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Optimization of Refrigeration Rate for a Thermoelectric Cooler in Restricted Space using Stochastic Algorithms

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Abstract*: *In the present study, a mathematical model of single stage thermoelectric cooler (TEC) is reported. This model is then employed to optimize the rate of refrigeration (ROR) which is one of the important performance measures of TEC. Two stochastic algorithms, namely, the genetic algorithm (GA) and simulated annealing (SA) are employed for optimizing the said performance of TEC for restricted space. The selected design variables are the geometric structural parameters of TEC elements and the input current. This study also includes the thermal resistance of hot side heat exchanger and electrical contact resistances into consideration. The results show that these design variables can be optimally set to maximize ROR within restricted space very significantly. The two algorithms for optimization attained almost the same values of design variables that lead to optimum ROR, though the GA could locate multi-modal optimum and hence can be used by the designer to choose among various options of design variables without compromising on the optimized value of ROR. .

Index Terms*: *Single-stage thermoelectric cooler, Rate of refrigeration, Genetic algorithm, Simulated annealing

I. INTRODUCTION

Thermoelectric technology has endeavoured to pave the way for green energy devices. A thermoelectric cooler (TEC) fundamentally works on the Peltier effect, a well-known principle discovered by Jean Peltier in 1834. Thermoelectric coolers are hassle-free solid state devices with no moving components and no refrigerants. These devices produce no harmful chlorofluorocarbons (CFCs) compared to traditional refrigeration or cooling systems. In thermoelectric coolers, electric current flows through p-and n-type semiconductor elements and a temperature gradient is established. The small size thermoelectric coolers have applications in fields such as optical, laser, radio-electronic devices. Modular design, high reliability, low maintenance, and noiseless operation provide benefice for new horizons of TEC applications [1], [2], [3], [4]. Eco-friendliness of thermoelectric cooling makes these devices suitable for the cooling method of the future. However, thermoelectric coolers are limited in applications due to their low rate of refrigeration (*ROR*) and low coefficient of performance (*COP*) compared to compressor-based systems. In the past, some researchers have worked to synthesize newer semiconductor materials with a high figure of merit. But comparable refrigeration rate and coefficient of performance are still not achieved. Hence, it is a challenge to achieve the highest possible *ROR* and *COP* with the available semiconductor materials. Further, to increase market penetration of thermoelectric coolers, the main challenge is to upgrade its *ROR* and *COP* within available space.

The conversion of electrical energy into a temperature gradient depends on the figure of merit (Z), which is managed by the properties of thermoelectric materials and determined by α^2/RK; α represents Seebeck coefficient, R represents electric resistance and K represents overall thermal conductance. Numerous research is carried out to maximize the figure of merit of the thermoelectric materials which provide optimum cooling effects [5], [6], [7], [8].

The performance of thermoelectric coolers with different arrangements has been reported by the researchers. Martnez et al. [9] used the thermoelectric system for temperature control without the use of any external electrical power source. The heat generated in the internal source was transformed into the electricity and supplied for cooling of the device. It was highlighted that thermal resistance between the source of heat and environment can be lowered by 25-30%. Chang et al. [10] reported the performance of thermoelectric air cooling module employed in the electronic devices. The results show that at a specific heat load, the module reaches the best cooling performance at an optimum input current. Commercially available solid-state thermoelectric devices may be used for their electrical power generation capabilities when coupled to a thermometric refrigerator or heat pump. The DC current provided to run a thermoelectric refrigerator can also be generated from solar cells. Many researchers have done experimental investigations of a TEC coupled with a solar cell. Dai et al. [11] investigated a thermoelectric refrigeration system driven by solar energy. The temperature in the refrigerator was successfully maintained between 5 to 10°C with a *COP* of 0.3.

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A solar driven thermoelectric cooling module with a waste heat refrigeration unit designed for green building application has been investigated by Cheng et al. [12]. It was found that the approach was able to produce a 16.2°C temperature difference between the ambient temperature and the air temperature in the model house. An analysis

of TEC performance has been conducted for high power electronic packages such as processors by Zhang et al. [13]. The cooling configurations and optimum current resulted in substantial thermal enhancements.

Huang et al. [14] used the conjugate-gradient method for TEC geometry optimization at a fixed current and fixed temperature difference to achieve optimized cooling rate. The effects of applied current and temperature difference on the optimal geometry were discussed. A review of cooling parameters and related performance along with possible approaches to improve *COP* of TEC has been presented by Enescu and Virjoghe [15]. Lin and Yu [16] indicated that trapezoid-type two-stage Peltier couples can reduce thermal resistance followed by improved cooling capacity and *COP*. Nain et al. [17] optimize structural parameters to improve *ROR* and *COP* of a single-stage TEC.

The importance of the development of high performance and cost controlled cooling systems for small enclosures is significant. In this study, a mathematical model of single stage thermoelectric cooler is used for establishing the rate of refrigeration in terms of design variables and material properties. Then this *ROR* is optimized by two stochastic optimization techniques, namely, genetic algorithm (GA) and simulated annealing (SA) independently [18], [19], [20], [21] [22]. Genetic algorithm and simulated annealing are the stochastic algorithms that are able to search large regions of the solution's space without being trapped in local optima. The design variables that maximize *ROR* within considered available space are reported.

II. MATHEMATICAL MODELLING

The physical model of a single stage thermoelectric cooler is shown in Fig. 1. It includes one thermoelectric module, corresponding cold, and hot side material, and external hot side heat exchanger.

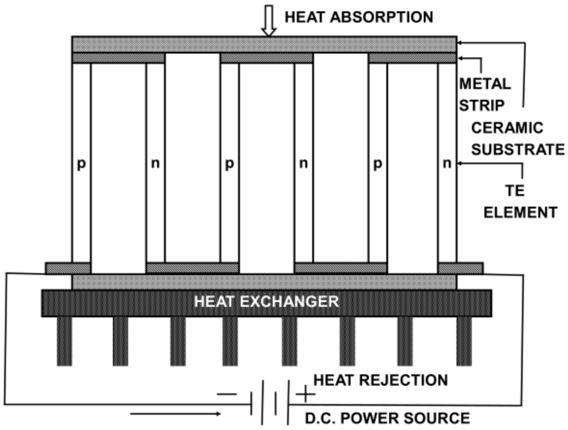

Fig. 1: Schematic Diagram Of A Single Stage

Thermoelectric Cooler

The present analysis starts with the phenomenological relations described in the literature [23], [24]. The Peltier effect is the basis for thermoelectric cooling. As a result of DC current flow through a number of pairs of n- and p-type thermoelectric elements linked together electrically in series and thermally in parallel, a temperature difference is created. The thermal conduction from the hot side of the module to the cold side. The Joule heat resulting from the current flow in the thermoelectric materials will also be generated. Half the Joule heat thus generated will flow on each side of the module. An electrically insulated and thermally conductive material is in thermal contact with a heat source on the cold side of the junctions and with a heat sink on the hot side of the junctions. The electrically conductive material is alternately attached to the cold and hot side to form an electrical circuit inside the thermoelectric module as shown in Fig.1. The external hot side heat exchanger dissipates heat to the ambient environment.

The equations for heat balance at the two junctions of the module are:-

$$\dot{Q}_c = 2N\left[I\alpha T_c - \frac{kA(T_h - T_c)}{L} - \frac{1}{2}I^2\left(\frac{\rho L}{A} + 2\frac{r_c}{A}\right)\right] \quad (1)$$

$$Q_h = 2N\left[I\alpha T_h - \frac{kA(T_h - T_c)}{L} + \frac{1}{2}I^2\left(\frac{\rho L}{A} + 2\frac{r_c}{A}\right)\right] \quad (2)$$

Where, Q_h is the rate of heat rejection from the hot junction and Q_c is the rate of heat absorption at the cold junction which is referred as the rate of refrigeration (*ROR*) in common usage. Thermoelectric material properties α, ρ, k and r_c are the Seebeck coefficient, electrical resistivity, thermal conductivity, and electrical contact resistance, respectively. T_h and T_c are the hot and cold junction temperatures and L and A are the length and cross-sectional area of thermoelectric elements, respectively. I is the input electric current and N is the number of thermoelectric couples in the module. Heat flows $kA\,(T_h - T_c)/L$ and ½ I^2 $(\rho L/A + 2r_c/A)$ refers to thermal conduction and Joule heat, respectively. $I\alpha T_h$ is the Peltier heat at the hot junction and $I\alpha T_c$ is the Peltier heat at the cold junction.

The material of the thermoelectric elements used in this study is bismuth telluride. The thermoelectric material properties are based on the average temperature (T_{ave}) of the cold and hot side temperatures. The equations for this temperature range are provided by Melcor [25] and described as follows:

$$\alpha = (22224 + 930.6\,T_{ave} - 0.9905\,T_{ave}^2) \times 10^{-9} \quad (3)$$

$$\rho = (5112 + 163.4\,T_{ave} + 0.6279\,T_{ave}^2) \times 10^{-10} \quad (4)$$

$$k = (62605 - 277.7\,T_{ave} + 0.4131\,T_{ave}^2) \times 10^{-4} \quad (5)$$

In this work, the thermal resistance of hot side heat exchanger is taken into account in order to discuss its effect on the hot junction temperature prediction and performance of the system in such a way that following equation is satisfied.

$$T_h = Q_h \times R_{th} + T_a \qquad (6)$$

Where R_{th} is the thermal resistance of hot side heat exchanger and T_a is the ambient temperature.

III. NUMERICAL OPTIMIZATION OF SINGLE STAGE TEC

In order to analyze the performance of a single stage TEC, the rate of refrigeration of the system is examined. The thermoelectric cooler has to be designed as compact as possible because the space in electronic equipment is limited. In this study, the cross-sectional area of the thermoelectric cooler (*S*) is fixed as 25 mm^2. Further, there is also a restriction on the maximum length of elements which makes the total space restricted. The optimization process starts by selecting the length of elements (*L*), cross-sectional area of elements (*A*), a number of thermoelectric couples (*N*) in TEC and input current (*I*) to be the design variables. The objective is now to find the values of *L*, *A*, *N* and *I* for achieving the best performance, i.e., max *ROR* using equation (1) for the thermoelectric cooling system in restricted space.

The optimization of the rate of refrigeration of thermoelectric cooler is stated as the following single objective constrained optimization problem

$$\left.\begin{array}{lc} \text{Maximize} & ROR \\ \text{subject to} & \\ & I_{min} \le I \le I_{max} \\ & L_{min} \le L \le L_{max} \\ & A_{min} \le A \le A_{max} \\ & N_{min} \le N \le N_{max} \end{array}\right\} \qquad (7)$$

The thermal resistance of hot side heat exchanger is fixed as 0.1°CW^{-1}. An increase in thermal resistance of hot side heat exchanger decreases the rate of refrigeration and increases the hot-side temperature. The other fixed design parameters are the electrical contact resistance as 1x10^{-8} Ω-m^2, cold junction temperature as 293.15 K and ambient temperature as 298.15 K. The packaging density is taken as 80% considering practical manufacturing limitation. The four design variables are allowed to vary within certain ranges as per real-world applications. The length and cross-sectional area of elements are in the range of 0.5-1.5 mm and 0.25-1.0 mm^2, respectively. The available cross-sectional area of TEC and the maximum length of elements makes the total space restricted as 25 mm^2 x 1.5 mm = 37.5 mm^3.

The number of thermocouples (*N*) is a dependent variable, as its value depends on the available cross-sectional area of thermoelectric cooler and the cross-sectional area of TE elements. The number of thermocouples is rounded down to the nearest integer which is obtained using equation (8).

$$N = \frac{S \times packaging\ density}{2 \times A} \qquad (8)$$

The available TEC cross-sectional area of 25 mm^2 with 0.8 packaging density allows the number of thermocouples to vary from 10 to 40 in this work. The input current is allowed to vary in the range of 0.1-3.0 ampere. The hot side temperature (T_h) is initially fixed to some guessed value so that the material properties can be obtained using equations (3), (4) and (5). Employing equations (1), (2) and (6), the new value of T_h is calculated. This difference of guessed and the new value of T_h is used to modify the guess value of T_h iteratively, till the difference becomes negligible. In this work, we have separately applied genetic algorithm & simulated annealing as optimization tools for the same optimization problem.

IV. OPTIMIZATION RESULTS

(A) Optimization results using GA

The real-variable genetic algorithm employing SBX operator developed by Deb and Agarwal is used for single objective optimization [20]. The algorithm is coded in C language. GA requires a population of data points to be evaluated over multiple generations to reach an optimal solution. Table I shows GA parameters, applied in the single objective optimization of *ROR*. Since the genetic algorithm is a stochastic algorithm, therefore, the simulation was run five times and the best one is reported. The parameters values which are used in optimization by GA are reported in Table I.

Table I. Parameters Settings for GA

Population size	20
Crossover probability	0.80
Mutation probability	0.25
Termination Criteria	1000 generations
Number of runs	5

The search process is completed very quickly with GA. The optimization history for best run in terms of the optimized average *ROR* for the entire population with generation number is shown in Fig. 2. Under the set conditions GA has achieved the optimum *ROR* as 1.144869 W at a current (*I*) of 2.360 A, TE element length (*L*) of 0.542 mm, cross-sectional area of TE element (*A*) of 1.0 mm^2. The optimal value of the length of TE elements is 0.542 mm and it demonstrates that it is unique while maximizing *ROR* and does not hit any variable bound of the permitted range of 0.5-1.5 mm. The optimal value of the cross-sectional area of TE elements hit the upper variable bound of 1.0 mm^2 and hence, it signifies the importance of exploring the upper limit of the permitted range of cross-sectional area of TE element. The optimal number of thermoelectric couples (*N*) is 10.

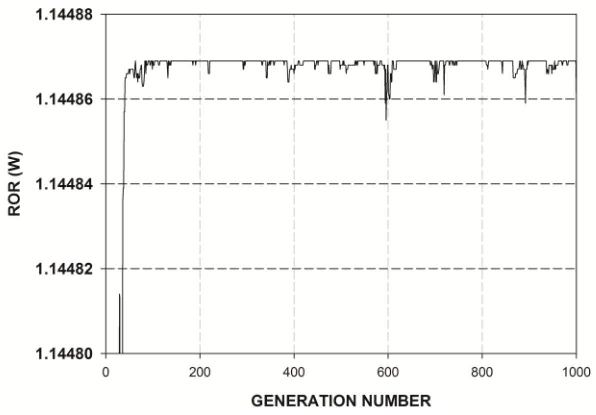

Fig. 2: GA Convergence Curve Of Optimized ROR

Additionally, in order to examine the possibility of multiple optima in the search space, this simulation was repeated many times with different settings. It was found that the present optimization problem is having three distinct optima with same values of *ROR*. Hence, it is a case of multimodal function optimization. The optimal parameters for these captured peaks are reported in Table II. These three captured peaks of the objective function are labeled as Optima-1, Optima-2, and Optima-3. If we observe all three optimum solution vectors reported in Table II, it is found that the optimal value of the length of TE elements '*L*' is 0.542 mm in all three cases.

Table II. GA optimization results

S. No.	I (A)	L (mm)	A (mm^2)	N	Optimized ROR (Watts)	Label
1	2.360	0.542	1.0	10	1.144869	Optima -1
2	1.180	0.542	0.5	20	1.144869	Optima -2
3	0.944	0.542	0.4	25	1.144869	Optima -3

(B) Optimization results using SA

These optimization results and searchability of GA is checked by applying another robust stochastic algorithm which is simulated annealing. Simulated annealing is a random search procedure for global optimization problems, and it resembles the annealing process to solve an optimization problem. The process uses the careful control of temperature and cooling rate that controls the search. The temperature parameter is high at the start and a number of iterations are executed before lowering the temperature at each instant till the convergence criteria is satisfied. The SA source code that works robustly for MATLAB® and able to handle non-linear constraints is developed by Xin-She Yang and used in this work [26]. The simulation was run with different settings of SA. Table III shows SA parameters with best-obtained performance, applied in optimizing *ROR*.

Table III. Parameters Settings for SA

Initial Temperature	1.0
Final Temperature	$1e^{-10}$
Cooling Factor	0.8
Boltzmann Constant	1
Number of iterations per temperature level	500

The Optimum *ROR* captured by SA is 1.144869 watts. The solution vector is identical to one obtained from GA and labeled as Optima-1 in Table II. This simulation was also carried out several times with different settings but SA is able to capture only single best peak as reported in Table IV. These identical results obtained from GA and SA doubly ensure the optimized value of ROR and corresponding optimal design parameters.

Table IV. SA optimization result

S. No.	I (A)	L (mm)	A (mm^2)	N	Optimized ROR (Watts)	Label
1	2.360	0.542	1.0	10	1.144869	Optima -1

The mathematical model of the present work has defined *ROR* as a function of *I*, *L*, *A*, and *N*. The variables *I*, *L* and *A* are the independent design variables whereas *N* is a dependent design variable because it is calculated by using the value of *A* and employing equation (8). We observe from Table II and Table IV that the optimal value of *L* is 0.542 mm in all the results discovered by GA and SA. Though *ROR* is a function of three independent variables namely *I*, *L* and *A*. If we fix *L* at 0.542 mm as found in all solutions of this optimization problem, it is possible to represent *ROR* with two remaining independent variables *I* and *A* in a 3D plot by using equation (1). Hence a 3D plot is drawn employing equation (1) with *L* fixed at 0.542 mm and shown in Fig. 3. The parameters *I*, *A* and *ROR* are shown on the X-axis, Y-axis, and Z-axis, respectively. Then, we have superimposed the three optima which were discovered by GA and reported in Table II. These are labeled as Optima-1, Optima-2, and Optima-3 in the same order as reported in Table II. If we refer to Table II and Table IV, we find that the Optima-1 is discovered by both, GA and SA while the Optima-2 and Optima-3 are discovered by GA only. Though the optimum *ROR* is 1.144869 W in all the solutions vectors, GA has discovered all three possible optima and hence, it is able to handle this multimodal optimization problem. SA was unable to find multiple optima though the simulation was run at repeated times with different settings. Hence, in the present multimodal problem, GA has partially outperformed SA.

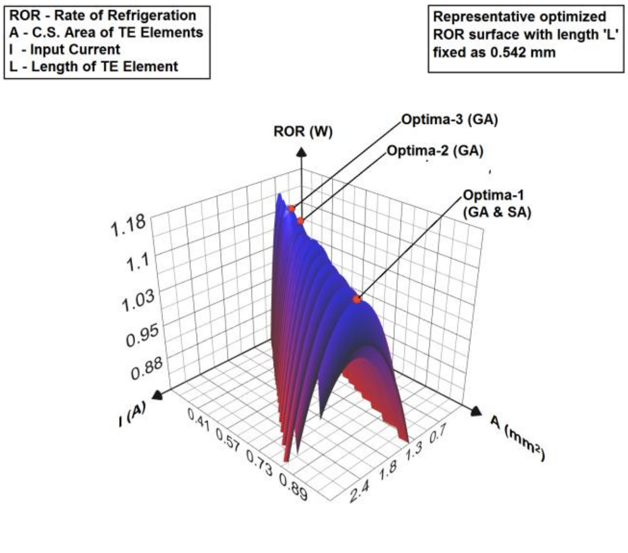

Fig. 3 Multiple optima labeled at the *ROR* surface with thermoelectric element length *L* = 0.542 mm

V. CONCLUSION

The importance of TECs is significant in the present scenario when the world is facing the challenge to provide devices which are ecologically green and the cost-effective. Thus, it is important to study these devices to derive their maximum performance within the technological limitation of manufacturing. This work attempted to find out the optimum design parameters to improve an important performance measure (*ROR*) of TEC using GA and SA. This work has demonstrated that the geometric structural parameters of the thermoelectric elements and the input current have an influence on the rate of refrigeration of TEC and can be suitably varied to enhance the performance of TEC. The *ROR* of thermoelectric cooler within the total restricted space of 37.5 mm^3 of TEC was successfully optimized in this work. The conspicuous observation of GA and SA results in present work is that GA search outperformed SA and located multiple optima. The combined selection of input current and the geometric structural parameters used as design variables in this work provides a guide to future research. The current optimization problem can also be attempted by using different optimization methods as Huang et al. have employed in their work [14]. Designers can handle space restriction for the case at hand and improve *ROR*.

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