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Topological Properties of Parallel Series Language

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Abstract*: *Parallelism is a process by which a sequential string is broken down into a number of alphabets and used to speed up the acceptance of a string. To identify the parallelisable string, we have used parallel operator || and defined the language as parallel series languages. Algebraic and recognition properties of series parallel posets have been studied by Lodaya in [8]. In this paper, we have introduced finite and infinite parallel series language (parallel strings are connected sequentially). We have considered the set of all parallel series language as topological space and prefix order relation (poset relation) has been used to relate two parallel series strings. Topological concepts like limit, sequence, open set, closed set and basis for parallel series languages and their properties have been derived.

Index Terms*: *Paralle Series language, Topological properties, Open and Closed languages.

I. INTRODUCTION

A deep analysis shows that the notion of limit generated by the natural topology is different from that used in the theory of ω-automata. Concepts of limit associated with a topology have been studied in [1, 2, 3, 5, 6] limit point is the most important concept.

Kuroda [7] has shown that it is possible to handle problems regarding syntactic analysis and translation using topological methods and concepts. General languages, continuous mappings, connectedness and compactness have been extended in [10] by an extension of Kuroda's concept. Also, the topological equivalence between grammars and languages have been derived.

Topology on subwords has been defined and the properties have studied using topological concepts, namely, compactness, closed sets, open sets and closure of a languages [5]. Topological characterization of the set of random sequences has been derived in [3] as a byproduct. Finite words and infinite words are very familiar in formal languages. Mathematically, topological concepts are needed to arrive at the transition between finite words and infinite words.

Topologies arise from comparison of words, languages and the multiplicative structure. Partial order is considered from the topological generation of prefix relation. Several partial orders on the set of finite words to infinite words have been considered in [4] and similar properties have been identified.

Infinite strings (words) can be considered [9] as limits of finite words and topology as a theoretical study of limits. Languages with infinite strings have been studied as subsets of a topological space.

In this paper, Section 2 gives the preliminary definitions and notations. Section 3 comprises the definition of limit of a set and its properties. Also, open sets and closed sets have been defined for parallel series infinite language in terms of limit of a set and derived their properties. Basis of parallel series infinite language has been derived and discussed about continuous mapping, sequences and convergence.

II. PRELIMINARIES

Let Σ_p^* be a collection of all parallel series finite strings over an alphabet Σ and Σ_p^ω the collection of all parallel series ω -strings or infinite strings over Σ. Consider $\Sigma_p^\infty = \Sigma_p^* \cup \Sigma_p^\omega$ the collection of all both finite and infinite parallel series strings that can be considered as a topological space over Σ.

A set $L \subseteq \Sigma^\infty$ is called a parallel series language. A relation $\leq$ on a language L is called partial order if $\leq$ is transitive, reflexive and anti-symmetric. The pair $(L, \leq)$ is called partially ordered set (poset). If $(L, \leq)$ is a poset or chain and $M \subseteq L$ then $(M, \leq)$ is also a chain. A relation $\leq$ on a language L is said to be a linear order if $\leq$ is a partial order and any two strings are comparable. That is, for all $v, w \in L$, either $v \leq w$ or $w \leq v$. In this case, the pair $(L, \leq)$ is called a linearly ordered set. For $v, w \in \Sigma_p^\infty$, if $v \leq w$ we say that v is a prefix of w. The collection of all prefixes of w is denoted by P(w). Consider $\Sigma = \{a, b, c\}$.

For example, if $w = (b||c)(a||b||c)(a||c)$ then

$P(u) = \{(b||c), (b||c)(a||b||c), (b||c)(a||b||c)(a||c)\}$ where

$P_1(u) = (b||c)$, $P_2(u) = (b||c)(a||b||c)$ and

$P_3(u) = (b||c)(a||b||c)(a||c)$.

If $u = (b||c)(a||b||c)(a||c)^\omega$ then

$P(u) = \{(b||c), (b||c)(a||b||c), (b||c)(a||b||c)(a||c), \ldots\}$

If $u = x_1 x_2 \ldots x_n$ where $x_k \in \Sigma^\oplus$ for $1 \leq k \leq n$ then the kth prefix of u is $P_k(u) = u_1 u_2 \ldots u_k$.

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Example 2.1. Let $\leq$ be a relation and L = {$(b||c)^k$: k > 0}.

(L, $\leq$) is an infinite chain. L = {(b||c), (b||c)(b||c),...}

If u = (b||c)(b||c)(b||c) and v = (b||c) then v $\leq$ u.

Example 2.2. Consider L = {(b||c)(a||b||c)(a||b), (b||c||a)}

and $\leq$ is a prefix relation. Then (L, $\leq$) is not a chain. If u = (b||c) and v = (b||c||a) then u $\nless$ v or v $\nless$ u.

Suppose $L \subseteq \Sigma_p^*$ we say that $u \epsilon \Sigma_p^*$ is an upper bound for L if $v \leq u$ for all $v \epsilon L$.

Example 2.3. Let L = {$(b||c||a)^k$: k > 0}. Then $(b||c||a)^\omega$ is an upper bound of the language L. We note that $(b||c||a)^\omega$ is the least upper bound of L, also known as Supremum of L denoted by Sup(L).

The complement of a language L is denoted as CL.

For example, if L={$(b||c||a)^k$: k>0} then CL=Σ_p^∞-L.

Fin(L) denotes the collection of finite subsets of a language L and Omega(L) the collection of all subsets which consist of infinite strings.

$$\text{Fin}(L)=L\cap\Sigma_p^*$$
$$\text{Omega}(L)= L\cap\Sigma_p^\omega$$

Example 2.4 If L={$(b||c||a)^k$, $(c||c)^k (b||c)^\omega$: k >0} then

Fin(L)={$(b||c||a)^k$: k>0}

Omega(L)={$(c||c)^k(b||c)^\omega$: k>0}

Let L$\subseteq\Sigma_p^\infty$ we define limit of L as

Lim(L)={Sup(M) : M$\subseteq$L is an infinite chain}

and colimit of the language L is defined as,

Clim(L)= Σ_p^ω-Lim(CL)

Example 2.5 Let L={$(b||c||a)^k$, $(c||c)^k (b||c)^\omega$: k >0} and M={$(b||c||a)^k$: k >0} is an infinite chain. Then

Sup(L)= $(b||c||a)^\omega$

Lim(L)= $(b||c||a)^\omega$

Property 2.6 For all L, M $\subseteq \Sigma_p^\infty$

(1) Lim(L) = {u $\in\Sigma_p^\omega$: P (u) $\cap$ L is infinite}

(2) Clim(L) = {u $\in \Sigma_p^\omega$: P (u)$-$L is finite}

(3) Lim(L) = Lim(Fin(L))

(4) Clim(L)= Clim(Fin(L))

(5) Lim(φ)=Clim(φ)= φ

(6) Lim(Σ_p^*)=Clim(Σ_p^*)=Σ_p^ω

(7) Clim(L) $\subseteq$ Lim(L) $\subseteq\Sigma_p^\omega$

(8) Lim(M$\cup$L)=Lim(M) $\cup$Lim(L)

(9) Clim(M$\cap$ L)=Clim(M) $\cap$ Clim(L)

(10) Lim(Lim(L)) $\subseteq$ L$\cup$Lim(L)

Proof: Properties are proved by giving following examples. Let L={$(b||c)^k$, $(c||a)^k (c||c)^\omega$: k>0} and M={$(b||c)^k$: k >0}

(1)Lim(L)=$(b||c)^\omega$ if u=$(b||c)^\omega$ and P (u) = {(b||c), (b||c)(b||c), (b||c)(b||c)(b||c), …} then P (u) $\cap$ L=$(b||c)^\omega$ is infinite.

(2) Lim(L)=$(b||c)^\omega$ and Clim(L)=$(b||c)^\omega$ if u=$(b||c)^\omega$ and P(u) = {(b||c), (b||c)(b||c), (b||c)(b||c)(b||c), …} then P(u)$-$L=$(b||c)^n$ is finite.

(3) Lim(L)=$(b||c)^\omega$ F in(L) = {$(b||c)^n$: n $\in$ N} and Lim(F in(L)) = $(b||c)^\omega$

(4) Clim(L) = {$(b||c)^k$: k $\in$ N}, Fin(L) = {$(b||c)^k$: k $\in$ N} and Lim(Fin(L)) = $(b||c)^\omega$.

(5) Lim(ϕ) = ϕ.

(6) Lim(Σ_p^*) = Σ_p^ω and Lim(L)=Σ_p^ω where L $\subseteq \Sigma_p^*$. Similarly, Clim(Σ_p^*)=Σ_p^ω.

(7) For a given L, Lim(L)=$(b||c)^\omega$ and Clim(L)=$(b||c)^\omega$ then Clim(L) $\subseteq$ Lim(L) $\subseteq \Sigma_p^\omega$.

(8) Lim(L)=$(b||c)^\omega$, Lim(M)=$(b||c)^\omega$ and Lim(L) $\cup$ Lim(M)=$(b||c)^\omega$. Also, if L $\cup$ M = {$(b||c)^k$, $(c||a)^k(c||c)^\omega$: k>0} then Lim(L$\cup$M) = $(b||c)^\omega$

(9) Clim(L)=$(b||c)^\omega$, Clim(M)=$(b||c)^\omega$ and Clim(L) $\cap$ Clim(M)=$(b||c)^\omega$. Also, Clim(L$\cap$M) = $(b||c)^\omega$

(10) Lim(L)=$(b||c)^\omega$ and Lim(Lim(L)) = $(b||c)^\omega$

Now, L $\cup$ Lim(L) = {$(b||c)^k$,$(b||c)^\omega$, $(c||a)^k(c||c)^\omega$: k > 0}

Definition 2.7. Let L $\subseteq\Sigma_p^\infty$. The derived set of L is the collection of limit points of L.

Example 2.8. Consider L = {$(b||c)^k$, $(c||a)^k(c||c)^\omega$: k > 0} then Lim(L) = $(b||c)^\omega$. Since, the derived set of L is the collection of limit points of L.

Definition 2.9. Let L $\subseteq\Sigma_p^\infty$. Closure of a language L is denoted by Cl(L) which is the union of L and all its derived set.

Example 2.10. Consider L = {$(b||c)^k$, $(c||a)^k(c||c)^\omega$: k > 0} then Lim(L) = $(b||c)^\omega$. Since, Cl(L) is the union of L and its derived set. Therefore, Cl(L) = $(b||c)^\omega$.

Definition 2.11. The interior of L $\subseteq\Sigma_p^\infty$ is the set In(L) = CC l(CL).

Example 2.12. Consider L = {$(b||c)^k$, $(c||a)^k(c||c)^\omega$: k > 0} then CL = Example 2.12. Consider L = {$(b||c)^k$, $(c||a)^k(c||c)^\omega$: k > 0} then CL =Σ_p^∞- L. Now, Cl(CL) = $(b||a)^\omega$ then CC l(CL) = {$(b||c)^k$: k > 0}. Therefore, In(L) = {$(b||c)^k$: k > 0}.

Definition 2.13. The boundary (frontier) of L $\subseteq \Sigma_p^\infty$ is the set F r(L) = Cl(L) $-$ In(L).

Example 2.14. Consider L = {$(b||c)^k$, $(c||a)^k(c||c)^\omega$: k > 0} then Cl(L) = $(b||c)^\omega$ and In(L) = {$(b||c)^k$: k > 0}. Therefore, F r(L) = $(b||c)^\omega$

III. TOPOLOGICAL PROPERTIES

Properties given in Section 2, provide an idea that there is a unique topology, where the collection of limit points of L is Lim(L) and topology T called as Lim-topology of the set Σ_p^∞ and consider T is the family (collection) of open sets.

Property 3.1. For each language L$\subseteq\Sigma_p^\infty$,the conditions given below are equivalent:

(1) L is a closed language

(2) Lim(L) $\subseteq$ Omega(L)

(3) For every string v $\in$ Omega(CL), there exists

some $k \in N$ such that $P_k(v) \subseteq CL$

Proof. A language contains its derived set if and only if it is closed.

Because $Lim(L) \subseteq \Sigma_p^\omega$ for every L. By definition, $Omega(L) = L \cap \Sigma_p^\omega$ we say that $Lim(L) \subseteq Omega(L)$. By property 2.6 (1), (2) is equivalent to every infinite string outer L has finite and many of prefixes in L, which can be derived by condition(3).

Property 3.2. For each language $L \subseteq \Sigma_p^\infty$, the conditions given below are equivalent:

(1) L is a open language

(2) $Omega(L) \subseteq Clim(L)$

(3) For every string $v \in Omega(L)$, there exists some $k \in N$ such that $P_k(v) \subseteq L$

Proof. A collection of parallel series strings is open if and only if its complement is closed. By the definition of C lim, we establish that condition (2) is equivalent to $Lim(CL) \subseteq Omega(CL)$. From property 3.1 (2), this leads to CL being closed. Then, by property 3.1 (3), we prove that there exists some $k \in N$ such that $P_k(v) \subseteq L$ for each $v \in Omega(L)$.

Property 3.3. A language is called Open-and-Closed iff $Omega(L) = Lim(L) = Clim(L)$.

Proof. By property 3.1 (2) and property 3.2 (2), if V is closed then $Lim((L) \subseteq Omega(L)$ and if L is open then $Omega(L) \subseteq Clim(L)$. This implies $Lim(L) \subseteq Omega(L) \subseteq Clim(L)$. Also by property 2.6 (7), we have $Clim(L) \subseteq Lim(L)$ and $Omega(L) \subseteq Lim(L)$. This implies $Omega(L) = Lim(L) = Clim(L)$.

Property 3.4. Each $u \in P_k(u)$, $u \in \Sigma_p^\infty$ and each language of the form $L\Sigma_p^\infty$ with $L \subseteq \Sigma_p^*$

Each language of the form $M \cup L\Sigma_p^\infty$ with $L, M \subseteq \Sigma_p^*$ is open.

Property 3.5. For each $L \subseteq \Sigma_p^\infty$,

(1)$Cl(L)=Fin(L) \cup (Omega(L) \cup Lim(L))$

(2) $In(L)= Fin(L) \cup (Omega(L) \cup Clim(L))$

(3) $Fr(L)=(Omega(L)-Lim(L)) \cup (Lim(L)-Clim(L)) \cup (Clim(L)-Omega(L))$

(4) $Cl(In(L))=Fin(L) \cup Lim(L)$

(5) $In(Cl(L))=Fin(L) \cup Clim(L)$

Definition 3.6. A language $L \subseteq \Sigma_p^\infty$ is a closed language domain if $L = Cl(In(L))$ and an open language domain if $L = In(Cl(L))$.

Property 3.7. For each $L \subseteq \Sigma_p^\infty$,

(1) L is a closed language domain iff $Omega(L) = Lim(L)$ (2) L is an open language domain iff $Omega(L) = Clim(L)$

Definition 3.8. For a string $v \in \Sigma_p^\infty$, a basis at the string v is a family of open languages consisting v, such that each open language $N \supseteq v$ consists a member of the family.

Property 3.9. For each $v \in \Sigma_p^\infty$, the family $B_v = \{P_k(v) : k > 0\}$ is a basis at v.

Proof. Each set $P_k(v)$ is open. If $v \in \Sigma_p^*$ the set of all $P_k(v)$ is in B_v is contained in each $O \supseteq v$. If $v \in \Sigma_p^\omega$, then B_v has the property that for every string $v \in Omega(B_v)$, there exists $k \in N$ such that $P_k(v) \subseteq B_v$.

Property 3.10. The family $B = \cup\{B_v : v \in \Sigma_p^\infty\}$ is a basis for the topology T.

Proof. We know that basis for topology T is a collection of open languages such that every language $O \in T$ is a union of members of basis B that can be derived as the union of bases with respect to all strings.

Definition 3.11. A subbasis for the topology T is a family of open languages such that every language $N \in T$ that can be written as finite intersections and an arbitrary unions of members of the family.

Let S be a topological space. A function $g : \Sigma_p^\infty \to S$ is continuous function if $g^{-1}(L)$ is open for each open language $L \subseteq S$. The definition of continuity as follows:

Property 3.12. If $g : \Sigma_p^\infty \to S$ is a continuous function then the conditions given below are equivalent.

(1) $g^{-1}(L)$ is open for every member L of a subbasis for S

(2) $g^{-1}(L)$ is closed language for every closed language $L \subseteq S$,

(3) g is a continuous function at each point $u \in \Sigma_p^\infty$, that is, for each member L of the basis at g(u), there is a member M of the basis at u such that $g(M) \subseteq L$.

Property 3.13. A function $g : \Sigma_p^\infty \to \Sigma_p^\infty$ is a continuous function iff for each $u\Sigma_p^\omega$ and natural number k, there exists an $l>0$ such that $g(v) \in P_k(g(u))$ wherever $v \in P_l(u)$.

Definition. 3.14. A function $g : \Sigma_p^\infty \to \Sigma_p^\infty$ is monotone if for every $u,v \in \Sigma_p^\infty$, $u \le v$ implies $g(u) \le g(v)$.

Property 3.15. A monotone function $g : \Sigma_p^\infty \to \Sigma_p^\infty$ then the following constraints are equivalent:

(1) g is a continuous function

(2) $g^{-1}(u)$ is closed for each $u \in \Sigma_p^*$

(3) $Sup(g(L)) = g(Sup(L))$ for each infinite chain L

Proof. (1)⇒(2): It derives from the property $g^{-1}(L)$ is closed for each closed language $L \subseteq S$.

(2)⇒(3): Consider $L \subseteq \Sigma_p^\infty$ is an infinite chain. g(L) is also a chain when g is monotone. Also, g(Sup(L)) is its upper bound. This gives $Sup(g(L)) \le g(Sup(L))$. If Sup(g(L)) is infinite then the unique possibility is $Sup(g(L)) = g(Sup(L))$.

If Sup(g(L)) is finite, g(L) must be a finite set. If the condition (2) holds, $g^{-1}(g(L)) \supseteq L$ is closed. Since finite union of closed languages is closed and it should contain Sup(L). Therefore, $g(Sup(L)) \in g(L)$ gives $g(Sup(L)) \le Sup(g(L))$.

(3)⇒(1): Suppose g is discontinuous. By the property, (A function $g : \Sigma_p^\infty \to \Sigma_p^\infty$ is continuous iff for every $u\Sigma_p^\omega$ and natural number k, there exists an $l>0$ such that $g(v) \in P_k(g(u))$ wherever $v \in P_l(u)$), there exists $u\Sigma_p^\omega$ such that $g(v) \notin P_k(g(u))$ for infinitely many strings $v \leq u$. All these strings v clearly form an infinite chain L with $Sup(L) = u$.

By definition, $g(L)$ is disjoint with $P_k(g(u))$, and thus should be a finite language set. This implies $Sup(g(L)) \in g(L)$. Since $g(Sup(L)) = g(u) \in P_k(g(u))$. Therefore $Sup(g(L)) = g(Sup(L))$.

Definition 3.16. Let S be any topological space. A mapping $\phi : N \to S$ is called a sequence of elements of S.

Definition 3.17. The sequence ϕ converges to $u \in S$, can be written as $\phi \to u$, if, for every open set $O \supseteq u$, there exists an $n \in N$ such that $\phi(i) \in O$ for all $i \geq n$. The point u is called the limit of ϕ.

Property 3.18. Every ascending sequence $\phi: N \to \Sigma_p^\infty$ converges to $Sup(\phi(N))$.

Property 3.19. A language $L \subseteq \Sigma_p^\infty$ contains the limit of each convergent sequence $\phi : N \to L$ if and only if L is closed.

Property 3.20. Let S be a topological space. A function $g : \Sigma_p^\infty \to S$ is continuous at $u \in \Sigma_p^\infty$ if and only if for each sequence ϕ of strings, $\phi \to u$ implies $g \circ \phi \to g(u)$.

Property 3.21. A function $g : \Sigma_p^\infty \to S$ is continuous at $u \in \Sigma_p^\omega$ if and only if for an increasing sequence ϕ of all the prefixes of u, $g \circ \phi$ converges to $g(u)$.

Proof. The necessary condition is true by property 3.18 and 3.20.

To prove the sufficient condition, consider for any open set O, $g(u) \in O \subseteq \Sigma_p^\infty$ If $g \circ \phi$ converges to $f(u)$, there exists $n \in N$ such that $g \circ \phi(i) \in O$ for all $i \geq n$.

It means that $g(P_n(u)) \subseteq O$ and g is continuous at u by property 3.12 (3).

IV. CONCLUSION

We have defined the definition of limit of a parallel series language and studied their properties. Also, open set(language) and closed set(language) have been defined for parallel series infinite language using limit and derived their properties. Basis of parallel series infinite language has been derived and discussed about continuous mapping, sequences and convergence. We target to explore more interesting properties namely, compactness, connectedness, disconnectedness, separation and so on.

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