λ_1	98.313	107.40	116.12	124.43	132.40	140.06
	λ_2	127.32	139.30	151.52	163.76	175.92	187.95
	λ_3	179.09	195.83	212.83	229.76	246.54	263.11

The computations have also been performed on the square plate as a special case of rectangular plate with aspect ratio is considered as a/b=1.0. The results have been computed for the different combinations of the boundary conditions and represented in the following table [2]. From the table, it is again observed that the frequencies are represented upto the five significant digits for different combinations of boundary conditions and these are in increasing form as the taper parameter is increasing.

Table 2 First few frequencies for the square plate (a/b=1.0)

	α	0.0	0.2	0.4	0.6	0.8	1.0
FFFF	λ_1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	λ_2	13.469	14.780	16.141	17.527	18.927	20.336
	λ_3	19.597	21.498	23.476	23.999	27.541	29.600
FFFS	λ_1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	λ_2	6.6438	7.3006	8.0110	8.7588	9.5346	10.332
	λ_3	14.903	16.324	17.818	19.351	20.905	22.472
FFFC	λ_1	3.4811	3.8218	4.1951	4.5915	5.0051	5.4316
	λ_2	8.5217	9.3486	10.229	11.143	12.080	13.035
	λ_3	21.317	23.312	25.342	27.366	29.371	31.354
FFSF	λ_1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	λ_2	6.6438	7.6422	8.6821	9.7442	10.819	11.904
	λ_3	14.903	16.697	18.525	20.369	22.221	24.079
FFSS	λ_1	3.3671	3.8131	4.2870	4.7811	5.2911	5.8140
	λ_2	17.317	18.909	20.531	22.165	23.804	25.446
	λ_3	19.293	21.188	23.149	25.145	27.163	29.194
FFSC	λ_1	5.3577	5.9060	6.4869	7.0915	7.7146	8.3530
	λ_2	19.083	20.990	22.941	24.911	26.886	28.862
	λ_3	24.690	26.596	28.539	30.501	32.474	34.457
FFCF	λ_1	3.4811	4.1940	4.9595	5.7590	6.5825	7.4244
	λ_2	8.5217	9.7224	10.959	12.212	13.473	14.738
	λ_3	21.317	23.972	26.676	29.373	32.039	34.663
FFCS	λ_1	5.3577	6.1516	6.9954	7.8717	8.7717	9.6904
	λ_2	19.083	20.668	22.293	23.936	25.588	27.246
	λ_3	24.690	27.507	30.407	33.335	36.269	39.200
FFCC	λ_1	6.9300	7.7499	8.6182	9.5191	10.444	11.389
	λ_2	23.942	25.977	27.953	29.884	31.787	33.672
	λ_3	26.598	29.209	31.992	34.859	37.765	40.688
FSFS	λ_1	9.6314	10.539	11.475	12.424	13.80	14.342
	λ_2	16.135	17.701	19.386	21.145	22.953	24.793
	λ_3	36.727	40.252	43.900	47.556	49.880	52.111

FSFC λ_1	15.211	16.592	17.920	19.198	20.444	21.673
λ_2	20.613	22.641	24.898	27.299	29.787	32.323
λ_3	39.773	43.583	47.537	51.556	55.601	59.649
FSSS λ_1	11.685	12.564	13.480	14.424	15.393	16.382
λ_2	27.756	30.406	33.133	35.896	38.673	41.458
λ_3	41.202	43.591	45.881	48.099	50.269	52.407
FSSC λ_1	16.807	17.889	18.993	20.115	21.253	22.408
λ_2	31.122	34.032	37.033	40.074	43.133	46.199
λ_3	51.432	54.233	56.836	59.313	61.709	64.054
FSCF λ_1	5.3577	6.1516	6.9954	7.8717	8.7717	9.6904
λ_2	19.083	20.668	22.293	23..936	25.588	27.246
λ_3	24.690	27.507	30.407	33.335	36.269	39.200
FSCS λ_1	12.687	13.676	14.707	15.770	16.859	17.969
λ_2	33.065	36.333	39.689	43.073	46.460	49.841
λ_3	41.707	44.000	46.223	48.394	50.530	52.644
FSCC λ_1	17.551	18.663	19.803	20.967	22.151	23.355
λ_2	36.032	39.476	43.008	46.566	50.126	53.676
λ_3	51.843	54.513	57.038	59.466	61.832	64.161
FCFC λ_1	22.200	24.081	25.693	27.159	28.562	29.941
λ_2	26.478	29.188	32.362	35.762	39.256	42.781
λ_3	43.630	47.817	52.205	56.700	61.251	65.830
FCSC λ_1	23.402	24.747	26.077	27.400	28.726	30.059
λ_2	35.592	38.863	42.254	45.699	49.168	52.646
λ_3	62.940	66.185	69.091	71.804	74.402	76.931
FCCC λ_1	23.949	25.247	26.549	27.857	29.175	30.507
λ_2	40.019	43.720	47.520	51.350	55.183	59.00
λ_3	63.282	66.365	69.190	71.857	74.429	76.941
SFFS λ_1	3.3671	3.5717	3.7912	4.0180	4.2481	4.4794
λ_2	17.317	18.992	20.707	22.417	24.105	25.766
λ_3	19.293	21.117	23.028	24.997	27.004	29.037
SFFC λ_1	5.3577	5.8404	6.3576	6.8941	7.4421	10.530
λ_2	19.083	20.810	22.580	24.358	26.131	35.647
λ_3	24.690	27.467	30.352	33.303	36.256	52.326
SFSF λ_1	9.6314	10.533	11.417	12.275	13.110	13.921
λ_2	16.135	17.637	19.204	20.802	22.413	24.030
λ_3	36.727	40.139	43.626	47.123	50.605	54.062
SFSS λ_1	11.685	12.778	13.883	14.986	16.079	17.162
λ_2	27.756	30.343	32.993	35.658	38.318	40.964
λ_3	41.202	45.065	48.967	52.848	56.687	60.478
SFSC λ_1	12.687	13.876	15.087	16.301	17.512	18.715
λ_2	33.065	36.150	39.285	42.420	45.535	48.624
λ_3	41.707	45.618	49.570	53.505	57.399	61.247

SFCF	λ_1	15.211	16.982	18.791	20.598	22.389	24.162
	λ_2	20.613	22.675	24.804	26.952	29.098	31.233
	λ_3	39.773	43.301	46.887	50.473	54.038	59.595
SFCS	λ_1	16.807	18.652	20.542	22.437	24.321	26.189
	λ_2	31.122	33.958	36.844	39.731	42.600	45.446
	λ_3	51.432	56.511	61.669	66.802	71.877	76.883
SFCC	λ_1	17.551	19.437	21.370	23.311	25.242	27.157
	λ_2	36.032	39.259	42.523	45.775	48.999	52.190
	λ_3	51.843	56.956	62.148	67.317	72.427	77.469
SSFS	λ_1	11.685	13.032	14.457	15.917	17.391	18.870
	λ_2	27.756	30.379	33.087	35.827	38.573	41.316
	λ_3	41.202	46.433	51.720	56.891	61.886	66.701
SSFC	λ_1	16.807	18.901	21.093	23.317	25.543	27.758
	λ_2	31.122	34.127	37.248	40.425	43.628	46.841
	λ_3	51.432	58.027	64.532	70.707	76.516	82.005
SSSS	λ_1	19.739	21.588	23.474	25.368	27.256	29.134
	λ_2	49.349	53.942	58.493	62.956	67.321	71.595
	λ_3	49.349	53.974	58.658	63.328	67.955	72.531
SSSC	λ_1	23.646	25.867	28.124	30.383	32.632	34.866
	λ_2	51.675	56.522	61.437	66.343	71.209	76.023
	λ_3	58.646	64.092	69.427	74.609	79.642	84.543
SSCS	λ_1	23.646	25.923	28.241	30.557	32.858	35.137
	λ_2	51.675	56.208	60.695	65.096	69.407	73.633
	λ_3	58.646	64.290	70.012	75.706	81.335	86.889
SSCC	λ_1	27.055	29.581	32.139	34.691	37.221	39.725
	λ_2	60.539	65.878	71.067	76.113	81.026	85.821
	λ_3	60.791	66.545	72.429	78.295	84.099	89.830
SCFC	λ_1	23.402	26.431	29.549	32.655	35.708	38.696
	λ_2	35.592	39.109	42.813	46.636	50.541	54.503
	λ_3	62.940	71.005	78.671	85.658	92.048	98.012
SCSC	λ_1	28.951	31.675	34.423	37.162	39.879	42.569
	λ_2	54.744	59.886	65.116	70.349	75.551	80.710
	λ_3	69.327	75.733	81.918	87.853	93.563	99.083
SCCC	λ_1	31.827	34.716	37.623	40.510	43.367	46.190
	λ_2	63.333	69.367	75.490	81.590	87.629	93.595
	λ_3	71.079	77.228	83.190	88.945	94.514	99.922
CFFF	λ_1	3.4811	3.4395	3.4065	3.3793	3.3562	3.3363
	λ_2	8.5217	8.9308	9.3375	9.7333	10.116	10.487
	λ_3	21.317	22.651	23.959	25.231	26.464	27.662
CFFS	λ_1	5.3577	5.5901	5.8411	6.1002	6.3621	6.6242
	λ_2	19.083	21.111	23.237	25.404	27.581	29.756
	λ_3	24.690	26.522	28.377	30.229	32.069	33.896

Retrieval Number: B3197078219/19©BEIESP
DOI: 10.35940/ijrte.B3197.078219
Journal Website: www.ijrte.org

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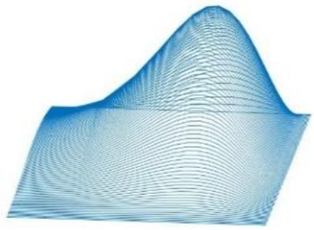
Blue Eyes Intelligence Engineering
& Sciences Publication

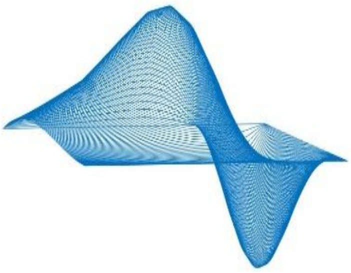
CFFC λ_1	6.9300	7.4366	7.9802	8.5427	9.1148	9.6920
λ_2	23.942	26.242	28.468	30.600	32.658	34.661
λ_3	26.598	29.182	32.007	34.976	38.016	41.088
CFSF λ_1	15.211	16.231	17.239	18.221	19.175	20.101
λ_2	20.613	22.344	24.129	25.928	27.726	29.515
λ_3	39.773	43.617	47.542	51.470	55.372	59.235
CFSS λ_1	16.807	18.054	19.310	20.555	21.782	22.990
λ_2	31.122	34.054	37.047	40.044	43.022	45.973
λ_3	51.432	55.878	60.352	64.780	69.139	73.423
CFSC λ_1	17.551	18.901	20.269	21.630	22.977	24.306
λ_2	36.032	39.501	43.026	46.544	50.030	53.476
λ_3	51.843	56.336	60.860	65.340	69.751	74.088
CFCF λ_1	22.200	24.204	26.250	28.288	30.300	32.281
λ_2	26.478	28.880	31.339	33.796	36.231	38.636
λ_3	43.630	47.656	51.742	55.814	59.845	63.826
CFCS λ_1	23.402	25.578	27.680	29.835	31.965	34.062
λ_2	35.592	38.863	42.178	45.476	48.735	51.949
λ_3	62.940	68.716	74.558	80.349	86.048	91.647
CFCC λ_1	23.949	26.119	28.335	30.545	32.730	34.884
λ_2	40.019	43.713	47.442	51.143	54.796	58.396
λ_3	63.282	69.093	74.970	80.797	86.534	92.173
CSFS λ_1	12.687	14.112	15.622	17.169	18.731	20.296
λ_2	33.065	36.009	39.029	42.066	45.092	48.099
λ_3	41.707	47.175	52.787	58.360	63.821	69.145
CSFC λ_1	17.551	19.758	22.081	24.449	26.825	29.193
λ_2	36.032	39.366	42.801	46.268	49.738	53.198
λ_3	51.843	58.737	65.704	72.488	79.002	85.239
CSSS λ_1	23.646	25.751	27.890	30.024	32.141	34.235
λ_2	51.675	56.761	61.837	66.831	71.726	76.520
λ_3	58.646	63.893	69.188	74.441	79.622	84.724
CSSC λ_1	27.055	29.564	32.108	34.646	37.161	39.648
λ_2	60.539	66.125	71.658	77.144	82.555	87.884
λ_3	60.791	66.676	72.609	78.419	84.087	89.617
CSCS λ_1	28.951	31.597	34.279	36.944	39.572	42.160
λ_2	54.744	59.828	64.883	69.850	74.713	79.476
λ_3	69.327	75.713	82.167	88.564	94.865	101.06
CSCC λ_1	31.827	34.752	37.705	40.635	43.523	46.365
λ_2	63.333	69.224	75.041	80.728	86.275	91.692
λ_3	71.079	77.635	84.262	90.835	97.312	103.68
CCFC λ_1	23.949	27.145	30.478	33.839	37.176	40.468
λ_2	40.019	43.869	47.866	51.937	56.046	60.178
λ_3	63.282	71.741	80.092	87.989	95.370	102.30

CCSC λ_1	31.827	34.885	37.974	41.047	44.087	47.089
λ_2	63.333	69.094	74.925	80.729	86.470	92.138
λ_3	71.079	78.125	85.027	91.716	98.188	104.46
CCCC λ_1	35.986	39.313	42.656	45.964	49.220	52.423
λ_2	73.399	80.180	86.914	93.414	99.725	105.86
λ_3	73.399	80.225	87.035	93.839	100.55	107.15

After computing the first three frequencies, the mode shapes have been demonstrated by computing the eigenvectors on the corresponding frequency. These are represented in the following figures [3] and [4] for the rectangular and square plate, respectively with taper parameter is considered as 0.5. The mode shapes represent the behaviour of the plate during vibrat



(a)



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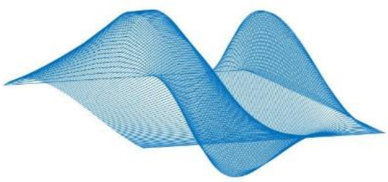
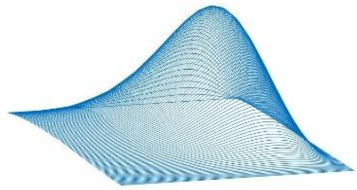


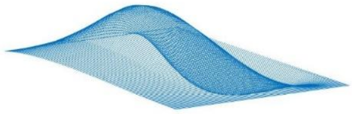
(c)

Fig. 3 First three mode shapes for rectangular clamped plate (α =0.5, ν =0.3)
(a) first frequency (b) second frequency (c) third frequency



(a)

(b)



(c)

Fig. 4 First three mode shapes for square clamped plate (α =0.5, ν =0.3)
(a) first frequency (b) second frequen0cy (c) third frequency

The computations are upto the five significant digits and computed after the study of the convergence of the result. In the Rayleigh-Ritz method, the convergence is so fast and represented in the following table [3]. The values of convergence have been obtained for the rectangular and square plate both with taper parameter as α=0.2. It is observed from the table that the values are converging upto the five significant digits.

Table 3 Convergence of the results for rectangular and square plate

N	Rectangular Plate (μ=a/b=1.0)			Square plate (μ=a/b=1.0)		
	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3
1	108.19	-	-	39.505	-	-
2	107.78	140.17	-	39.332	81.613	-
3	107.78	140.17	286.36	39.332	81.534	81.613
4	107.59	139.40	200.94	39.328	80.989	81.534
5	107.59	139.40	200.94	39.328	80.989	81.265
6	107.42	139.40	200.23	39.328	80.989	81.265
7	107.42	139.35	199.04	39.328	80.365	81.265
8	107.42	139.35	199.04	39.328	80.365	81.083
9	107.42	139.35	199.04	39.328	80.239	81.083
10	107.42	139.35	199.04	39.328	80.239	80.266
11	107.42	139.33	196.31	39.328	80.214	80.266
12	107.42	139.33	196.31	39.328	80.214	80.266
13	107.41	139.33	196.19	39.317	80.211	80.266
14	107.41	139.33	196.19	39.317	80.211	80.263
15	107.41	139.33	196.19	39.316	80.211	80.263
16	107.41	139.32	195.99	39.316	80.199	80.263
17	107.41	139.32	195.99	39.316	80.199	80.258
18	107.41	139.30	195.98	39.316	80.186	80.258
19	107.41	139.30	195.98	39.316	80.186	80.243
20	107.41	139.30	195.98	39.316	80.181	80.243
21	107.41	139.30	195.98	39.316	80.181	80.225
22	107.41	139.30	195.88	39.316	80.180	80.225
23	107.41	139.30	195.88	39.316	80.180	80.225
24	107.40	139.30	195.84	39.315	80.180	80.225
25	107.40	139.30	195.84	39.315	80.180	80.225
26	107.40	139.30	195.83	39.313	80.180	80.225
27	107.40	139.30	195.83	39.313	80.180	80.225
28	107.40	139.30	195.83	39.313	80.180	80.225

The comparisons of results have also obtained from the literature [22, 24, 27] for uniform thickness variation for the rectangular and square plate and represented in the following Table [4] with different combinations of the boundary conditions. It is observed from the table that the available results are in excellent agreement.

Table 4 Comparisons of the Results for Rectangular and Square Plate (α=0.0)

Boundary Condition	Reference	Available Results			Present Result		
		λ_1	λ_2	λ_3	λ_1	λ_2	λ_3
Rectangular Plate							
FFFF	[12]	-	21.789	-	0.0	21.467	26.585
CFCF	-	-	-	-	22.076	36.018	60.897
CCFF	[24]	17.347	36.449	-	17.142	36.423	73.516
CSFF	[27]	15.851	-	49.103	15.082	31.310	49.310
SSFF	[27]	-	27.801	38.079	9.512	27.522	38.531

Square Plate

FFFF	[22][27]	-	13.468	19.596	0.00	13.469	19.597
CCCC	[22]	35.989	-	-	35.986	73.399	73.399
CFCF	[22]	22.165	-	43.589	22.200	26.478	43.630
SFFF	[27]	-	6.641	14.899	0.00	6.643	14.903
SFSF	[27]	3.470	8.498	21.281	3.481	8.521	21.317
CCFF	[24]	9.630	16.127	36.705	9.631	16.135	36.727
CSFF	[24]	6.832	23.418	26.096	6.930	23.942	26.598
SSFF	[24]	-	20.685	-	15.211	20.613	39.773
		3.208	17.510	-	3.367	17.317	19.293

V. CONCLUSION

The above problem is based upon the study of the transcendental thickness variation applied on the thin plate. It is observed that the Rayleigh-Ritz method is an efficient method for obtaining the results for the first few frequencies for the thin plate. The behaviour of the plate is represented through the computations of the mode shapes. As the taper parameter controlling the thickness variation is increasing, the frequencies of the plate are increasing due to increment in the stiffness of the plate. The results have been obtained for the rectangular and square plates correct upto the five significant digits along with the convergence.

REFERENCES

1. A. W. Leissa, “Vibration of plates”, *Washington, D.C, U.S Gov. Printing Office NASASP*, 160, 1969.
2. J. Ramachandran, D. V. Reddy, “Nonlinear vibrations of rectangular plates with cut-outs”, *American Institute of Aeronautics and Astronautics*, vol. 10, 1972, pp. 1709-1710.
3. A.W. Leissa, “The free vibration of rectangular plates”, *Journal of Sound and Vibration*, vol. 31, 1973, pp. 257-293.
4. A. W. Leissa, “Recent Research in Plate Vibration”, *The shock and vibration digest*. Vol. 9, 1977, pp. 13-24, 1973-1976: Classical Theory.
5. A. W. Leissa, “Recent Research in Plate Vibrations”, *The shock and vibration digest* vol. 10, 1978, pp. 21-35, 1973-1976. Complicating Effects.
6. A. W. Leissa, “Plate Vibration Research”, *The shock and vibration digest* vol. 13, 1981, pp. 11-22, 1976-1980: Classical Theory.
7. A. W. Leissa, “Plate Vibration Research”, *The shock and vibration digest*, vol. 13, 1981, pp. 19-36, 1976-1980: Complicating Effects.
8. A. W. Leissa, “Recent Studies in Plate Vibrations”, *The shock and vibration digest*, vol. 19, 1987, pp. 11-18, 1981-1985, Part I, Classical Theory.
9. A.W. Leissa, “Recent Studies in Plate Vibration”, *The shock and vibration digest* vol. 19, 1987, pp. 10-24, 1981-1985, Part II, Complicating Effects.
10. A. W. Leissa, “Vibration of plates”, *Acoustical Society of America*, 1993.
11. P. A. A Laura, R. O. Grossi, “Transverse vibration of rectangular plates with edges elastically restrained against translation and rotation”, *Journal of Sound and Vibration*, vol. 75, 1981, pp. 101-107.
12. L. shuyu, “Study on the flexural vibration of rectangular thin plates with free boundary condition”, *Journal of Sound and Vibration*, vol. 239, 2001, pp. 1063-1071.
13. D. J. Gorman, “Plane free in plane vibration analysis of rectangular plates by the method of superposition”, *Journal of Sound Vibration*, vol. 272, 2004, pp. 831-851.
14. D. J. Gorman, “Free in plane vibration analysis of rectangular plates with elastic support normal to the boundaries”, *Journal of Sound and Vibration*, vol. 285, 2005, pp. 941-966.
15. J. H. Wu, A. Q. Liu, H.L. Chen, “Exact solution for free vibration analysis of rectangular plates using functions”, *Journal of Applied Mechanics*, vol. 74, 2007, pp. 1247-1251.
16. Y. Xing, B.Liu, “Closed form solutions for free vibration of rectangular Mindlin plates”, *Acta Mechanica Solid Sinica* vol. 25, 2009, pp. 689-698.
17. L. Dozio, “Free in-plane vibration analysis of rectangular plates with arbitrary elastic boundaries”, *Mechanics Research Communications*, vol. 37, 2010, pp. 627-635.
18. B. Liu, Y. Xing, “Exact solutions for free in plane vibration of rectangular plates”, *Acta Mechanica Solid Sinica*, vol. 24, 2011, pp. 556-567.
19. D.G. Zhang, “Modeling and analysis of FGM rectangular plates based on physical neutral surface and high order shear deformation theory”, *Int. J. of Mechanical Sci.*, vol. MS2379, 2013, pp. 00007-6.
20. R. Li, B. Wang, P. Li, “Hamiltonian system based benchmark bending solutions of rectangular thin plates with a corner point-supported”, *Int. J. of Mechanical Sci.*, 2014, pp. 00176-3
21. Y. Zhang, J. Du, T. yang, Z. Liu, “A series solution for the in-plane vibration analysis of orthotropic rectangular plates with elastically restrained edges”, *Int. J. of Mechanical Sci.*, vol. 79, 2014, pp.15-24.
22. J. R. Banerjee, S.O. Papkov, X. Liu, D. Kennedy, “Dynamic stiffness matrix of a rectangular plate for the general case”, *Journal of Sound and Vibration*, vol.342, 2015, pp. 177-199.
23. M. N. Danilovic, M. petrojjevic, “In-plane free vibration and response analysis of isotropic rectangular plates using dynamic stiffiness method”, *Computers and Structures*, vol. 152, 2015, pp. 82-95.
24. R. Li, B. Wang, G. Li, J. Du, “Analytic free vibration solution of rectangular thin plates point-supported at a corner”, *Int. J. of Mechanical Sci.*, vol. 96-97, 2015, pp. 199-205.
25. B. Wang, R. Li, Rui Li, “Symplectic superposition method for new analytic buckling solutions of rectangular thin plates”, *Int. J. of Mechanical Sci.*, vol. 119, 2016, pp. 432-441.
26. R. Li, Y. Tian, P. Wang, Y. Shi, B. “Wang, New analytic free vibration solutions of rectangular thin plates resting an multiple point supports”, *Int. J. of Mechanical Sci.*, 2016, pp. 00073-4.
27. X. Wang, Z. yuan, “Discrete singular convolution and Taylor series expansion method for free vibration analysis of beams and rectangular plates with free boundaries”, *Int. J. of Mechanical Sci.*, vol. 122, 2017, pp. 184-191.
28. Y.Zhou, Q.Wang, D.Shi, Q.Liang, “Exact solutions for the free in plane vibration of rectangular plates with arbitrary boundary conditions”, *Int. J. of Mechanical Sci.*, vol. MS3711, 2017, pp. 31014-1.
29. Y. Xing, Q. Sun, B. Liu, Z. Wang, “The overall assessment of closed from solution methods for free vibrations of rectangular thin plates”, *Int. J. of Mechanical Sci.*, vol. S0020-7403, 2018, pp. 30292-3.

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