PAPR Performance Analysis of low Complexity SLM for Various Phase Sequences in MIMO-OFDM System

Srinu Pyla, K. Padma Raju, N. Balasubrahmamyan

Abstract Multiple Input Multiple Output Orthogonal Frequency Division Multiplexing (MIMO-OFDM) is a key technology for contemporary communication systems due to its spectral efficiency, higher data rates, better diversity gain, good link reliability and both inter symbol interference (ISI) and multipath fading free transmission. However, due to the presence of OFDM, MIMO-OFDM suffers from high peak to average power ratio (PAPR). Even though, several schemes are available to mitigate PAPR, there is no standard solution. Selective Mapping (SLM) significantly reduces the PAPR in OFDM systems at the cost of computational complexity (CC). The CC of SLM can be reduced by proper design of SLM. This paper considers a low complexity SLM (LC SLM) scheme in which both the CC and length of the index of selected phase sequence are significantly reduced. The PAPR of an SLM-OFDM depends on the number of subcarriers in OFDM, the number of candidate blocks in SLM and selected phase sequence and this paper investigate various phase sequences and analyses their PAPR performance. Simulation results show the superior performance of Riemann sequence over the other phase sequences.

Keywords: LC SLM, MIMO-OFDM, PAPR, phase sequences, Riemann sequence

I. INTRODUCTION

The main challenges of contemporary communication systems are providing high-data-rate transmission, optimum utilization of the spectrum, both multipath fading and inter-symbol interference-free transmission, and link reliability for quality of service. MIMO wireless technology improves spectral efficiency and link reliability through spatial multiplexing and antenna diversity gain respectively. OFDM not only mitigates both the ISI and the multipath fading, but also supports higher data rate transmission. Hence, the combination of MIMO and OFDM popularly known as MIMO-OFDM plays a vital role in modern communication systems. Spatial multiplexing enhances the system capacity with no additional power or bandwidth. The spatial gain is good at rich channel scattering. An increase in diversity increases the link reliability by providing less channel fading, moreover, the system becomes more robust against co-channel interference. Transmission of the data over multiple uncorrelated fading dimensions in frequency, time, and space increases the diversity gain. Space-time codes can be considered for improvement of spatial diversity gain in MIMO systems without having channel knowledge. OFDM provides spectral efficiency, both multipath fading, and ISI free transmission and also supports high data rate transmission. Because of numerous advantages, OFDM has been preferred in several applications such as HDTV, WLAN, WiMAX, digital audio broadcasting, mobile broadband wireless access, long-term evolution (LTE) and LTE-A. In spite of the numerous advantages, OFDM suffers from high PAPR due to the coherent addition of subcarriers and high PAPR increases complexity by increasing the dynamic range in analog to digital conversion, creates signal distortion and spectral broadening, reducing amplifier efficiency. Popular methods for PAPR mitigation are signal clipping, clipping - recursive filtering, coding, tone injection, tone reservation, partial transmit sequence, active constellation and selective mapping (SLM) so on and so forth. SLM reduces the PAPR significantly in OFDM without creating distortion and spectral broadening.


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* Correspondence Author
Srinu Pyla*, Department of ECE, Gayatri Vidya Parishad College of Engineering (A), Visakhapatnam, India. Email: srimupyla@gvpe.ac.in
K. Padma Raju, department of ECE, J. N. T. University, Kakinada, Kakinada, India. Email: padmaraju_k@yahoo.com
N. Balasubrahmamyan, Department of ECE, Gayatri Vidya Parishad College of Engineering (A), Visakhapatnam, India. Email: nbs360@yahoo.co.in

Correspondence Author
K. Padma Raju
Srinu Pyla*

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Murthy et al. [9] discussed the effects of PAPR reduction using clipping and filtering, turbo coding of clipped OFDM signal, pre-distorter and active channel extension on MB OFDM UWB signals. Mizhou Tan et al. [10] discussed about the most efficient solution to reduce PAPR in MIMO OFDM over conventional OFDM. P. Van Eetvelt et al. [11] proposed a selective scrambling technique for reduction of PAPR in QPSK-OFDM systems and listed out the reasons for the need of PAPR reduction in OFDM in [12]. R. W. Bauml et al. [13] proposed the selective mapping (SLM) method in which multiple blocks with the same information are array multiplied with phase sequence, then processed through IFFT and the block with least PAPR is considered for transmission. N. Ohkubo et al. [14] stated that for same side information length, SLM is better than PTS. C. L. Wang et al. [15] proposed a method to determine the PAPR of OFDM signal by oversampling the OFDM signal with a factor of four. Stephane et al. [16] proposed a selected mapping method to minimize the PAPR of OFDM. Ehab et al. [17] proposed a new semi-blind SLM scheme to reduce PAPR. Sukchel Yang et al.[18] proposed FSLM scheme to reduce the computational complexity in SLM. S. H. Han et al. [19] proposed a method to improve the PAPR reduction by embedding phase sequence. H. B. Jeon et al. [20] proposed an SLM scheme with low complexity. H. B. Jeon et al. [21] proposed bit based SLM to mitigate PAPR in OFDM. Matthias Gay et al. [22] presented a PAPR reduction method using SLM and clipping and sparse reconstruction at transmitter and receiver respectively. Neil Carson et al. [23] presented a novel approach for PAPR reduction using modified repeat accumulate (RA) code, signal clipping, and SLM.

II. PAPR IN OFDM

In an OFDM, M information bits are grouped into N symbols (X_u) and the symbols are modulated on N mutually orthogonal subcarriers. Each subcarrier bandwidth is equal to total bandwidth (W) of the system over the total number of subcarriers (N) and the N should be selected such that, the subcarrier bandwidth (W/N) should be less than the coherence bandwidth (B_c). This process increases the symbol duration by N times and makes signal flat fading. OFDM signal can be defined as (1)

\[ x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi fk t}, \quad 0 \leq t \leq NT \]

Due to orthogonality, the real and imaginary components of x(t) are not correlated and the distribution becomes Gaussian for high value of N (central limit theorem) with mean zero and variance \( \sigma^2 = E[(Re{x(t)})]^2 + |Im{x(t)}|^2]/2 \). Then the probability density of an OFDM signal is (2)

\[ p_f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{|x|^2}{2\sigma^2}} \]

(2)

For four times oversampled information signal, the continuous PAPR and the discrete PAPR are almost same and hence, the signal is oversampled in the IFFT process. Moreover, the complexity of SLM is less over PTS for oversampled IFFT OFDM systems [24]. The oversampled OFDM signal is (3)

\[ x(n) = \frac{1}{\sqrt{4N}} \sum_{k=0}^{4N-1} X_k e^{j2\pi f_k n}, \quad 0 \leq n \leq 4NT \]

(3)

Before the IFFT process in an OFDM, input sequence average power is equal to the autocorrelation peak value. For fixed N, maximum value of the system varies with the input data sequence. In IFFT process, the input sequence is multiplied with sinusoidal functions, summing and sampling the results. Due to the strong correlation of the input of IFFT, the sinusoidal are in in-phase form. The summation of in-phase functions result in large amplitude, hence, the peak power is far greater than the average and the large PAPR moves the OFDM signal into the saturation region of power amplifier and creates signal distortion and radiation. The PAPR of OFDM signal x(t) is (4)

\[ \frac{P_{\text{peak}}}{P_{\text{average}}} = \frac{\max\{|X(n)|^2\}}{E[|X_n|^2]} \]

where E[] is expected value. OFDM signal peak increases with N and probability exceeds a threshold \( P_{\text{peak}} = \frac{c_0^2}{\sigma^2} \).

The peak power is proportional to number of subcarriers. Max \( \frac{1}{\sqrt{N}} \max\{|X_0 + X_1 + X_2 + \cdots + X_{N-1}|\} \]

III. PAPR REDUCTION USING LOW COMPLEXITY SLM

Popular PAPR reduction schemes are mentioned in the introduction and SLM is considered in this work. Almost all schemes have their own merits and demerits. From the literature, it is found that SLM reduces the PAPR more significantly without information loss. In an SLM, U alternative complex data blocks (X_u) with the same information of length N are array multiplied with phase sequences (P^u) of length N. The resultant data blocks are processed through IFFT and will be determined their PAPRs. The minimum PAPR sequence will be considered for transmission. The new block for the kth sequence X^u_k is X_u^k \in \{1, 2, ..., M\}, X^u_k is converted into time domain signal x^k via IFFT. Applying SLM to OFDM signal, (3) becomes

\[ x^k(t) = \frac{1}{\sqrt{4N}} \sum_{k=0}^{4N-1} X_k^u P_k^u e^{j2\pi f_k t}, \quad 0 \leq t \leq 4NT \]

(7)

\[ x^k = \text{IFFT}(x^u) \text{ = IFFT}(X^u \cdot P^u) \]

(8)

Due to its linear operation, SLM does not create nonlinear distortion. In SLM, the PAPR reduction is proportional to the number of alternate data blocks (U).
However, \( U \) increases the overhead information (\( \log_2 U \)) which is used for decoding the data at the receiver and results in low data rate. In addition, \( U \) number of IFFTs are required and thereby complexity increases. S J Heo et al. [25] proposed a modified SLM in which both length of the side information index and computational complexity of SLM are reduced with marginal loss of PAPR reduction. As PAPR reduction depends on phase sequence, the phase sequence selection is also critical. Popular phase sequences to reduce PAPR in SLM OFDM are described in succeeding section.

**Figure 1:** schematic diagram of low complexity SLM scheme for MIMO-OFDM system

If \( x_1 \) and \( x_2 \) are the OFDM signal sequences generated by traditional SLM, from the Fourier transform linear property, the sequences can be written as

\[
x_{1k} = c_1 \text{IFFT}(x_1 \otimes V_k) + j c_2 \text{IFFT}(x_2 \otimes V_k)
\]

\[
x_{2k} = \text{IFFT}[a \otimes (c_1 V_k + j c_2 V_k)]
\]

where \( c_1 \) and \( c_2 \) are complex numbers. If \( (c_1 V_k + c_2 V_k) \) sequences have unit magnitude, \( (c_1 V_k + j c_2 V_k) \) can also be considered as phase sequences. With \( x_1 \) and \( x_2 \), alternative sequence \( x_{1k} \) can be generated without performing IFFT. Here, the sequences \( (c_1 V_k + j c_2 V_k) \) are not statistically independent to \( V_i \) and \( V_k \). Each element of \( (c_1 V_k + j c_2 V_k) \) should have unit magnitude as \( V_i \) and \( V_k \). The new sequences \( (c_1 V_k + j c_2 V_k) \) have unit magnitude if: i) every element of \( V_i \) and \( V_k \) considers \(+1,-1\); ii) \( c_1 = \frac{1}{\sqrt{2}} \) and \( c_2 = \frac{1}{\sqrt{2}} \). Since \( |c|^2 = |c|^2 \), the expected power of \( x_{1k} \) is equal to half of the sum of average powers of \( x_1 \) and \( x_2 \). In this process, \( 2U \) additional phase sequences can be generated. \( U^2 \) sequences can be generated from \( U \) number of IFFTs as shown in fig. 1.

**IV. PHASE SEQUENCES FOR SLM SCHEME**

In an SLM, for constant \( N \), the PAPR reduction is proportional to value of \( U \) and type of phase sequence. An increase in \( U \), improves PAPR performance at the cost of computational complexity. Hence, the selection of a phase sequence to achieve better PAPR reduction capability is a crucial aspect in SLM OFDM. However, the PAPR reduction performance comparison and reasons are rarely given.
The phase sequence with low average and large variance has better PAPR reduction capability. Several phase sequences are available to reduce PAPR in SLM OFDM. Hence, for identification of optimum phase sequence, determination of PAPR for phase sequences becomes critical. If the phase sequence is considered as the data sequence, the OFDM signal using u\textsuperscript{th} phase sequence can be given as

$$s_{k}^{(u)}(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} b_{k}^{(u)} e^{j2\pi ft_k} , \quad 0 \leq t \leq NT$$  \hspace{1cm} (10)

In an OFDM signal, the average power \( \mu \) is same for all the phase sequences. Let the power variance of the OFDM signal using the \( u \)\textsuperscript{th} phase sequence be \( \sigma_{u}^2 \). Then the PAPR of phase sequence is

$$PAPR_{u} = \frac{\max P_{b}^{(u)}(t)}{\mu} = \frac{P_{b_{\text{max}}}^{(u)}}{\mu}$$  \hspace{1cm} (11)

where \( P_{b_{\text{max}}}^{(u)}(t) \) is the OFDM signal peak power for the \( u \)\textsuperscript{th} phase sequence and \( P_{b}^{(u)}(t) \) is the average power and can be expressed as (13)

$$P_{b}^{(u)}(t) = \left| s_{k}^{(u)}(t) \right|^2 = \frac{1}{N} \sum_{n=0}^{N-1} |b_{n}^{(u)}|^2 \exp\left(\frac{2\pi inm}{N}\right)$$  \hspace{1cm} (12)

From equation (3), for large number \( N \), \( P_{b}^{(u)}(t) \) can be considered as a Gaussian variable with mean \( \mu \) and variance \( \sigma_{u}^2 \). Hadamard sequence is a well-known phase sequence to reduce PAPR in SLM-OFDM. This sequence is applied to data symbols before IFFT process to reduce the input sequence correlation.

$$H_{1} = [1], \quad H_{2} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, \quad \text{and} \quad H_{2N} = \frac{1}{\sqrt{2}} \begin{bmatrix} H_{N} & H_{N} \\ H_{N} & -H_{N} \end{bmatrix}$$

Where \( H_{N} \) is the binary complement of \( H_{N} \). Riemann sequence can be given as (15)

$$R(\varphi, q) = \begin{cases} p-1 & \text{if } p \text{ divides } q \\ -1 & \text{otherwise} \end{cases}$$  \hspace{1cm} (14)

In Riemann matrix, the first row and column are discarded for effective PAPR reduction. The elements of \( v^h \) row are either \( v \) or \(-1\). In this row, \( C = \left[ \begin{array}{c} \frac{2}{v^h+1} \\ \end{array} \right] \) number of elements have magnitude \( v \) and other elements have magnitude \(-1\). Hence, not only phase but also the magnitude of the modulated symbols are changed. Due to this reason, the average power of the signal will not be same as average power of the original signal. From [25], the M-ary chaotic sequence \( C_{n} \in \{0,1,2,...,M-1\} \), \( 0 \leq n \leq N-1 \) and \( C_{n} \) is given by

$$C_{n} = \left[ \frac{M+1}{2} \right] + z, \quad y_{n+1} = f(y_{n}) = 1 - ay_{n}^{2} \quad \alpha \in [1.0151.99] , \quad y_{n} \in (-1,1) \hspace{1cm} (15)$$

where \( z \) is greatest integer not exceeding to \( z \) and from \( C_{n} \), the first phase sequence \( v^{h} \) element becomes

$$P_{v}^{h} = \exp\left(\frac{2\pi C_{n}v}{M}\right) = \Phi_{v}$$  \hspace{1cm} (16)

The first sequence is formed as

$$P_{v} = [\Phi_{v}, \Phi_{\epsilon v}, \Phi_{\epsilon^{2}v},..., \Phi_{\epsilon^{n-1}v}]$$

and \( u \)\textsuperscript{th} phase sequence can be obtained by applying \( u - 1 \) circular shifts to \( P_{1} \). Shapiro Rudin sequence consists of either \(+1\) or \(-1\) and the \( n \)\textsuperscript{th} term of the sequence is

$$a_{n} = \sum_{i=1}^{n} e_{i} \quad \text{and} \quad b_{n} = (-1)^{\epsilon_{n}} \hspace{1cm} (17)$$

where \( e_{i} \) indicates the binary form digits of \( n \) and \( a_{n} \) represents the sub-string 11 in the binary expansion. \( b_{n} = +1 \) or \(-1\) for \( a_{n} \) is even or odd respectively and can be generated by

$$b_{2n} = b_{n} \quad \text{and} \quad b_{2n+1} = (-1)^{\epsilon_{n}}b_{n} \hspace{1cm} (18)$$

The Chu phase sequence is defined as \( B^{(u)} = [B_{0}^{(u)}, B_{1}^{(u)}, \ldots, B_{N-1}^{(u)}] \). \( B^{(u)} \) is the \( k \)\textsuperscript{th} term of the sequence as (20)

$$b_{k}^{(u)} = \begin{cases} \exp\left(\frac{2\pi n}{N} \left[ \frac{uk}{N} \right] \right), & N \text{ even} \\ \exp\left(\frac{2\pi n}{N} \left[ \frac{uk(k+1)}{N} \right] \right), & N \text{ odd} \end{cases}$$  \hspace{1cm} (19)

Pseudorandom sequence can be generated as (21)

$$p(u) = \exp\left(\frac{-\gamma}{\pi} \right) \quad (20)$$

where \( \gamma \) indicates random angle.

**RESULTS AND DISCUSSION**

The probability of PAPR of an OFDM above a threshold value \( \gamma \) is \( P_{l}[PAPR(x)] > \gamma = 1 - (1 - e^{-\gamma N})^{N} \). In this paper, the PAPR performance improvement using SLM is analyzed for various phase sequences for \( N=128 \) and 16 QAM. From figure 2, the PAPR reduction improvement of the Riemann sequence is superior over other phase sequences. If total subcarriers is \( N = 2^{n} \) and \( U \) is the candidate blocks (IFFTs), the number of multiplications and additions for \( U \) IFFTs in a standard SLM are \( \left(\frac{N}{2}\right)nU \) and \( NnU \), respectively and \( NU^2 \) additional multiplications to find peak power for \( U^2 \) alternative OFDM signal sequences are required. Hence, \( \left(\frac{N}{2}\right)nU + NU^2 \) multiplications and \( N(U^2 - U) \) additions are required in the low complexity SLM scheme. For \( N = 128 \), \( n = 7 \), and \( U = 8 \), the total multiplications and additions required in standard SLM are 7, 168 and 14, 336 respectively. Under the same conditions, in modified SLM, total multiplications and additions required are 3,584 and 11, 776 respectively. From the figure 2, superior PAPR reduction performance of Riemann sequence is observed over the other phase sequences.

Figure 3 depicts the improved PAPR performance of low complexity SLM over traditional SLM and also it is observed that almost same performance of low complexity SLM with less number of IFFTs (\( U=4 \)) in comparison with the standard SLM (\( U=8 \)) as shown in figure 4. Figure 5 and figure 6 illustrate the improved PAPR performance with increase in both \( U \) and order of Riemann sequence. Figure 7 depicts the PAPR performance comparison of various phase sequence in low complexity SLM and superior of Riemann sequences also observed. Table 1 illustrates the PAPR performance of various phase sequences in both traditional and low complexity SLM-OFDM system.
Fig. 2. PAPR performance of various phase sequences in SLM-OFDM

Fig. 3. PAPR performance comparison between conventional and modified SLM in OFDM

Fig. 4. PAPR performance comparison between traditional SLM with U=8 and modified SLM with U=4

Fig. 5. PAPR performance variation with order of Riemann sequence 16x16, 32x32 and 64x64 at U=8

Fig. 6. PAPR performance variation of Riemann sequence with U=4, U=8, and U=15 at order 16x16

Fig. 7. PAPR performance comparison of various phase sequences in modified SLM-OFDM
In this paper, the reasons for high PAPR in OFDM are discussed and different methods for PAPR reduction in OFDM are mentioned. Computational complexity in SLM is discussed and a low complexity SLM is considered. Various phase sequences for PAPR reduction are investigated and their PAPR performance is analyzed. The Riemann sequence reduces the PAPR more significantly over the other phase sequences. Hence, in this paper, Riemann sequence is considered in low complexity SLM to reduce PAPR in OFDM. Considered low complexity SLM significantly reduces the number of additions and multiplications over traditional SLM. PAPR reduction improves with number of candidate blocks (U) and order of Riemann sequence (RxR).

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AUTHORS PROFILE

SrinuPyla is working as Assistant Professor from 2006 in the department of Electronics and Communication Engineering in Gayatri Vidya Parishad College of Engineering (Autonomous) Visakhapatnam. He received B E and M.Tech from Andhra University and Kakatiya University in the year 2006 and 2008 respectively. He is presently pursuing PhD from Jawaharlal Nehru Technological University Kakinada, India. He has published 10 technical papers in National/International Conferences and journals. His fields of interest are Communications and Signal Processing.

K. Padma Raju. received B. Tech, M. Tech and Ph. D from Nagarjuna University, National Institute Technology, Warangal, and Andhra University, India respectively and Post Doctoral Fellowship at Hoseo University, South Korea. He worked as Digital Signal Processing Software Engineer in Signom Systems Pvt. Ltd., Hyderabad, India, before joining Jawaharlal Nehru Technological University Kakinada, India. He has 24 years of teaching experience and is Professor of Electronics and Communication Engineering, Jawaharlal Nehru Technological University Kakinada, India. Presently he is Director of Academics (DA), Jawaharlal Nehru Technological University Kakinada, India. He worked as Research Professor at Hoseo University, South Korea during 2006-2007. He has published more than 50 technical papers in National/International Journals/Conference proceedings and guiding 15 research students in the area of Antennas, EMI/EMC and Signal Processing. His fields of interest are Signal Processing, Microwave and Radar Communications and EMI/EMC.

Table 1. PAPR performance comparison for various phase sequences in SLM OFDM at cdf 10-2

<table>
<thead>
<tr>
<th>Sl. no</th>
<th>Phase sequence</th>
<th>PAPR with conv. SLM (in dB)</th>
<th>PAPR with modified SLM (in dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Riemann</td>
<td>5.1</td>
<td>5.1</td>
</tr>
<tr>
<td>2</td>
<td>Hadamard</td>
<td>7.8</td>
<td>5.6</td>
</tr>
<tr>
<td>3</td>
<td>Chaotic</td>
<td>8.9</td>
<td>9.0</td>
</tr>
<tr>
<td>4</td>
<td>Shapiro Rudin</td>
<td>8.0</td>
<td>6.5</td>
</tr>
<tr>
<td>5</td>
<td>Chu Square</td>
<td>7.5</td>
<td>5.6</td>
</tr>
<tr>
<td>6</td>
<td>Pseudorandom</td>
<td>7.8</td>
<td>6.2</td>
</tr>
</tbody>
</table>

5. CONCLUSION

V.
N. Balasubrahmanyam, received B.E and M. E from Birla Institute Technology Ranchi and Ph.D from Andhra University, India. He has 25 years of teaching experience and is presently Chairman of Board of Studies and Professor of Electronics and Communication Engineering, Gayatri Vidy Parishad College of Engineering (Autonomous) Visakhapatnam, India. He has published more than 20 technical papers in National/International Journals/Conference proceedings. His fields of interest are Antennas, Communications, Signal Processing, Microwave Communications and EMI/EMC.